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Building a New Reverse Mortgage Model for Elderly People with Low Price House

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Abstract

To increase the amount of monthly payments for the elderly people who have low price housing in reverse mortgage program, the Korean government decided to subsidize the insurance premium corresponding to the amount of 2.25% of housing value. As a result, the borrowers who have low price housing could receive about 9%~17% increased monthly payments since April 2016. However, the current method of just subsidizing the insurance premium to increase the amount of monthly payments has the problem of long term sustainability because the government's financial situation would become worse as the number of subscribers increases. To increase the amount of monthly payments for elderly people who have low price housing innovatively without causing any financial burden for the government, it is necessary to consider a totally different type of reverse mortgage model. Under the current Korean reverse mortgage program, the government guarantees that the borrower will not owe more than the value of housing at the time it is sold due to non-recourse limit. On the contrary, if the value of housing is larger than the outstanding loan balance at the time the loan is terminated, the heirs can receive leftover on the sale. So, as long as we calculate the amount of monthly payments under the structure of the current actuarial model, the possible maximum amount of monthly payments cannot be increased innovatively although there is some government's subsidy to the program. Therefore, we suggest a totally different type of reverse mortgage program. In this new model, we allow the guarantor to receive or share the residual housing equity on the sale if the home value is larger than the mortgage balance at the time the loan is terminated. As a result, the borrowers can get considerably increased amount of monthly payments because the guarantor can take on more losses in the program under similar burden of risk. So, if the elderly people want to receive more money to supplement their living costs, they can choose our new model although there is no chance that they can receive leftover on sale or have to share it with guarantor if the home value is larger than the mortgage balance at the time the loan is terminated.

Keywords: reverse mortgage, low price housing, guarantor's risk, money's worth, residual housing equity

Introduction

In the current KRM(Korean reverse mortgage program), the level of monthly payments the borrowers can receive is mainly influenced by the value of housing as well as borrowers' age. So, the elderly people who have low price housing can receive relatively small amount of monthly payments from the KRM. To increase the amount of monthly payments for the elderly people who have low price housing, Korean government decided to provide a special reverse mortgage program which subsidizes the insurance premium corresponding to the amount of 2.25% of housing value when the borrower's housing value is under 150 million KRW[1]. As a result, the borrowers who have low price housing could receive about 9%~17% increased monthly payments in accordance to their ages from a special KRM model since April 2016. However, the current method of just subsidizing the insurance premium to increase the amount of monthly payments has the problem of long term sustainability because the government's financial situation could become worse as the number of subscribers increases.

To increase the amount of monthly payments for elderly people who have low price housing innovatively without causing any financial burden for the government, it is necessary to consider a totally different type of reverse mortgage model. The purpose of this research is to introduce a new reverse mortgage model which can increase the amount of monthly payments innovatively without causing any financial burden for the government. Under the current Korean reverse mortgage program, the government guarantees that the borrower will not owe more than the value of housing at the time it is sold due to non-recourse limit. On the contrary, if the value of housing is larger than the outstanding loan balance at the time the loan is terminated, the heirs can receive leftover on the sale. So, as long as we calculate the amount of monthly payments under the structure of the current actuarial model, the possible maximum amount of monthly payments cannot be increased innovatively although there is some government's subsidy in the program.

In this paper, to increase the amount of monthly payments for elderly people who have low price housing innovatively without causing any financial burden for the government, we suggest a totally different type of reverse mortgage model. In this new model, we allow the guarantor to receive a proportion or all of the residual housing equity on the sale if the home value is larger than the mortgage balance at the time the loan is terminated. As a result, in this new program, the borrowers can get considerably increased amount of monthly payments than the current KRM because the guarantor can take on more losses in the program. Although the elderly people who die early can be disadvantaged in this type of model, we can solve this problem by adding the option of 10 years, 15years or 20years payment guarantee to the program as applied in the private annuities[2]. In this case, the heirs of early deceased can get the remainder of monthly payments until the guaranteed term expires.

It is expected that providing a new reverse mortgage program can increase the right of choice among varied reverse mortgage programs. As a result, the borrowers can select the proper one for them according to their

[1] The average price for borrowers' homes in the Korean reverse mortgage program is confirmed to be around 280 million KRW(Korea Housing Finance Corporation, Feb., 2016). On April 11, 2017, US 1.0 dollar was equivalent to Korean 1,146.50 KRW.

[2] Refer to Ma and Kim(2015).

needs. In this case, if the elderly people who have low price housing want to receive more money to supplement their living costs, they can select our new model.

The remainder of the paper is organized as follows: section 2 presents actuarial model to determine the level of monthly payment, section 3 presents methodologies focusing on calculating methods of money's worth for borrowers, net guarantee loss and explains stochastic models to generate future interest rates, housing prices, and mortality rate of subscribers, section 4 reports the results of the analysis, and section 5 presents our conclusions.

Actuarial Model to Determine the Level of Monthly Payment

Current Korean Reverse Mortgage Program

In the KRM, monthly payments can be made in pre-determined monthly amounts to be paid as long as the borrower resides in the house. The borrower does not have to make repayments on the loan as long as she or he continues to live in it, but when the borrower becomes deceased and no longer occupies the house, the lender takes over the property. Reverse mortgages therefore allow equity rich but cash poor elderly homeowners to convert all or part of their home equity into tax-free cash without having to sell their homes. It gives the seniors financial stability and independence, and also allows them the opportunity to retain control of the homes they live in (Ma and Deng, 2013). Under the current KRM, the government guarantees that the borrower will not owe more than the value of housing at the time it is sold due to non-recourse limit. On the contrary, if the value of housing is larger than the outstanding loan balance at the time the loan is terminated, the heirs can receive leftover on the sale.[3]

We determine the breakeven level of monthly payments in current KRM under the condition that the present value of the mortgage insurance premium (PVMIP) is equal to the expected loss (PVEL) according to the traditional principle of insurance pricing (Ma and Deng, 2013).

$$PVMIP = PVEL \tag{1}$$

$$but, \quad PVMIP = Up_0 + \sum_{t=1}^{T(a)} \left\{ \frac{mip_t \cdot \ {}_tp_a}{(1+i)^{t-1}} \right\},$$

$$PVEL = \sum_{t=1}^{T(a)} \left\{ \frac{max[(OLB_t - H_t), 0] q^*_{a+t} \cdot \ {}_tp_a}{(1+i)^t} \right\}$$

Where, Up_0 is the upfront mortgage insurance premium, $T(a)$ is the remainder of the payment period until loan termination, mip_t is the monthly mortgage insurance premium, i is expected interest rate, ${}_tp_a$ is the

[3] It is known that the model of KRM just imitated that of HECM (Home Equity Conversion Mortgage) in the U.S. when it was first launched in July 2007 (Ma and Kim, 2016). According to the HECM, we can refer to Chen et al. (2010) and Szymanoski (1990).

probability that a borrower at age a will survive at $a+t$, H_t is the house value, OLB_t is the outstanding loan balance, q^*_{a+t} is modified mortality rate after considering the prepayment rate at age $a+t$.

New Reverse Mortgage Program

To increase the amount of monthly payments for elderly people who have low price housing innovatively without causing any financial burden for the government, we suggest a totally different type of reverse mortgage model. In this new reverse mortgage program, the level of monthly payments can be determined under the condition that the present value of expected leftovers (PVLO) is equal to that of expected losses (PVEL) in accordance with traditional principle of insurance pricing. In this new program, we substitute PVLO for PVMIP in the current KRM. In this new program, because the elderly people who died early can be disadvantaged, we added the option of 5 years, 10 years, 15years or 20years payment guarantee to the program. In this case, the heirs of early deceased can get the remainder of monthly payments until the guaranteed term expires.

$$PVLO = PVEL \tag{2}$$

$$PVLO = \left\{ \sum_{t=1}^{n} \left[\frac{max(H_t - OLB_t, 0) \cdot er}{(1+i)^t} \right] - \left[PVCpmt_{n-1} - \sum_{t=1}^{n} \frac{pmt}{(1+i)^{t-1}} \right] + \sum_{t=n+1}^{T(a)} \left[\frac{max(H_t - OLB_t, 0) \cdot er}{(1+i)^t} \right] \right\} q^*_{a+t} \cdot {}_tp_a$$

$$but,\ \ PVCpmt_{n-1} = \sum_{t=1}^{n} \frac{pmt}{(1+i)^{t-1}}$$

Where, er is the ratio that the guarantor can get from residual housing equity at the time the loan is terminated ($er = 1.0\ or\ 0.5$), $PVCpmt_{n-1}$ is the present value of cumulative monthly payments at $t = n-1$, pmt is the amount of monthly payment.

To compare the new model we suggested in this paper with the current KRM, we will evaluate the guarantor's risk as well as money's worth for borrowers. To do this, we use Monte Carlo simulation method which considers the stochastic processes of interest rates, housing prices and loan termination probabilities concurrently.

Criteria for the Evaluation of New Reverse Mortgage Model

Money's worth for the Borrowers

Money's worth (MW) is a ratio of the present value of pension payments to the lump sum insurance premium (Mitchell et al., 1999; Brown et al., 2000). So, we use this concept to measure the changes in the reverse mortgage borrower's benefit under different structures of model. In the reverse mortgage model, if we use equation (3) or (4), we can calculate money's worth for borrowers.

$$MW^C = \sum_{t=1}^{T(a)} \left[\frac{(PVCPmt_t + PVLO_t) q^*_{a+t} \cdot {}_tp_a}{PVH_t} \right] \quad (3)$$

$$MW^N = \left\{ \sum_{t=1}^{n} \left[\frac{PVCpmt_n + PVLO_t(1-er)}{PVH_t} \right] + \sum_{t=n+1}^{T(a)} \left[\frac{PVCpmt_t + PVLO_t(1-er)}{PVH_t} \right] \right\} q^*_{a+t} \cdot {}_tp_a \quad (4)$$

Where, MW^C is money's worth for borrowers in the current KRM, MW^N is money's worth for borrowers in the new reverse mortgage model, $PVCpmt_t$ is the present value of cumulative monthly payments at time t, $PVLO_t$ is the present value of residual housing equity (leftover) at time t, PVH_t is the present value of housing at time t, n is term of payment guarantee.

Net guarantee loss

We calculate net guarantee loss (NL) to measure the cost side of reverse mortgage programs. We can evaluate the guarantor's risk through calculating the present value of NL. Under the current KRM, present value of net guarantee losses (NL^C) can be calculated by subtracting present value of expected mortgage insurance premiums (PVMIP) from that of expected losses (PVEL).

$$NL^c = PVEL - PVMIP \quad (5)$$

On the other hand, in the new reverse mortgage model, present value of net guarantee losses (NL^N) can be calculated by subtracting present value of residual housing equity (PVLO) from PVEL.

$$NL^N = PVEL - PVLO \quad (6)$$

Stochastic Models

In the actuarial model to determine the level of monthly payments, for convenience, all the major risk factors are assumed to have fixed values although they have stochastic processes in real world. Thus, the money's worth and net guarantee losses can have different values according to the realized future values of major risk factors. In this analysis, we consider the stochastic processes of interest rates, discount rates, housing prices, and mortality rates concurrently.

Interest Rate and Discount Rate

To generate the stochastic processes of actual interest rate and discount rate, we use the Vasicek model as below.

$$\Delta i_t = \alpha(\mu - i_t)\Delta t + \varepsilon_t \sigma \sqrt{\Delta t} \quad (7)$$

In the formula above, i_t is the interest rate at time t and Δi_t means $i_{t+1} - i_t$. α and μ means the speed of

reversion and the mean reversion level respectively. σ implies the volatility of i_t and ε_t is a random variable which follows the standard normal distribution. We estimated the parameters of eq(7) applying the maximum likelihood estimation method using the time series data for rate observed from 09.2004 to 12.2014. The Table 1 below shows the estimation results.

Table 1. Estimation Results for the Vasicek Model

	α	μ	σ
CD Rate	0.1317	0.0241	0.0068
10 Year Treasury Bond Rate	0.1624	0.0334	0.0075

Housing Appreciation Rate

We use the geometric brownian motion model to generate the stochastic processes of future housing values. In the model, the housing price at time t follows as below.

$$H_{t+\Delta t} = H_t \cdot exp\left[\left(\mu_H - \frac{\sigma^2}{2}\right)\Delta t + \sigma_H \varepsilon_t \sqrt{\Delta t}\right] \quad (8)$$

Where H_t is the housing price at time t. μ_H and σ_H means the expected value and volatility of housing appreciation rate respectively. ε_t is a random variable which follows the standard normal distribution. Table 2 below shows the estimation results.

Table 2. Parameter Estimation for the Geometric Brownian Motion Model

	μ_H	σ_H
Housing Appreciation Rate	0.0374	0.0192

Mortality Rate of Subscribers

The life table applied in the current 2017 actuarial model in the KRM is the 2015 period life table, released by the Statistics Korea. However, to conduct a more precise evaluation of the criteria adopted in this study, a cohort life table should be applied to reflect the trend of life expectancy extension. To generate a cohort life table, long-term forecast for mortality rates in life tables should be obtained and the Lee-Carter (LC) model can be used for this purpose. The LC model is represented in the following formula.

$$\ln(q_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t} \quad (x = 1, 2, \cdots, n\,;\ t = 1, 2, \cdots, T) \quad (9)$$

In this formula, $q_{x,t}$ means the mortality rate in year t for the age group x. a_x is a constant which reflects the average pattern of age group-wise mortality rate and b_x is the speed of mortality rate changes according to the changes in k_t. k_t is the mortality index and $\varepsilon_{x,t}$ is an error term.

In LC model, k_t has a stochastic process and affects the future forecasted values of the mortality rate. So, we

used the following models to generate the future processes of k_t for male or female according to the results of random walk model estimated by using the data from 1997 to 2014.

- Male : $k_t = -1.46277 + k_{t-1} + 0.293406 \times N(0,1)$
- Female : $k_t = -1.79676 + k_{t-1} + 0.402014 \times N(0,1)$

In actual analysis, we used multiple life model[4] using both cohort mortality rates for male and that for female which we have generated through LC models.

Results of Analysis

Reverse mortgage program may serve as a complementary source of income for the elderly, but the amount of monthly payments for the elderly people who have low price housing is in sufficient to cover their living costs. With this point of view, we introduce a new reverse mortgage model to increase the amount of monthly payments innovatively. We then evaluated the values of MW and NL respectively to confirm the effects of changing reverse mortgage model on the subscribers' benefits and guarantor's risk. If the guarantor's risk does not change significantly after changing the model, the new model can be a good substitute for the current model. In addition, when we check MW for the borrowers, if MW does not change significantly or the magnitude of MW increases after changing the model, we can say that the new model can be a good substitute for the current model.

In this paper, we introduce two new reverse mortgage models(New Model Ⅰ and New Model Ⅱ) in which we substitute PVLO for PVMIP in the current KRM. In the New Model Ⅰ, we assume that the guarantor can take all of the residual housing equity on the sale if the home value is larger than the mortgage balance at the time the loan is terminated. And in the New Model Ⅱ, the guarantor can take 50% of the residual housing equity on the sale if the home value is larger than the mortgage balance at the time the loan is terminated.

Analysis Results of New Model Ⅰ

The Level of Monthly Payments

Table 3 shows the amounts of monthly payments in the current KRM and its special model. As we know, the special KRM is a model in which government subsidizes mortgage insurance premium corresponding to the amount of 2.25% of housing value when the borrower's housing value is under 150 million KRW. As a result, the borrowers who select this special KRM can receive about 9%~17% increased monthly payments.

[4] KRM is an annuity whose monthly payments terminate at the time the last survivor dies. Thus, we used a last-survivor life table with cohort effect.

Table 3. Current KRM: Comparing Monthly Payments (Unit: 1,000KRW)

Age	Basic Model: A	Special Model: B	Ratio: B/A
60	204	223	1.09
65	247	271	1.10
70	302	334	1.11
75	376	421	1.12
80	476	545	1.14
85	626	731	1.17
90	911	1,063	1.17

Note: 1. Housing price: 100 million KRW

2. We assumed constant monthly payments by borrower's ages in tenure advances.
3. We calculated the level of monthly payments in the KRM using our private spreadsheet model, so the values could be a little bit different from the actual level of monthly payments in the actual KRM.

Due to the embedded structural problem in the current KRM model, it is difficult to increase the amount of monthly payments without government's subsidy. However, the current method of just subsidizing the insurance premium to increase the amount of monthly payments for elderly people with low price housing has the problem of long term sustainability because the government's financial situation could become worse as the number of subscribers increases.

Therefore, we introduced a totally different type of reverse mortgage model to increase the amount of monthly payments for elderly people who have low price housing innovatively without causing any financial burden for the government. In this new model, we allow the guarantor to receive a proportion or all of the residual housing equity on the sale if the home value is larger than the mortgage balance at the time loan is terminated. As a result, in this program, the borrowers can get considerably increased amount of monthly payments because the guarantor can take more losses in the program. In this new model, we adopted the option of 5years, 10 years, 15years or 20years payment guarantee as applied in the private annuities because the elderly people who died early can be disadvantaged in this type of model. In this case, the heirs of early deceased can get the remainder of monthly payments until the guaranteed term expires.

First, we confirmed the amount of monthly payments in a New Model under the assumption that the guarantor takes all of the residual housing equity on the sale (that is $equity\ ratio = 1.0$) if the home value is larger than the mortgage balance at the time loan is terminated. Table 4 shows the amount of monthly payments in the new reverse mortgage model when the guarantor's $equity\ ratio = 1.0$ (New Model Ⅰ).

Table 4. Monthly Payments in the Current KRM Model and the New Model Ⅰ (Unit: 1,000KRW)

Age	Current Model ($i = 5.24\%,\ g = 2.20\%$)	New Model Ⅰ ($i = 5.24\%,\ g = 2.20\%,\ equity\ ratio = 1.0$)

	Basic Model	Special Model	No-Guarantee	5 year Guarantee	10 year Guarantee	15 year Guarantee	20 year Guarantee
60	204	223	264	253	260	257	251
65	247	271	334	333	328	318	304
70	302	334	440	435	420	395	-
75	376	421	599	583	538	-	-
80	476	545	851	795	669	-	-
85	626	731	1,255	1,063	-	-	-
90	911	1,063	1,890	1,333	-	-	-

Note: 1. House price: 100 million KRW

2. We assumed the same expected interest rate and housing appreciation rate in both the current model and the new model, that is $i = 5.24\%,\ g = 2.20\%$.
3. Guarantor takes all of the residual housing equity on the sale (that is $equity\ ratio = 1.0$) in the new model

Table 5 shows ratios of monthly payments in the new reverse mortgage model to the current KRM model (Basic Model=1.0). As we can see in Table 5, if there is no payment guarantee option, the subscribers who choose the new reverse mortgage model can get 1.29~2.07 times increased monthly payments than the current basic KRM. However, if the borrowers do not select payment guarantee option, they can be disadvantaged when they die early because the guarantor takes all of the residual housing equity on the sale. To solve this problem, we adopted payment guarantee option in the New Model. So, the borrowers can select among 5 years, 10 years, 15 years, and 20 years payment options according to their needs and their ages. If there is payment guarantee option, the subscribers who choose the New Model can get 1.23~1.70 times increased monthly payments than the current basic KRM according to the guarantee periods.

Table 5. Ratio of Monthly Payments in the New Model Ⅰ to the Current KRM Model (Unit: 1,000KRW)

Age	Current Model ($i = 5.24\%,\ g = 2.20\%$)		New Model Ⅰ ($i = 5.24\%,\ g = 2.20\%,\ equity\ ratio = 1.0$)				
	Basic Model	Special Model	No-Guarantee	5 year Guarantee	10 year Guarantee	15 year Guarantee	20 year Guarantee
60	1.00	1.09	1.29	1.24	1.27	1.26	1.23
65	1.00	1.10	1.35	1.35	1.33	1.29	1.23
70	1.00	1.11	1.46	1.44	1.39	1.31	-
75	1.00	1.12	1.59	1.55	1.43	-	-
80	1.00	1.14	1.79	1.67	1.41	-	-
85	1.00	1.17	2.00	1.70	-	-	-

90	1.00	1.17	2.07	1.46	-	-	-

On the other hand, we can increase the level of monthly payments by reducing the level of expected interest rates in the New Model. In this paper, we applied the expected interest rate as 3.15% to refer to the current fixed interest rate in 30 year's mortgage loan[5].

When we applied 3.15% as an expected interest rate, the monthly payments that the borrowers can receive are increasing as in Table 6.

Table 6. Comparing Monthly Payments between the Current Model and the New Model Ⅰ (Unit: 1,000KRW)

Age	Current Model ($i = 5.24\%,\ g = 2.20\%$)		New Model Ⅰ ($i = 3.15\%,\ g = 2.20\%,\ equity\ ratio = 1.0$)				
	Basic Model	Special Model	No-Guarantee	5 year Guarantee	10 year Guarantee	15 year Guarantee	20 year Guarantee
60	204	223	348	347	344	339	330
65	247	271	415	413	407	395	374
70	302	334	516	511	494	461	-
75	376	421	669	652	600	-	-
80	476	545	913	855	713	-	-
85	626	731	1307	1111	-	-	-
90	911	1,063	1935	1362	-	-	-

Note: House price: 100 million KRW

Table 7 shows ratios of monthly payments in the New Model to the current KRM model (Basic Model=1.0) when we applied 3.15% as an expected interest rate in the New Model. As we can see from the Table, when we applied 3.15% as an expected interest rate, if there is no payment guarantee option, the subscribers who choose the New Model can get 1.71~2.12 times increased monthly payments than the current basic KRM. In this case, if there is payment guarantee option, the subscribers who choose the new reverse mortgage model can get 1.50~1.80 times increased monthly payments than the current basic KRM according to the guarantee periods.

Table 7. Ratio of Monthly Payments in the New Model Ⅰ to the Current KRM Model (Unit: 1,000KRW)

Age	Current Model ($i = 5.24\%,\ g = 2.20\%$)		New Model Ⅰ ($i = 3.15\%,\ g = 2.20\%,\ equity\ ratio = 1.0$)				
	Basic Model	Special Model	No-Guarantee	5 year Guarantee	10 year Guarantee	15 year Guarantee	20 year Guarantee

[5] Refer to Korea Housing Finance Corporation (http://hf.go.kr).

60	1.00	1.09	1.71	1.70	1.69	1.66	1.62
65	1.00	1.10	1.68	1.67	1.65	1.60	1.51
70	1.00	1.11	1.71	1.69	1.64	1.53	-
75	1.00	1.12	1.78	1.73	1.60	-	-
80	1.00	1.14	1.92	1.80	1.50	-	-
85	1.00	1.17	2.09	1.77	-	-	-
90	1.00	1.17	2.12	1.50	-	-	-

Evaluation of Net guarantee loss

To confirm the changed magnitude of guarantor's risk resulting from adopting New Model Ⅰ, we calculated the values of NL(net guarantee loss). Table 8 shows the values of NL in the current KRM and that of the New Model Ⅰ respectively.

Table 8. Comparison of NL (Unit: 1,000KRW)

		pmt	median	5% VaR	1% VaR	1% CVaR	P(NL>0)
Current Model	Basic KRM	302	-8334	-4134	278	2802	1.07%
	Special KRM	334	-10975	-3678	1444	4720	1.61%
New Model Ⅰ ($i = 5.24\%, er = 1.0$)		420	-20888	3228	11265	15449	8.07%
New Model Ⅰ ($i = 3.15\%, er = 1.0$)		494	-5861	17946	26858	31678	35.23%

Note: 1. House price: 100 million KRW, Age: 70

2. Simulation: 30,000 trials

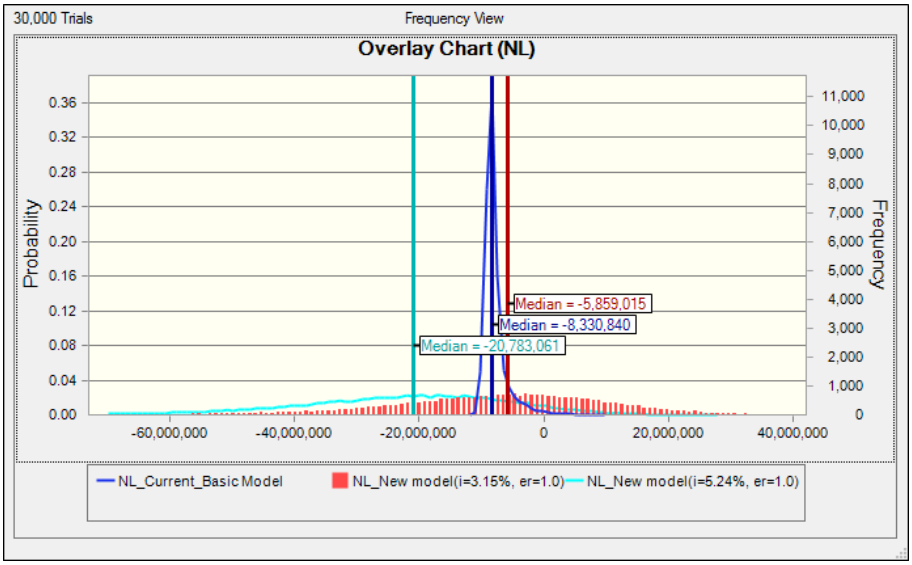
3. CVaR(Conditional Value at Risk); the expected value of losses beyond the threshold level[6]

In the current KRM, the median values represent negative values and these values tell us that the guarantor can get net profit from the guarantee of KRM when we estimate it under the basis of median values. In this case, the probabilities that the values of NL are larger than zero (P(NL>0)) show only 1.07% (basic KRM) or 1.61% (special KRM).

In the New Model Ⅰ, the median values also represent negative values and these values tell us that the guarantor can get net profit from the guarantee of the New Model when we estimate it under the basis of median values. However, in this case, the probabilities that the values of NL are larger than zero (P(NL>0)) are increased compared to the current KRM. When we apply the expected interest rate as 5.24%, the probability increased to 8.07% and when we apply the expected interest rate as 3.15%, the probability increased to 35.23% in New Model Ⅰ.

[6] Refer to Charnes(2012).

Figure 1. Comparison of NL between Basic KRM and the New Model Ⅰ



As we can see in Figure 1, the range of probability distributions of NL in the New Models are significantly larger than that of KRM. This result means that the magnitude of guarantor's risk in the New Models is larger than that of KRM.

In the New Model, we substituted PVLO for PVMIP in the current KRM. So, the level of monthly payments can be determined under the condition that the present value of expected leftovers (PVLO) is equal to that of expected losses (PVEL) in this New Model. So, the increased range of probability distributions of NL in the New Model is totally resulting from the fact that the variability of PVLO is significantly larger than that of PVMIP.

Evaluation of Money's Worth

We calculated the values of MW (money's worth) for borrowers. Table 9 shows the values of MW in the probability distributions under 30,000 trials of Monte Carlo simulation.

Table 9. Comparison of MW

		pmt	Lower	Median	Upper
Current Model	Basic KRM	302,000	0.80	0.91	1.01
	Special KRM	334,000	0.80	0.91	1.03
New Model ($i = 5.24\%, er = 1.0$)		420,000	0.57	0.78	1.10

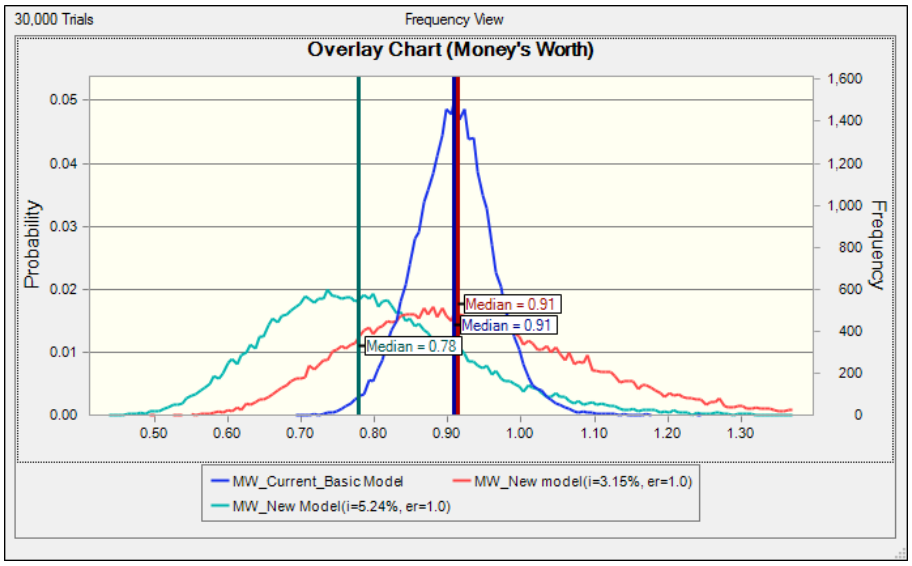
New Model ($i = 3.15\%, er = 1.0$)	494,000	0.67	0.91	1.29

Note: Lower and Upper values represent the values under 95% certainty in the probability distributions under 30,000 trials of Monte Carlo simulation.

As we can see in Table 9, it is confirmed that the values of MW in the New Model Ⅰ are smaller than that of the current KRM when we estimate them under the basis of median values in the probability distributions.

Figure 2 shows the probability distributions of MW in the New Model and KRM respectively. Because the variability of PVLO is significantly larger than that of PVMIP, the probability distributions of MW in the New Models also showed wide ranges than that of KRM.

Figure 2. Probability Distributions of MW



Analysis Results of New Model Ⅱ

Monthly Payments

Secondly, we can consider the method of sharing leftover between guarantor and borrower to reduce the guarantor's risk in the New Model Ⅱ. In this case, if the guarantor and borrower share the same amount of leftover when the loan is terminated (guarantor's equity ratio is 0.5), the level of monthly payments decreases compared to when guarantor's equity ratio is 1.0 as below. In the New Model Ⅱ, we considered only the model which applies the level of expected interest rate as 3.15%. Table 10 shows monthly payments in the current KRM and the New Model Ⅱ respectively.

Table 10. Monthly Payments in the Current KRM and the New Model Ⅱ (Unit: 1,000KRW)

Age	Current Model ($i = 5.24\%,\ g = 2.20\%$)		New Model Ⅱ ($i = 3.15\%,\ g = 2.20\%,\ equity\ ratio = 0.5$)				
	Basic KRM	Special KRM	No-Guarantee	5 year Guarantee	10 year Guarantee	15 year Guarantee	20 year Guarantee
60	204	223	321	320	317	311	300
65	247	271	379	377	370	355	330
70	302	334	464	459	438	400	-
75	376	421	590	571	513	-	-
80	476	545	787	724	570	-	-
85	626	731	1,100	895	-	-	-
90	911	1,063	1,599	1,022	-	-	-

Note: House price: 100 million KRW

Table 11 shows ratios of monthly payments in the New Model Ⅱ to the current KRM model (Basic KRM=1.0). As we can see in Table , if there is no payment guarantee option, the subscribers who choose the New Model can get 1.53~1.76 times increased monthly payments than the current basic KRM. If there is payment guarantee option, the subscribers who choose New Model Ⅱ can get 1.12~1.57 times increased monthly payments than the current basic KRM according to the guarantee periods.

Table 11. Ratio of Monthly Payments in the New Model Ⅱ to the Current KRM Model (Unit: 1,000KRW)

Age	Current Model ($i = 5.24\%,\ g = 2.20\%$)		New Model Ⅱ ($i = 3.15\%,\ g = 2.20\%,\ equity\ ratio = 0.5$)				
	Basic Model	Special Model	No-Guarantee	5 year Guarantee	10 year Guarantee	15 year Guarantee	20 year Guarantee
60	1.00	1.09	1.57	1.57	1.55	1.52	1.47
65	1.00	1.10	1.53	1.53	1.50	1.44	1.34
70	1.00	1.11	1.54	1.52	1.45	1.32	-
75	1.00	1.12	1.57	1.52	1.36	-	-
80	1.00	1.14	1.65	1.52	1.20	-	-
85	1.00	1.17	1.76	1.43	-	-	-
90	1.00	1.17	1.76	1.12	-	-	-

Evaluation of Net guarantee loss

Table 12 shows the values of NL in the current KRM and that of the New Model respectively. As we can see in Table 12, the probability that the values of NL would be larger than zero (P(NL>0)) in New Model Ⅱ is

decreased compared to New Model Ⅰ (35.23%→23.12%). This result tells us that if the guarantor and borrower share the amount of leftover when the loan is terminated, the guarantor's risk can be reduced.

Table 12. Comparison of NL (Unit: 1,000KRW)

		pmt	median	5% VaR	1% VaR	1% CVaR	P(NL>0)
Current Model	Basic KRM	302	-8334	-4134	278	2802	1.07%
	Special KRM	334	-10975	-3678	1444	4720	1.61%
New Model Ⅰ $(i = 3.15\%, er = 1.0)$		494	-5861	17946	26858	31678	35.23%
New Model Ⅱ $(i = 3.15\%, er = 0.5)$		438	-7667	10349	17854	21526	23.12%

Note: 1. House price: 100 million KRW, Age: 70

2. Simulation: 30,000 trials

3. CVaR(Conditional Value at Risk); the expected value of losses beyond the threshold level

Figure 3 shows the probability distributions of NL more concretely. As we confirmed from the previous analysis of New Model Ⅰ, the increased range of probability distributions of NL in the New Model is totally resulting from the fact that the variability of PVLO is significantly larger than that of PVMIP. When we compare the probability distributions of NL between New Model Ⅰ and New Model Ⅱ, we can see the range of distribution in New Model Ⅱ is relatively narrowed compared to that of New Model Ⅰ due to sharing the amount of leftover when the loan is terminated.

Figure 3. Comparison of NL between Basic KRM and the New Model

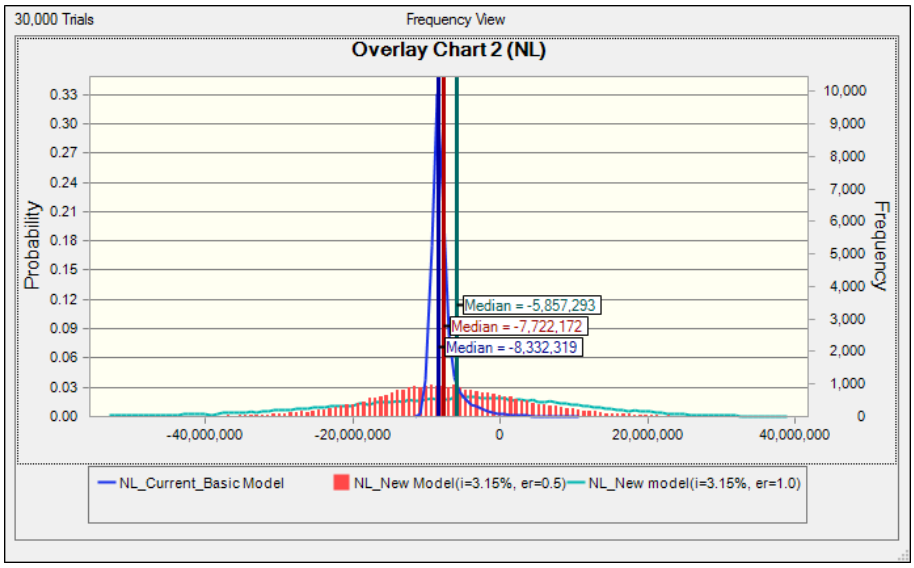


Table 13 shows the values of MW in the probability distributions under 30,000 trials of Monte Carlo simulation. We can see that all the median values in Table 13 show 0.91, the same value.

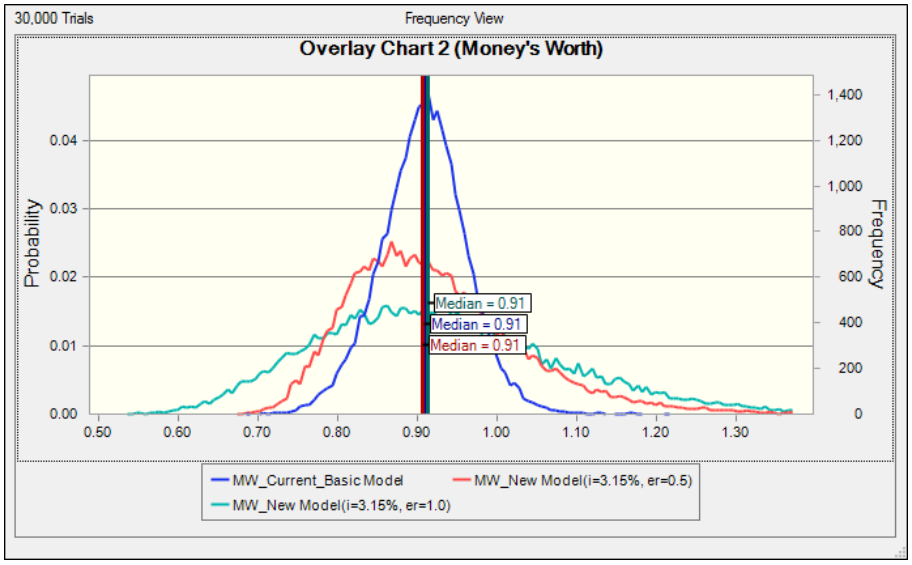
Table 13. Comparison of MW

		pmt	Lower	Median	Upper
Current Model	Basic KRM	302,000	0.80	0.91	1.01
	Special KRM	334,000	0.80	0.91	1.03
New Model Ⅰ $(i = 3.15\%, er = 1.0)$		494,000	0.67	0.91	1.29
New Model Ⅱ $(i = 3.15\%, er = 0.5)$		438,000	0.76	0.91	1.18

Note: Lower and Upper values represent the values under 95% certainty in the probability distributions

Figure 4 shows the probability distributions of MW more concretely. In this Figure, we can confirm that the variability of MW in New Model Ⅱ is relatively reduced compare to New Model Ⅰ.

Figure 4. Comparison of MW between Basic KRM and the New Model



Suggestion of New Model

Finally, we considered the way to reduce the level of actual loan interest rate to build the New Model more reasonably. As we can see from the results of previous analysis in this paper, the reverse mortgage guarantor in the New Model can get significantly large amount of leftover when the loan is terminated because this model allows the guarantor to receive a proportion or all of the residual housing equity on the sale if the home value is larger than the mortgage balance at the time loan is terminated.

For reference, in the forward mortgage loans, the lenders allow very low loan interest rate ranging from only 1%~2% fixed interest rate when they want to share the housing equity with homeowners when the loan is

terminated.[7] So, considering this aspect, we can also apply a very low actual loan interest rate in the New Model instead of sharing housing equity with borrowers. As we know, the actual loan interest rate is determined by adding 1.1% margin to CD (certificate of deposit) rates in the current KRM.

In this analysis, we suggested a New Model which applied CD rate+margin 0.1% as an actual loan interest rate. When we decrease the level of actual loan interest rate into CD+margin 0.1% then, the probability that the values of NL would be larger than zero (P(NL>0)) in the New Model can be reduced significantly.

Table 14 shows, in this case, P(NL>0) can be decreased to 10.30% in New Model Ⅰ and decreased to 5.02% in New Model Ⅱ. If actual loan interest rate is decreased, the increasing speed of outstanding loan balance also decreased. As a result, P(NL>0) can be decreased significantly as we can see in Table 14. When we evaluate NL under the median values, the guarantor in the New Model can get large values of net profit (negative values of net guarantee loss) compared to the current KRM.

Table 14. Comparison of NL (Unit: 1,000KRW)

		pmt	median	5% VaR	1% VaR	1% CVaR	P(NL>0)
Current Model	Basic KRM	302	-8334	-4134	278	2802	1.07%
	Special KRM	334	-10975	-3678	1444	4720	1.61%
New Model Ⅰ ($i = 3.15\%, er = 1.0$)		494	-18583	4740	13218	17180	10.30%
New Model Ⅱ ($i = 3.15\%, er = 0.5$)		438	**-13931**	**24**	**6536**	**10041**	**5.02%**

Note: 1. House price: 100 million KRW, Age: 70

2. Simulation: 30,000 trials

3. Actual loan interest rate: CD+margin 0.1%

Table 15 shows the values of MW. As we can see in Table 15, the value of MW in New Model Ⅰ does not change although actual loan interest rate is decreased. It is because the value of PVLO in New Model Ⅰ does not affect the value of MW. But, in New Model Ⅱ, we can confirm that the values of MW increased significantly to 0.95 when we evaluate it under the median value in the probability distribution.

Table 15. Comparison of MW

		pmt	Lower	Median	Upper
Current Model	Basic KRM	302,000	0.80	0.91	1.01
	Special KRM	334,000	0.80	0.91	1.03
New Model Ⅰ ($i = 3.15\%, er = 1.0$)		494,000	0.67	0.91	1.29

[7] Refer to shared equity mortgage program in Korea (http://nhuf.molit.go.kr).

New Model Ⅱ $(i = 3.15\%, er = 0.5)$	438,000	**0.80**	**0.95**	**1.20**

Note: Lower and Upper values represent the values under 95% certainty in the probability distributions

From the results of this analysis, we can say that the New Model can be more reasonable if we reduce the level of actual loan interest rate in addition to share the housing equity (PVLO) between borrower and guarantor.

Conclusions

The current method of just subsidizing the insurance premium to increase the amount of monthly payments for the elderly who have low price housing has the problem of long term sustainability because the government's financial situation could become worse as the number of subscribers increases. To increase the amount of monthly payments for elderly people who have low price housing innovatively without causing any financial burden for the government, we considered a totally different type of reverse mortgage model. As we confirmed in this analysis, as long as we calculate the amount of monthly payments under the structure of the current actuarial model in the KRM, the possible maximum amount of monthly payments cannot be increased innovatively although there is some government's subsidy to the program.

However, in the New Model, we allowed the guarantor to receive or share the residual housing equity on the sale if the home value is larger than mortgage balance at the time loan is terminated. As a result, the borrowers can get considerably increased amount of monthly payments because the guarantor can take on more losses in the program under similar burden of risk. So, if the elderly people want to receive more money to supplement their living costs, they can choose this newly suggested reverse mortgage model although there is no chance that they can receive leftover on sale or have to share it with guarantor if the home value is larger than the mortgage balance at the time the loan is terminated.

It is expected that providing a new reverse mortgage program can increase the borrowers' right of choice among varied reverse mortgage programs. As a result, the borrowers can select the proper one for them according to their needs. In this case, especially, if the elderly people who have low price housing want to receive more money to supplement their living costs, they can choose this New Model although there is no chance that they can receive the leftover on sale or have to share it with guarantor when the loan is terminated.
The results of this study can be a useful reference when the government considers to introduce a new reverse mortgage program for elderly people who have low price housing.

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