Lee–Carter Model and Select Period Effect

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Abstract

Age and gender are the two most common risk factors considered in the life insurance products. Other risk factors can also be used, depending on the feasibility and marketing, as well as data availability. Previous studies showed that the newly insured, likely passed certain health exams, have lower mortality rates than those who are already insured. Select and ultimate tables are often used by insurance companies to deal with the mortality discrepancy between the insured with different policy years. However, the effect of policy year is easily confused with the mortality improvement over years for a longer study period. In this study, we propose a modification of Lee-Carter model which can include the effect of policy year, as well as the mortality improvement. We first use computer simulation to evaluate the proposed approach, especially on the parameter estimation, and then apply it to the experienced data from Taiwan's largest insurance company. Results from both studies support the proposed approach and including the select effect (i.e., policy year) in the Lee-Carter model is a feasible approach.

Keywords: Lee–Carter Model, Select and Ultimate Tables, Mortality Risk, Mortality Improvement, Simulation

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1. Preface

Mortality improvement has been a global trend since the end of World War II. In addition to exploring whether the human longevity has a limit, many studies focus on searching the risk factors related to mortality rates. Age and gender are the two most common risk factors considered in the life insurance products. Tobacco use is another risk factor, but it is not easy to verify whether a person smokes and previous studies showed that many smokers tend to lie on life insurance applications. Other than these factors, it is believed that the newly insured, likely passed certain health exams, have lower mortality rates than those who are already insured. This phenomenon is often called as the select effect and it appears not only in life insurance products but also in health products. The insurance companies usually use select life tables to differentiate the mortality risk of the insured with respect to the policy year.

The select effect is well-known and often appears as a standard topic in the textbooks of actuarial mathematics (e.g., Bowers et al., 1997; Geber, 1995). However, surprisingly, not many past studies focus on modeling the select effect. For example, Carriere (1994) proposed a parametric model based on a linear combination of survival functions, and Renshaw and Haberman (1997) used a mixture of generalized linear and non-linear models. These studies focused mainly on the methodology for constructing the select mortality tables, especially on the connection between mortality rates of different policy years. Considerations of graduating mortality rates in two dimensions (e.g., age and policy year) and higher dimensions are more complicated, comparing to the traditional one-dimensional graduation with respect to age (London, 1985).

Quite a lot of factors contribute to with few methods proposed for modelling the select effect. The data availability is one of them. In order to evaluate whether the newly insured have lower mortality rates, a longer observation period is often required (10-year and longer periods are recommended). This is probably the main reason why there are not many select mortality tables available. For example, since the 1975-80 Basic Tables, the Society of Actuaries (SOA) in U.S. published only three studies of select period: 2001, 2008, and 2014 Valuation Basic Tables (VBA) (Klein

and Krysiak, 2014). The length of select period is quite different for different insurers and different insurance products, as reported in 2014 Select Period Mortality Survey, and it ranges from 10 years to more than 30 years for term products. The size of select effect also varies a lot.

However, on the other hand, a longer observation period would further complicate the modelling of select effect. The mortality rates of later policy years can be affected by the mortality improvement and thus the select effect would likely be under-estimated, assuming that the mortality rates of all ages decrease with time. To avoid the possibility of under-estimation, both the factors of select effect and mortality improvement need to be considered. Note that most of the past studies regarding the select effect did not incorporate the mortality improvement[2]. In fact, it is still not considered in pricing products now for many countries (including Taiwan), even for annuity products.

In this study, we use the Lee-Carter model (Lee and Carter, 1992) as the base for handling the mortality improvement and propose adding an extra factor for the select effect to the model. The estimation of mortality improvement and select effect is done via two-stage iteration, like most modifications of Lee-Carter model. We will use computer simulation to evaluate the proposed approach, showing that it can provide unbiased estimates to the parameters. We also use empirical data to demonstrate the estimates of select effects are underestimated if the mortality improvement is not considered.

The proposed mortality model can be used to identify the mortality risk regarding the policy year and life insurance companies can use it in underwriting and pricing products. The rest of this manuscript is organized as follows. Section 2 provides the description of proposed method, as well as related references. The computer simulation is given in Section 3, with the parameters of mortality improvement and select effect from the largest insurance companies, Cathay Life Insurance Company ltd. (CLI), in Taiwan. Empirical study is in Section 4 and the data are also from Cathay. The results from simulation and empirical study support the

[2] Mortality improvement is an important factor for modelling mortality rates today but it was not in the late 1990's.

use of proposed approach and it can provide stable and accurate estimates of parameters and mortality rates.

2. Methodology

As mentioned in the previous section, the size and the length of select effect vary a lot, depending on the merit of insured populations and insurance products. For example, the select period is 5 years for smokers (SCOR, 2016) in term life products and the size of select effect is more than 60% (i.e., mortality rates 40% or less than that of standard group). The results of estimation for the size and length of select effect should not be influenced by the estimation methods, the size and structure of insured population, or other factors (e.g., lapse rate, underwriting). Intuitively, the grand averages of mortality (& incidence) rates of different policy years can be used to estimate the size and length of select effect.

However, the mortality improvement would distort the estimation of select effect and this is the reason why we need to use the Lee-Carter (or LC) model to model the mortality reduction. The LC model (Lee and Carter, 1992) is a popular mortality model and it has been used in estimating and forecasting mortality years for more than 20 years. The LC model assumes that

$$\log(m_{xt}) = \alpha_x + \beta_x \kappa_t + \varepsilon_{xt} \tag{1}$$

where m_{xt} is the central mortality rate for age x and time t, and the error ε_{xt} is assumed to be normally distributed. It is like fitting a group of linear regression equations simultaneously with same predictor (i.e. time), and each regression equation has its own slope and intercept, which are constant of time. The slope of the regression equation can be interpreted as mortality improvement over time for each age, since the time variable κ_t is usually a linear function of time.

We propose adding the select effect to the LC model, or

$$\log(m_{xts}) = \alpha_x + \beta_x \kappa_t + C_{xs} I\{x \le x_s\} + \varepsilon_{xts} \tag{2}$$

where m_{xts} is the central mortality rate at age *x*, time *t*, and policy year *s*. Also, C_{xs} is the size of select effect at age *x* and policy year *s*, and x_s is the length of select period. In other words, the proposed model has 3 coordinates (age, time, policy year), similar to that of the cohort LC model (Renshaw and Haberman, 2006). However, the proposed approach does not have the problem of linear dependency, or time = cohort + age, and the estimation process would be straightforward. The two-stage estimation can be used for the proposed approach, like most of the modified LC models. Also, in this study, we assume that the parameters of select effect do not change with time.

We first obtain the parameter estimates $\hat{\alpha}_x$, $\hat{\beta}_x$, and $\hat{\kappa}_t$ via the SVD (Singular Value Decomposition), and then the select effect (size C_{xs} and length x_s). Intuitively, if the proposed model in (2) is true, the difference between $\log(m_{xts})$ and $\hat{\alpha}_x + \hat{\beta}_x \hat{\kappa}_t$ can be used to estimate the select effect. However, as mentioned earlier, the mortality improvement and select effect are confounded to each other, and the initial estimates $\hat{\alpha}_x$, $\hat{\beta}_x$, and $\hat{\kappa}_t$ would be biased as well. Thus, we can use the difference between $\log(m_{xts})$ and $C_{xs} I\{x \le x_s\}$ to revise the parameter estimates $\hat{\alpha}_x$, $\hat{\beta}_x$, and $\hat{\kappa}_t$. In other words, the two-stage estimation of is done recursively until all parameters' estimates converge. The criterion of convergence can be chosen as the difference of estimates between two iterations smaller than a selected threshold, which is usually set as 10^{-4} or 10^{-6}. The estimation process can be summarized as follows.

Step 0. Let $\log(m_{xt})^*$ be the average of $\log(m_{xts})$ over all policy years.

Step 1. Apply SVD to $\log(m_{xt})^*$, obtaining estimates $\hat{\alpha}_x$, $\hat{\beta}_x$, and $\hat{\kappa}_t$.

Step 2. Compute the difference between $\log(m_{xts})$ and $\hat{\alpha}_x + \hat{\beta}_x \hat{\kappa}_t$, and define the residuals of select effect $e_{xts} = \log(m_{xts}) - \hat{\alpha}_x - \hat{\beta}_x \hat{\kappa}_t$. Then, let the estimate of select effect $\hat{C}_{xs}$ equal the average of e_{xts} over *t*.

Step 3. Apply the parameters' estimate to obtain the estimate of central death rates

$$\log(\hat{m}_{xts}) = \hat{\alpha}_x + \hat{\beta}_x \hat{\kappa}_t + \hat{C}_{xs} I\{x \le \hat{x}_s\}.$$

Step 4. Let $\log(m_{xt})^*$ be the average of $\log(\hat{m}_{xts})$ over all policy years.

Step 5. Apply SVD to $\log(m_{xt})^*$, obtaining estimates $\hat{\alpha}_x$, $\hat{\beta}_x$, and $\hat{\kappa}_t$.

Step 6. Compute the difference between $\log(m_{xts})$ and $\hat{\alpha}_x + \hat{\beta}_x \hat{\kappa}_t$, and define the residuals of select effect $e_{xts} = \log(m_{xts}) - \hat{\alpha}_x - \hat{\beta}_x \hat{\kappa}_t$. Then, let the estimate of select effect $\hat{C}_{xs}$ equal the average of e_{xts} over *t*.

Step 7. Repeat Steps 3 to 6 until the differences of parameters' estimates between two consecutive iterations are smaller than a selected threshold.

We will use computer simulation and empirical data to evaluate the proposed model. First, in the next section, we use computer simulation to check the two-stage estimation process and verify if it can provide unbiased and stable estimates of mortality rates. Since the preceding iteration process is simple and easy to use, usually it would converge in a few seconds. We will compare the parameters' estimates between consecutive iterations.

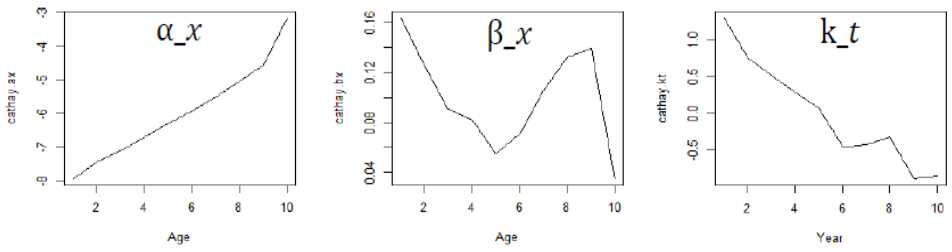

Figure 1. Parameters of the LC model (Taiwan Data)

3. Computer Simulation

For the simulation study, we need two sets of parameters: one for the mortality reduction (i.e., parameters in the LC model) and the other for the select effect. The parameters of mortality reduction are from plugging into the Taiwan mortality data

(2005-2014) and suppose that the mortality rates follow the Lee-Carter model and the population structure is the same as that in Taiwan. We treat the estimates of α_x, β_x, and κ_t as the true values (Figure 1). The parameters of select effect partly refer the experienced values of CLI, with some modifications making the size of select effect a smooth function.

Basically, we assume that the size of select period is a linear function of policy years and Table 1 shows the values of size of select effect, given 10 age groups and 10 groups of policy years. The mortality rates of each policy years need to multiply the values in Table 1. For example, the value 0.8 for ages 0~29 at policy year 2 indicates that the mortality rate at this age and policy combination is 80% of the standard rate. Also, the length of select effect is a non-decreasing function of age, with 3-year to 8-year select period. Note that the simulation setting suggests that the size of select effect is much larger than the mortality improvement. The age group 0~29 has the largest annual mortality reduction (about 3%) but it is still smaller than the select effect between two policy years (e.g., policy years 1 to 2).

Table 1. The Size of Select Effect (Simulation)

Ages	Policy Year									
	1	2	3	4	5	6	7	8	9	10+
0~29	0.7	0.8	0.9	1	1	1	1	1	1	1
30~34	0.6	0.7	0.8	0.9	1	1	1	1	1	1
35~39	0.6	0.7	0.8	0.9	1	1	1	1	1	1
40~44	0.5	0.6	0.7	0.8	0.9	1	1	1	1	1
45~49	0.5	0.6	0.7	0.8	0.9	1	1	1	1	1
50~54	0.4	0.5	0.6	0.7	0.8	0.9	1	1	1	1
55~59	0.4	0.5	0.6	0.7	0.8	0.9	1	1	1	1
60~64	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1	1
65~69	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1	1
70+	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1

Note that we apply the real exposures of age and policy combination from the CLI (2005-2014), in order to make the simulation close to the reality. There are

approximately 23 million people in Taiwan and 2/3 of them purchase life insurance products (Yue and Huang, 2011). The CLI is the largest life insurance company in Taiwan and about 8 million people purchased life insurance policies from the CLI. It is believed that the LC model would have unstable parameter estimates when the population size is small, especially for the case when the size is not larger than 200,000 (Yue et al., 2017). Since the exposures from the CLI are fairly large, we use the singular value decomposition (SVD) to obtain the parameters' estimates of the LC model. Also, we generate the numbers of deaths via Poisson distribution and then dividing them to the exposures to acquire the simulated mortality rates.

In addition to the proposed model, we also consider a modified version of the LC model to estimate the select effect (or reduced form in Equation (2)),

$$\log(m_{xs}) = \alpha_x^* + \beta_x^* \kappa_s^* + \varepsilon_{xs}^* \tag{3}$$

where m_{xs} is the central mortality rate for age x and policy year t. The reduced model in (3) only includes the factor of policy year. We should check if the reduced model can provide acceptable estimates of select effect and evaluate whether it is necessary to include both the mortality reduction and select effect in the mortality model.

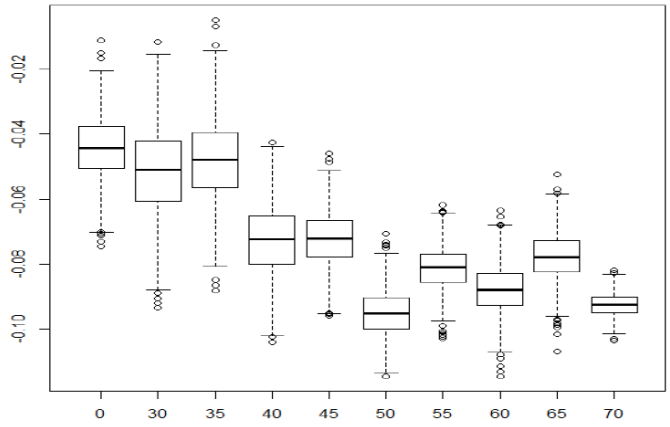

Figure 2. Bias of α_x Estimate for the Full Model (1st Iteration)

The computer simulation is repeated 1,000 times and the estimates of all parameters are recorded. We first evaluate the full model in Equation (2) and use the

estimates of α_x as a demonstration. Figure 2 shows the bias of α_x estimate for the first iteration and apparently it is under-biased. The estimates of parameters β_x, κ_t, and select effect are also under-biased for the first iteration. Note that the estimates of α_x, β_x, and κ_t at the first iteration is exactly the same as those of the LC model, and this confirms that the mortality improvement and the select effect are confounded with each other. On the other hand, the estimates of all parameters improve with the number of iterations and they become very stable at the 8th iteration. Appendix A shows the bias of estimates for parameters α_x, β_x, and κ_t and they are close to 0 at the 8th iteration.

Table 2. The Estimates of Select Effect (8th Iteration, Full Model)

Ages	Policy Year									
	1	2	3	4	5	6	7	8	9	10+
0~29	0.700	0.801	0.899	1	1	1	1	1	1	1
30~34	0.601	0.695	0.802	0.902	1	1	1	1	1	1
35~39	0.601	0.702	0.798	0.901	1	1	1	1	1	1
40~44	0.499	0.601	0.701	0.799	0.902	1	1	1	1	1
45~49	0.500	0.601	0.701	0.801	0.899	1	1	1	1	1
50~54	0.401	0.500	0.601	0.700	0.800	0.901	1	1	1	1
55~59	0.399	0.500	0.599	0.699	0.800	0.900	1	1	1	1
60~64	0.299	0.401	0.501	0.601	0.701	0.800	0.899	1	1	1
65~69	0.300	0.402	0.501	0.600	0.698	0.801	0.901	1	1	1
70+	0.200	0.300	0.401	0.500	0.600	0.700	0.801	0.899	1	1

The estimates of select effect behave similarly and the bias are very smaller for all combinations of age and policy year at the 8th iteration, as shown in Table 2. It seems that the proposed estimation process does provide stable estimates to the parameters for the full model in Equation (2). The evaluation of reduced model in Equation (3) can be conducted in a similar way and we only show the estimates of select effect for comparison. Table 3 lists the estimates of reduced model and

obviously they are under-biased. It seems that the reduced model, i.e., without considering the mortality reduction, would over-estimate the mortality rates inside the select period.

Table 3. The Estimates of Select Effect (Reduced Model)

Ages	Policy Year									
	1	2	3	4	5	6	7	8	9	10+
0~29	0.703	0.803	0.921	1	1	1	1	1	1	1
30~34	0.611	0.704	0.819	0.925	1	1	1	1	1	1
35~39	0.612	0.711	0.814	0.919	1	1	1	1	1	1
40~44	0.525	0.629	0.737	0.840	0.951	1	1	1	1	1
45~49	0.526	0.632	0.739	0.843	0.948	1	1	1	1	1
50~54	0.437	0.545	0.657	0.765	0.876	0.988	1	1	1	1
55~59	0.424	0.531	0.642	0.751	0.866	0.977	1	1	1	1
60~64	0.328	0.439	0.552	0.666	0.785	0.909	1	1	1	1
65~69	0.320	0.426	0.539	0.660	0.772	0.896	1	1	1	1
70+	0.236	0.354	0.472	0.592	0.711	0.832	0.956	1	1	1

We can compare the estimates of length of select periods for the full and reduced models as well. Table 4 shows the true and estimated length of select period for two mortality models. The estimated length of select periods are obtained from the Monte Carlo p-value, depending on if the estimated size of select effect is significantly different from 1. The full model provides accurate estimates of the length of select period but the reduced model under-estimates not only the size of select period but also the length of select period. It seems that the select effect is confounded with the mortality reduction and we should include both factors in modelling mortality rates.

Table 4. The Estimates of Length of Select Effect (Simulation)

Ages	0~29	30~34	35~39	40~44	45~49	50~54	55~59	60~64	65~69	70+
True	3	4	4	5	5	6	6	7	7	8
Full	3	4	4	5	5	6	6	7	7	8
Reduced	3	4	4	4	5	5	5	6	6	6

Note: The cells with gray background are under-estimated.

MAPE (Mean Absolute Percentage Error) of mortality rates can also be used to evaluate the proposed model, where the MAPE is defined as

$$\text{MAPE} = \text{Average of } \left(\sum_{x,t,s} \frac{(\hat{q}_{xts} - q_{xts})^2}{q_{xts}} \right) \times 100\%. \quad (4)$$

Table 5 shows the MAPEs of the LC model, full model in (2), and reduced model in (3). Again, the full model has the smallest error. However, surprisingly, the LC model fits fairly well, although the true model is the LC model plus the select effect. Probably the sizes of select effect are linear functions of policy year and the select effects are larger than the mortality reductions are the main causes, which can also explain why the MAPE of reduced model is very small as well. We will continue the empirical study, using the data from the CLI, in the next section.

Table 5. MAPE of Mortality Estimates (Simulation)

	LC model	Full model	Reduced model
MAPE	5.47%	1.92%	2.24%

4. Empirical Study

For the empirical study, we adapt the same setting in the previous section: 10 age groups and 10 groups of policy years for the study period 2005-2014, using the actual data (including age-specific exposures & numbers of deaths) from the CLI. These data are from the policies of whole life and term life for more than 10 years. Note that the claim system of the CLI was under a major reorganization in the early 2000's and the sales of life insurance policy also increased significantly at the turn of 21st century. Thus, we only choose the experienced data of the last 10 years to avoid the problem of data inhomogeneity.

Again, we use the CLI data to evaluate the fitting of mortality rates for the LC, full, and reduced models. There is a difference in determining the size and length of select effect. In the previous section, we can use t-test or similar test to decide the select effect based on 1,000 simulation runs, i.e., using the variance from 1,000

replications for hypothesis testing, but no data replications in the empirical study. Of course, we can use bootstrap simulation to estimate the variances of estimates for the select effect but it requires quite a lot of computation time. Instead, we suggest using the value 0.95 as the threshold, based on the experience from simulation study. If the estimated values of sizes of select effect are smaller (or larger) than 0.95, then there are (or there are no) select effects.

First, we compare the estimation results of select effect for the full and reduced models. It seems that the estimation results are pretty similar to those in the simulation study and, in general, the reduced model has shorter and smaller estimates of select effects. The estimated lengths of select effect are shown in Table 6. The full model also has longer select effects than the reduced model in all age groups. Similarly, the estimated sizes of full model are larger than those of reduced model, especially for the younger ages and middle age groups, around ages 50~64.

Table 6. The Estimates of Length of Select Effect (Empirical)

Ages	0~29	30~34	35~39	40~44	45~49	50~54	55~59	60~64	65~69	70+
Full	9	8	6	7	8	9	9	9	9	9
Reduced	4	6	5	6	6	6	6	6	7	8

Next, we compare the mortality fitting of three mortality models and the MAPE is also used in the empirical study. The full model still has the smallest MAPE and all three models have satisfactory estimation results. Even though there are select effects, the LC model is still a fine candidate of mortality model and it seems that the trend of mortality improvement is more obvious than the select effect. On the other hand, the empirical results also suggest that the size of select effect is much larger than the mortality improvement, similar to the setting in the simulation study. Thus, the reduced model also has smaller MAPE than that of the LC model as well.

Table 7. MAPE of Mortality Estimates (Empirical)

	LC model	Full model	Reduced model
MAPE	7.44%	2.49%	3.64%

5. Conclusion and Discussions

Mortality improvement has been one of the major considerations, as well as the factors of age and gender, in modelling mortality rates since the life prolonging becomes our consensus. Thus, stochastic mortality models are a popular choice in pricing life insurance and annuity products although scholars have different opinions about the trend and speed of mortality reduction. However, past studies found that the estimates of mortality improvement can be influenced by other factors (and vice versa), and the cohort effect is one of them (Renshaw and Haberman, 2006). The select period is another possible factor which can be confounded with the mortality improvement, but only a few studies focused on handling both the effects of mortality improvement and select period.

We proposed a mortality model and its estimation method for handling both the effects of mortality improvement and select period. We used computer simulation to confirm that the proposed approach and its estimation method does provide stable and reliable estimates. On the other hand, the regular LC model would have biased estimates on the parameters of mortality reduction. Similarly, without including the mortality improvement (i.e., the reduced model in Equation (3)) would also produce biased estimates. The results of empirical study support the proposed approach as well. It appears that both the mortality improvement and the select effect exist and they should be included in the mortality model.

The proposed estimation is via a two-stage iteration process and it becomes fairly stable after the 4^{th} iteration and converges before the 10^{th} iteration (i.e., estimates between two consecutive iterations $|\hat{\theta}_{i+1} - \hat{\theta}_i| < 10^{-6}$ for $i \geq 10$). Interestingly, the convergence rate is not the same for all parameters and it seems that the estimates of β_x and κ_t would be more sensitive. However, this is not the case, based on the simulation and empirical studies. Although the role of parameters α_x and select effects is like intercept and dummy variables, but they requires more iteration steps to become stable, especially for the select effects. This quite contradicts to our intuition

but it matches to our research experience of the LC model when the population size is small (Yue and Wang, 2017).

Although the mortality reduction and select effect are easily mixed with each other, they do not produce the problem of linear dependency, like the cohort modification of LC model (Renshaw and Haberman, 2006) and Age-Period-Cohort model (Wang and Yue, 2015). Perhaps this is the reason why the parameters of proposed approach fairly quickly. If we want to consider more factors in the mortality model, then we need to pay more attention to the estimation methods. We expect the iteration process would be unstable when there are possibilities of linear dependency or similar problems. The situation can be even more tricky if the exposures (or size) of target population is small (Yue and Wang, 2017) and we may need techniques in variance reduction to acquire stable parameter estimates.

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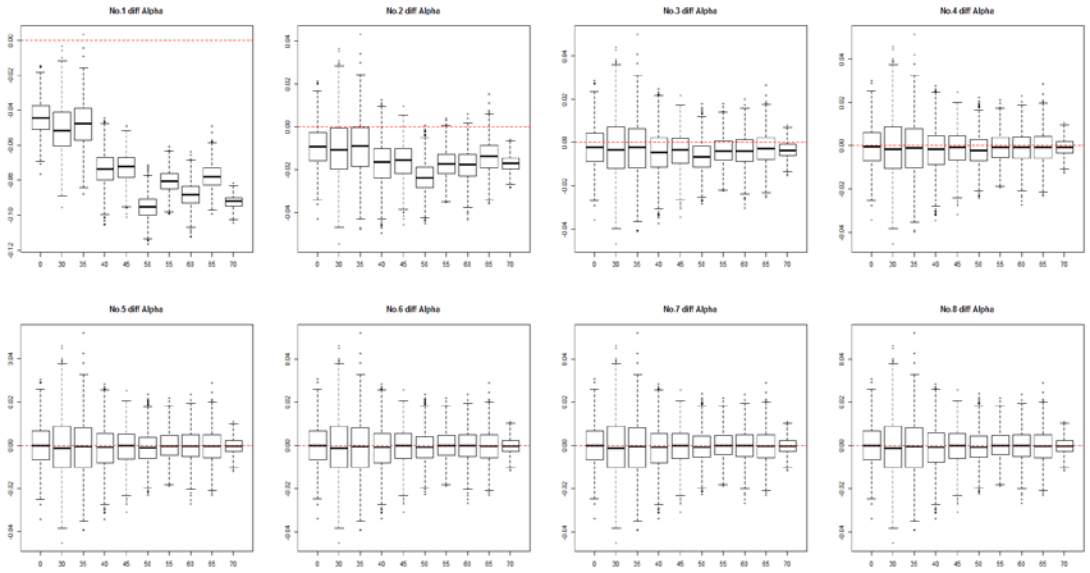
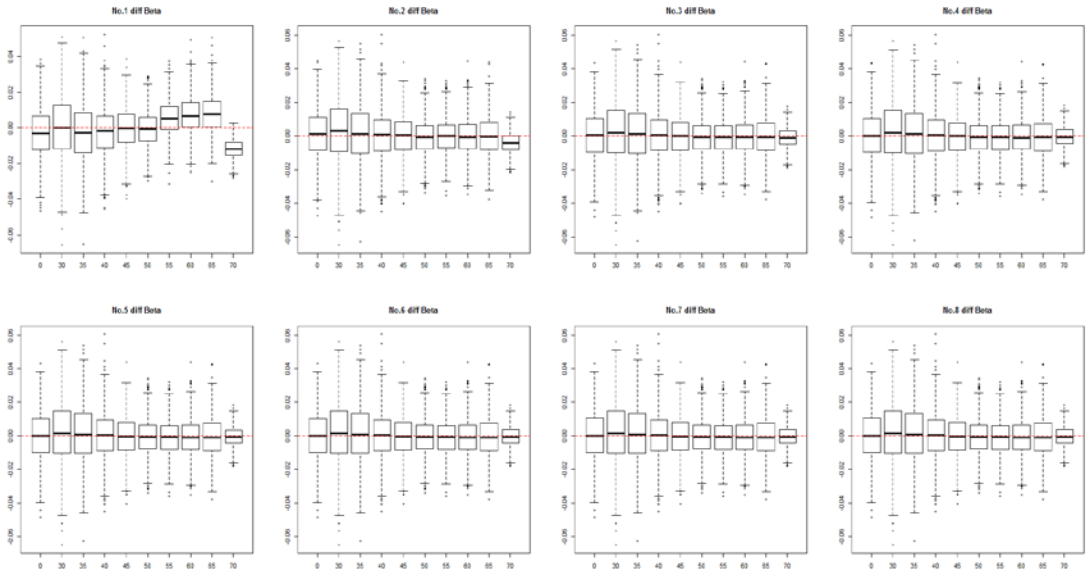
Appendix A. Bias of Estimates for Parameters α_x, β_x, and κ_t

1. Parameter α_x



2. Parameter β_x



3. Parameter κ_t