Longevity risk of Japanese population - Past,Present and Future -

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Abstract

In the next 50 years, beyond the whole history of mankind, we might encounter an unprecedented super-aged society. Three out of ten females in her 30's at present may live to 100 years old. A social structure of Japan will be obliged to change fundamentally. This paper aims at examining this assertion by using the typical stochastic mortality models which has rapidly developed in recent years with the past 100-years mortality data.

Key words: Japanese population forecast Stochastic mortality model GAPC family

1 Introduction

In the next 50 years, beyond the whole history of mankind, we might encounter an unprecedented super-aged society. Three out of ten females in her 30's at present may live to 100 years old. A social structure of Japan will be obliged to change fundamentally. This paper aims at examining this assertion by using the typical stochastic mortality models which has rapidly developed in recent years with the past 100-years mortality data.

1.1 Data on Japanese Mortality and Population

Complete Experience Life Tables are available from National Census basically investigated every 5 years since 1981 in Meiji Era. Population statistics are obtainable from Japanese Government published data since 1891, except fiscal 1920 before and 1944-1946 during the WW2. "Report on the estimated population of Japan in the future"[2] published in 2012 from National Institute of Population and Social Security Research (IPSS in abbreviation)contains future population predicts and life tables. In particular, the population of older-olds over aged 75, super-seniors over 85, and centenarians will grow rapidly and remarkably.

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1.1.1 Experience cohort population（1880-2105）

Tracking the birth cohorts between 1900 and 1980, more recent cohorts live longer.

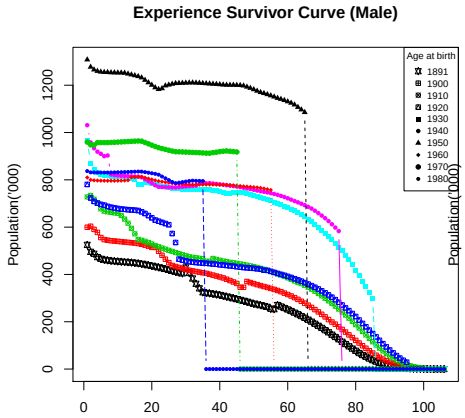

Figure 1: Cohort Survivorship up to 2060 (Male)

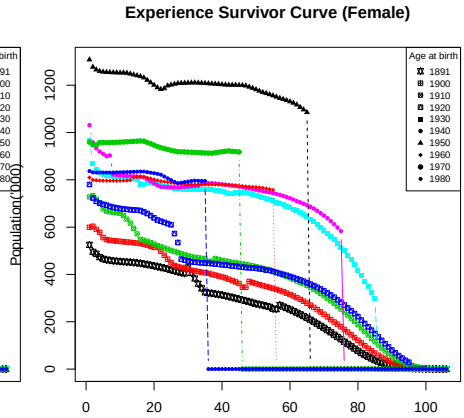

Figure 2: Cohort Survivorship up to 2060 (Female)

1.1.2 Trends in Japanese Mortality rates（1880-2105）

Mortality rates have been and improving year by year and will be improving up to 2060 in IPSS's forecast. After 2060, mortality rates are the same as the 2060 life table. So, expected life spans in generations after 1960 are possiblly underextimated. Mortaity model called TVF(Tangent Vector Field) used in IPSS's forecast was developed by Ishii[21].

1.1.3 2012 IPSS's mortality forecast

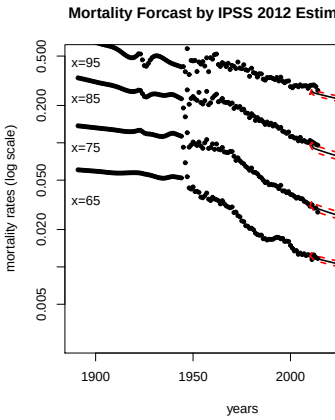

Figure 3: IPSS's mortality forecast (Male)

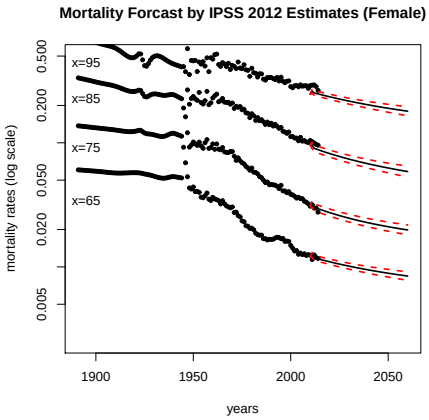

Figure 4: IPSS's mortality forecast (Female)

1.1.4 Survivorship probabilities up to 90 and 100

Accrding to IPSS's forecast, survivorship probabilities up to 90 and 100 are shown below. Female cohorts after 1970 birth year live to 100 with gradually increasing probablity near to 20 %, under 20 % in middle scenarios.Male cohorts with probability under 10 %.

Birth Year	sex/age('15)	Surv.90	Surv.100	sex/age('15)	Surv.90	Surv.100
1930	Male85	55.8 [52.4,59.1]	5.3 [4.3,6.4]	Female85	71.7[68.6,74.6]	14.0 [11.8,16.4]
1940	Male75	35.7 [32.8,38.5]	4.1 [3.2,5.1]	Female75	58.9[55.9,61.8]	13.5 [11.3,16.0]
1950	Male65	35.4 [32.4,38.5]	4.7 [3.7,6.0]	Female65	62.7[59.4,65.7]	16.3 [13.5,19.5]
1960	Male55	33.7 [30.6,36.6]	5.1 [3.9,6.5]	Female55	59.9[56.7,63.0]	17.4 [14.2,20.8]
1970	Male45	34.9 [31.6,38.1]	5.5 [4.2,7.1]	Female45	60.7[57.4,63.9]	18.2 [14.9,21.9]
1980	Male35	35.7 [32.3,39.2]	5.7 [4.3,7.3]	Female35	61.2[57.8,64.4]	18.4 [15.0,22.1]

Table 1: Cohort population forecast based on 2012 IPSS Report

(Remark) [a,b] indicates 5 % (a) and 95 % (b) confidence interval.

These results depend on the mortality estimation presition in superseniors or centenarians. Other approrches might predict more pessimistic future outcomes. In this paper, we try to investigate this problem by applying several recently-developed stochastic mortality models.

2 Preceding researches

Many various researches are conducted by government population bureau, population economists and statisticians, public health academicians or actuaries.

Public Population Reports Every 5 years, IPSS publish Population forecast report after National Census investigation years.

（4times :1997,2002,2007,2012 and 2017 in the next).

Statistics Japanese Statistical Associations 75's memorial book[5] deals with Population Statistics.Shibuya et al[4]'s "extreme value theory" applications.

Public Health Many researches published from medical science perspectives.（Horiuchi[5],Freeman et al[8],Shimizu et al[9] etc.）

Actuarial Science US Society of Actuaries has formed symposiums named "Living to 100" every 3 years from 2002(the most recent 2014). UK actuaries developed many stochastic mortality models, and apply them to mortality derivatives or risk management.

3 Models

Various types of stochastic mortality model have been earnestly studied derived Lee-Carter(1992)'s since 2000s particularly in UK. Frequently cited are Cairns,Blake and Dowd(2006),Renshaw-Haberman(2003,2006),Cairns et al(2009), that are generically called GAPC (Generalized Age-Periodic Cohort) family. In such models, future mortality rates are assumed to be explainable by age, periodic and cohort effect. In this study, we selected major 7 models, namely LC,CBD,CBD2,APC,RH,M7,PLAT. Though 3 models (LC,CBD,CBD2) only contain age and period as explanetory variables, the remaining 4 models contain cohort effect. (see Hunt-Blake[3])

GAPC family has the model structure similar to GLM (Generalized Linear Model).

1. (ramdom component) $D_{xt} \sim Poisson(E^c_{xt}\mu_{xt}) or \sim Binomial(E^0_{xt}, q_{xt})$.
2. (systematic component)

$$\eta_{xt} = \alpha_x + \sum_{i=1}^{N} \beta_x^{(i)} \kappa_t^{(i)} + \beta_x^{(0)} \gamma_{(t-x)} \tag{1}$$

3. (link function: g)

$$g(\mathbb{E}\left[\frac{D_{xt}}{E_{xt}}\right]) = \eta_{xt}$$

4. (a set of parameter constraints) $\theta = (\alpha_x, \beta_x^{(i)}, \gamma_{t-x})$ should be uniquely determined by some constraints.

Typical models are well explainable by *systematic component* η_{xt}.

Model	Predictor
LC	$\eta_{xt} = \alpha_x + \beta_x^{(1)}\kappa_t^{(1)}$
CBD	$\eta_{xt} = \kappa_t^{(1)} + (x - \bar{x})\kappa_t^{(2)}$
APC	$\eta_{xt} = \alpha_x + \kappa_t^{(1)} + \gamma_{(t-x)}$
RH	$\eta_{xt} = \alpha_x + \beta_x^{(1)}\kappa_t^{(1)} + \gamma_{(t-x)}$
M7	$\eta_{xt} = \kappa_t^{(1)} + (x - \bar{x})\kappa_t^{(2)} + ((x - \bar{x})^2 - \hat{\sigma_x^2})\kappa_t^{(3)} + \gamma_{(t-x)}$
PLAT	$\eta_{xt} = \alpha_x + \kappa_t^{(1)} + (x - \bar{x})\kappa_t^{(2)} + \gamma_{(t-x)}$

4 Data and models used

Mortality rates are often mathematically expressed as a function of ages in history.

Deterministic models coined by De Moivre(1729),Gomperts(1825) etc. are used to compile life tables even now.

Lee-Carter(1992), as a relational statistical model,is the first successful mortality forcast model, which led to stocastic mortality models. The next figure shows a few list of famous mortality models.

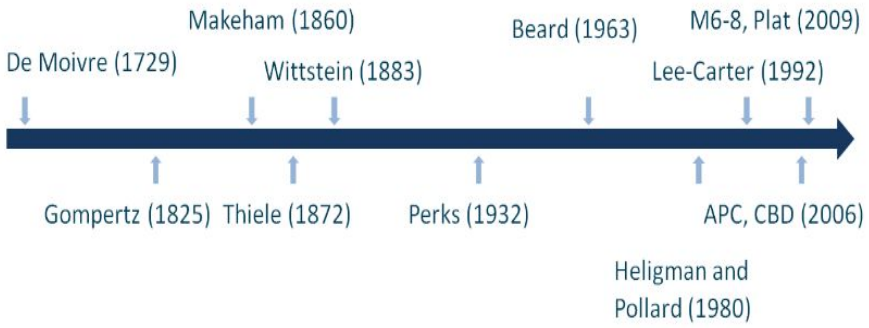

Figure 5: Development in mortality modeling (Soource: R in Insurance: Olga Mierzwa & Frankie Gregorkiewicz)

Mortality improvement factors(κ) are often assumed to follow multivariate random walks with drifts as in Cairns et al.(2006, 2011),Haberman and Renshaw(2011) etc.

$$\kappa_t = \delta + \kappa_{t-1} + \xi_t,\ \kappa_t = \begin{pmatrix} \kappa_1 \\ \vdots \\ \kappa_N \end{pmatrix},\ \xi_t \sim N(0, \Sigma)$$

Here,δ is N-dimensional vector,Σ are ξ's N-variance covariance matrix.

Model	Frequently used time series model
APC	ARIMA(1,1,0)
RH	ARIMA(1,1,0) with drift
M7	ARIMA(2,0,0) with non-zero intercept
PLAT	ARIMA(2,0,0) with non-zero intercept

These models have their own characteristics, shown in the table below. Models can be compared and evaluated by several characters, such as Popularity, Complexity ,Quality ,Objectivity etc.

	Generalised Age-Period-Cohort Stochastic Mortality Models							
Characteristic	LC	APC	CBD	RH	M6	M7	M8	Plat
General Shape of Mortality α_x	Y	Y	N	Y	N	N	N	Y
Cohort Effect γ_{t-x}	N	Y	N	Y	Y	Y	Y	Y
Mortality Trend κ_t	Y	Y	Y	Y	Y	Y	Y	Y
Number of Age-Period Terms N	1	1	2	1	2	3	2	2
Age Modulating Terms β_x	Non-Parametric Need to be estimated	Non-Parametric Static	Mixed	Non-Parametric Need to be estimated	Mixed	Mixed	Mixed	Mixed

Requirement	LC	APC	CBD	RH	M6	M7	M8	Plat
Popularity	1	3	2	3	3	3	3	4
Complexity	1	3	2	2	3	4	3	4
Quality	3	2	3	2	2	2	2	3
Objectivity	2	2	2	2	2	2	3	2

1 2 2 3 4
Very Good Bad

Figure 6: Comparisons in mortality models (R in Insurance: Olga Mierzwa & Frankie Gregorkiewicz)

5 Model selection

5.1 Model selection by AIC,BIC

For whole age interval (0-110), the most fitted model selected by AIC,BIC is RH for males and females ,followed by PLAT,M7.

	LC	CBD	APC	M7	PLAT	RH
Male(AIC)	1546907	53584801	1654125	15349103	945384.4	612159.7
Male(BIC)	1549492	53586665	1657583	15353590	949758.7	616451.4
Female(AIC)	1079986	47099955	1303467	13811130	654768.4	526920.2
Female(BIC)	1082575	47101821	1306928	13815621	659146.8	531216.2

For age interval 35-89, RH,PLAT,CBD2 are selected, and for over 90,APC,CBD are relatively good.

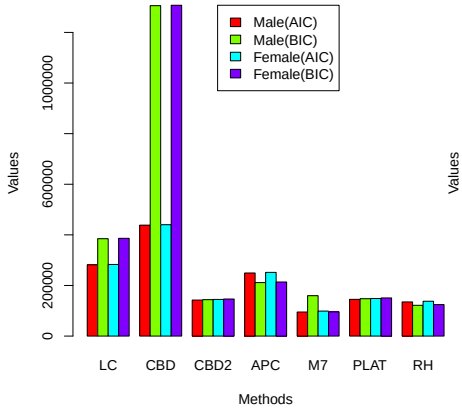

Figure 7: model goodness-fit test for aged 35-89

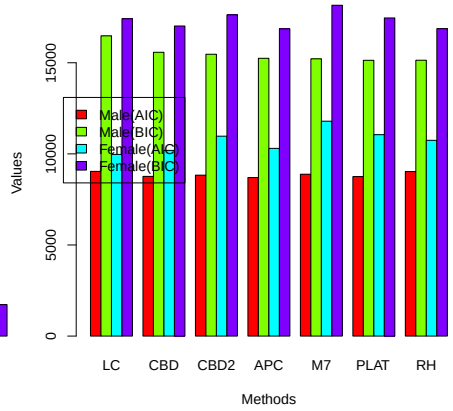

Figure 8: model goodness-fit test for over aged 90

5.1.1 Estimation errors in cohort effect（Male : APC,M7）

Cohort effects are sometimes difficult to estimate. We can observe pathological phenonmena in the figures below.

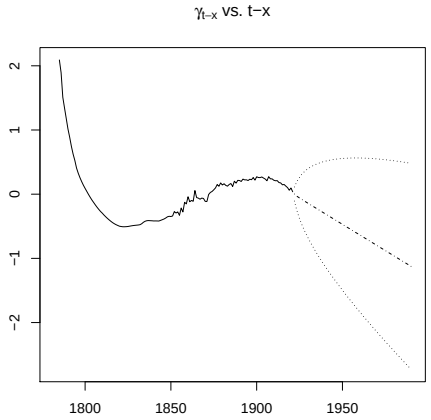

Figure 9: Cohort effect in APC(γ_{t-x})

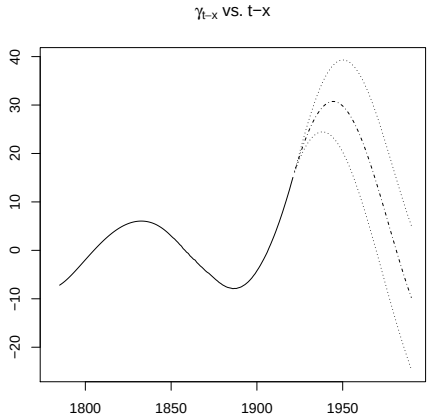

Figure 10: Cohort effect in M7(γ_{t-x})

5.1.2 Comparison in mortality forecasts by various models for aged 85

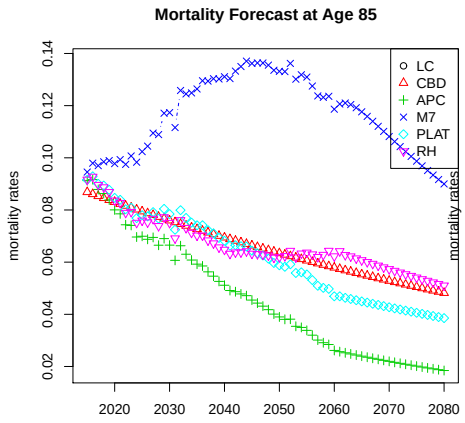

Figure 11: Male:LC,CBD,APC,PLAT,RH

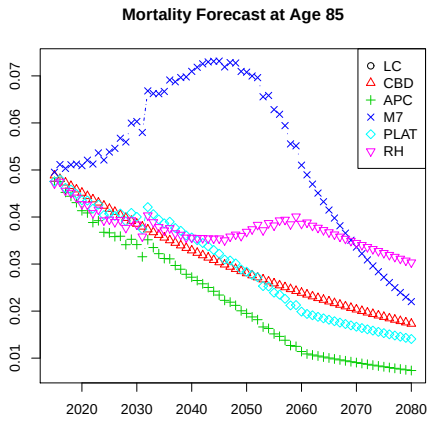

Figure 12: Female:LC,CBD,APC,PLAT,RH

5.2 Goodness-of-fit

Goodness-of-fit test investigate the fitting errors between theoretical values and sample data. Usually, model selection by some Information Criteria like AIC (Akaike's IC) or BIC (Bayesian IC) is used.

5.2.1 Goodness-of-fit (Male)

In males aged over 90, dispersion in mortality rates is very large.

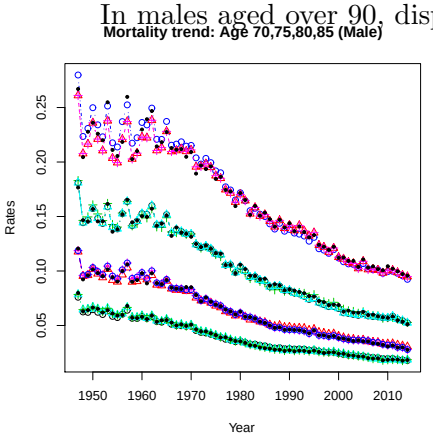

Figure 13: Ages:70,75,80,85, Models: RH,CBD2,M7 (Male)

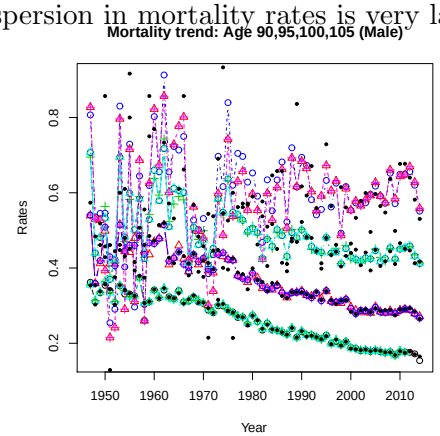

Figure 14: Ages : 90,95,100,105, Model: RH,CBD2,M7 (Male)

5.2.2 Goodness-of-fit (Female)

The same in females over 90.

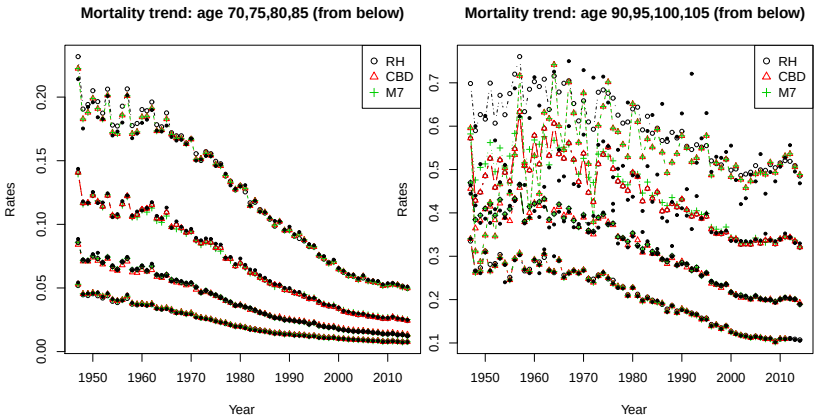

Figure 15: Ages : 70,75,80,85,Model: RH,CBD2,M7 (Female)

Figure 16: Ages : 90,95,100,105,Model: RH,CBD2,M7 (Female)

5.3 Backtest

- We predict the mortality rates for the period 1981-2014 by use of the sample mortality data from 1947 to 1980, and calculate prediction errors.
- Model comparison: LC,CBD,APC,PLAT,M7,RH; Population:Males and females, aged 90-109,Prediction performance: RSME (Root Mean Square Errors)
- The best model selected is CBD for both sexes, nearly followed by APC.
- We have selected both CBD and APC for over 90 zone.

	LC	CBD	APC	PLAT	M7	RH
Male	0.2350	0.1828	0.1875	0.3500	0.5944	0.1902
Female	0.1832	0.1309	0.1326	0.2478	0.6409	0.2874

Table 2: Standard deviations of Mean Square Errors

5.3.1 Backtest (Male)

We found the theoretical rates for males by CBD model have upward bias, though those by APC model have downward bias.

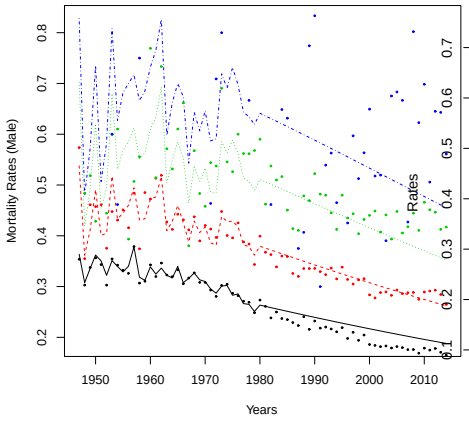

Figure 17: Model:CBD(Male), Ages : 90,95,100,105

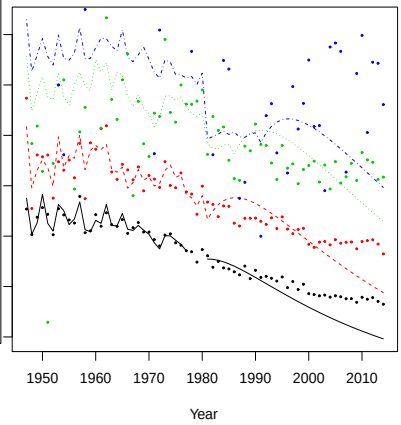

Figure 18: Model:APC (Male), Ages : 90,95,100,105

5.3.2 Backtest (Female)

We found the theoretical rates for males by CBD model have upward bias, though those by APC model have irregular movements.

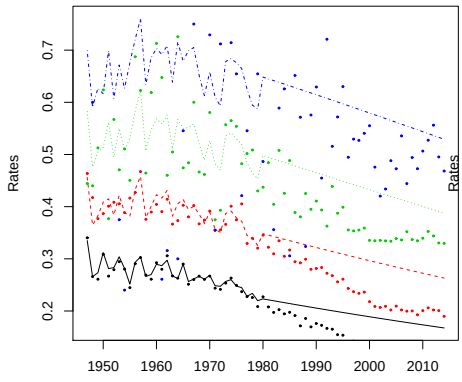

Figure 19: Model:CBD (Female), Ages : 90,95,100,105

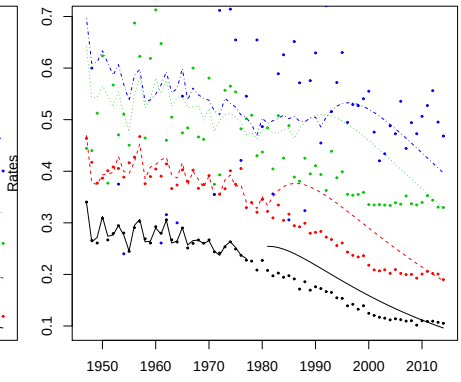

Figure 20: Model:APC (Female), Ages : 90,95,100,105

Based on the empirical investigation on Japanese mortality data, there is no model to fit all ages well from model selection criteria, AIC and BIC. In addition, we conducted backtesting to verify forcasting ability of each model.We found CBD2 best fitted to the mortality data in ages 35-89, and CBD or APC those in 90 and over.

Compared with the IPSS's published mortality forecasts, we pointed out the possibility that the aging problem is more serious than generally believed. In particular, the predicted distribution by CBD2+APC model that females born in 1980 survive to 100 is estimated with mean 30 %, and with median 45 %.

6 Cohort survivorsip forcast by stochastic models

Mortality forecast by CBD2 (Male) Mortality forecast by CBD2 for males is conducted in order to predict cohort survival probability.

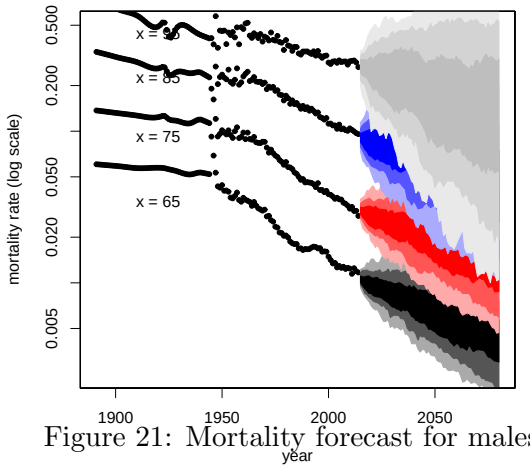

Figure 21: Mortality forecast for males aged 65,75,85,95

6.1 Cohort survivorsip forecast by hibrid CBD2-CBD model

Birth year 1970 cohort survivorsip forecasts by hibrid CBD2-CBD model for aged 90 males and females are 31 % and 65 % respectively, for aged 100 males and femeles 4 % and 15 % respectively. Birth year 1980 cohort for aged 90 males and females are 33 % and 68 %,for aged 100 males and femeles 5 % and 17 % respectively. The difference between males and females is very large.

Birth Year	sex/age('15)	Surv.90	Surv.100	sex/age('15)	Surv.90	Surv.100
1930	Male85	55.5[46.6,66.6]	5.0[0.1,21.3]	Female85	71.7[66.0,77.5]	12.7 [2.4,41.5]
1940	Male75	33.7[20.6,55.3]	3.6[0.0,26.6]	Female75	58.7[43.2,71.2]	11.6 [1.4,46.6]
1950	Male65	29.1[12.5,59.4]	3.5[0.0,37.4]	Female65	59.0[41.8,75.1]	13.0 [1.2,56.6]
1960	Male55	28.8[8.8,63.1]	3.8[0.0,43.0]	Female55	62.3[44.4,79.6]	14.4 [1.1,62.7]
1970	Male45	30.9 (38.0) [9.6,70.1]	4.1 (4.0) [0.0,53.1]	Female45	65.2 (67.4) [45.0,83.1]	15.0 (23.5) [0.6,69.3]
1980	Male35	32.6 (43.8) [9.6,76.0]	4.8 (7.3) [0.0,60.8]	Female35	67.8 (71.3) [44.5,87.1]	16.8 (31.0) [0.6,75.8]

(Remark) [a,b] indicates 5 % (a) and 95 % (b) confidence interval and (c) indicates the median value.

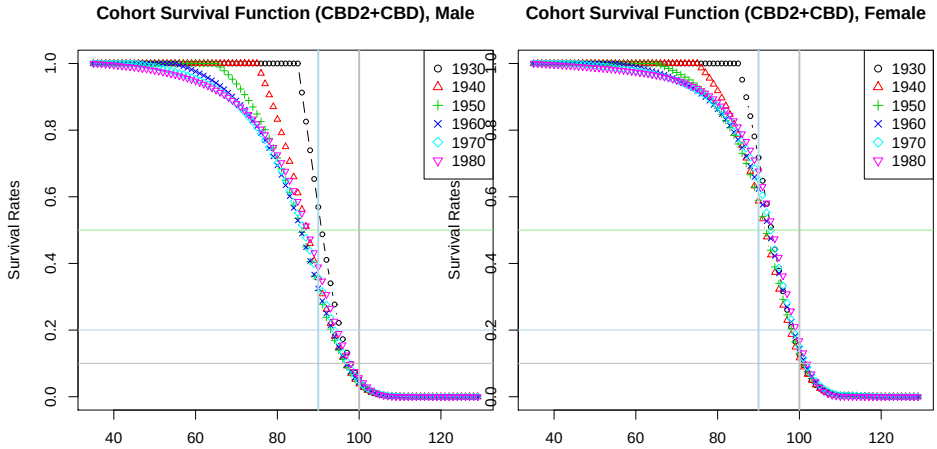

Figure 22: Survival Probability of Male aged 90 and 100 born in 1930-1980

Figure 23: Survival Probability of Female aged 90 and 100 born in 1930-1980

6.2 Cohort survivorsip forcast by hibrid CBD2-APC model

Birth year 1970 cohort survivorsip forecasts by hibrid CBD2-APC model for aged 90 males and females are 31 % and 65 % respectively, for aged 100 males and femeles 11 % and 29 % respectively. Birth year 1980 cohort for aged 90 males and females are 33 % and 68 %,for aged 100 males and femeles 12 % and 31 % respectively. The difference between males and females is very large and for aged 100, CBD2-APC model forecast high survival probability.

Birth Year	sex/age('15)	Surv.90	Surv.100	sex/age('15)	Surv.90	Surv.100
1930	Male85	55.5 [46.6,66.6]	7.0 [0.1,22.3]	Female85	71.7[66.0,77.5]	15.9[6.1,37.0]
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1970	Male45	30.9 (38.0) [9.6,70.1]	11.2 (25.9) [0.0,69.7]	Female45	65.2 (67.4) [45.0,83.1]	28.7 (36.9) [5.6,72.1]
1980	Male35	32.6 (43.8) [9.6,76.0]	12.2 (34.7) [0.0,75.9]	Female35	67.8 (71.3) [44.5,87.1]	30.6 (45.7) [2.9,80.7]

(Remark) [a,b] indicates 5 % (a) and 95 % (b) confidence interval and (c) indicates median.

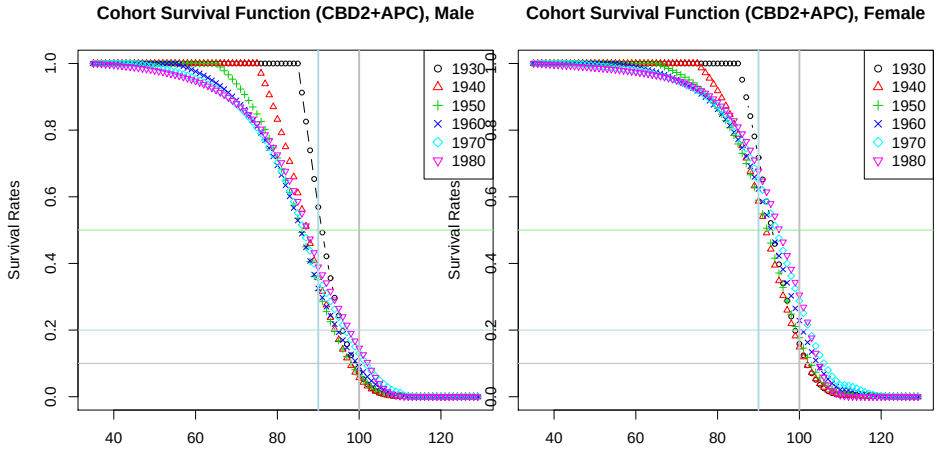

Figure 24: Survival Probability of Male aged 90 and 100 born in 1930-1980

Figure 25: Survival Probability of Female aged 90 and 100 born in 1930-1980

7 Results and Conclusions

Based on the empirical investigation on Japanese mortality data, there is no model to fit all ages well from model selection criteria, AIC and BIC. In addition, we conducted backtesting to verify forcasting ability of each model.We found CBD2 best fitted to the mortality data in ages 35-89, and CBD or APC those in 90 and over.

Compared with the IPSS's published mortality forecasts, we pointed out the possibility that the aging problem is more serious than generally believed. In particular, the predicted distribution by CBD2+APC model that females born in 1980 survive to 100 is estimated with mean 30 %, and with median 45 %.

However, over aged 90, sample mortality rates are extremely volatile, and difficult to estimate accurately. We need more efforts including alternative approaches, public health, required care staus or small area analysis.

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