Ambiguity Aversion and Insurance Demand When Indemnity is Uncertain

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Abstract: We analyze the demand for insurance under ambiguity aversion when indemnity is uncertain. We find that the effect of ambiguity aversion on the insurance demand may be positive or negative in general. We propose a sufficient condition for the increase in insurance demand. It is shown that ambiguity aversion affects the demand in three ways. The first effect is captured by absolute ambiguity aversion (AAA). When AAA is non-increasing, ambiguity aversion affects positively insurance demand. Second, ambiguity aversion increases the precautionary motive, leading to the increase in insurance demand. Lastly, ambiguity aversion negatively affects the demand for uncertain assets.

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I. Introduction

Most studies on the demand for insurance assume that indemnity is certain, as insurers are presumed to observe the loss and determined indemnity on the basis of the loss. In reality, however, indemnity is uncertain in several reasons. First, different claim adjusters may have different beliefs about the loss size. Although the claim adjusters can measure the loss accurately on average, they are exposed to the measurement errors in individual cases. This can be called an observation error of insurers. Second, insurers sometimes tend to lower the indemnity payment even when the policyholders are not accused for inappropriate behavior. One reason for this nitpicking activity is to prevent a possible fraudulent claim. Third, the insurance contract is not complete in that insurance policy fails to consider all possible contingencies in the loss states. This incompleteness allows insurers to exercise some flexibility in the process of claim adjustment. Furthermore, insurers are legally exempt for indemnity payment in many situations as a result of the exclusions in policy conditions. Oftentimes, it is not clear whether or not an event is of exclusions. In addition, policyholders may not fully understand the policy conditions, which results in the indemnity differing from the policyholder's expectation.

Indemnity uncertainty brings about a risk additional to the loss risk, which is referred to as ambiguity. Individuals face not only uncertain indemnity but also the uncertain distribution of the uncertain indemnity. It is well documented that individuals are averse to ambiguity as exemplified by the Ellsberg paradox. The aim of this paper is to analyze the effect of ambiguity aversion on the demand for insurance, given that ambiguity is present in the uncertain distribution of indemnity. We model ambiguity aversion using the smooth concave transformation following Klibanoff, Marinacci and Mukerji (2005), as the standard expected utility theory fails to distinguish between ambiguity and non-ambiguity.

We find that the effect of ambiguity aversion on the insurance demand may be positive or negative in general. We propose a sufficient condition for the increase in insurance demand. It is shown that ambiguity aversion affects the demand in three ways. The first effect is captured by absolute ambiguity aversion (AAA). When AAA is non-increasing, ambiguity aversion affects positively insurance demand. Second, ambiguity aversion increases the precautionary motive, leading to the increase in insurance demand. Lastly, ambiguity aversion negatively affects the demand for uncertain assets.

Our analysis will further help understand the insurance demand by providing a theoretical framework for ambiguity aversion. Among others, the existing studies report that the effects of ambiguity aversion on insurance demand are positive. Our analysis provides a counter case in which ambiguity aversion does not necessarily increase insurance demand when indemnity is uncertain. We provide a sufficient condition for the increase/decrease in insurance demand under ambiguity aversion. We also provide a comparative statics analysis regarding the degree of ambiguity aversion.

The remainder of the paper is organized as follows. Section II reviews the existing literature and summarizes the findings of our analysis. Section III shows a benchmark model under expected utility theory. Section IV suggests the main model with ambiguity aversion. Section V does comparative statics analysis. Section VI concludes.

II. Literature Review

There are two strands of literatures. The first one is about indemnity uncertainty.

According to Mossin (1968) and Schlesinger (2000), the risk averse individual purchase full insurance when the premium is fair. However, when the indemnity is uncertain, the uncertain wealth in loss state affects insurance demand in two ways (Lee, 2012). On the hand, a prudent individual purchases more insurance because of the precautionary demand. On the other hand, risk aversion will affect to reduce the demand for insurance as risk assets. A lot of studies refer to the uncertain indemnity as moral hazard problem such as fraudulent claim of policyholders. Above mentioned, policyholders can inflate or deflate their loss and insurers cut or delay to pay the indemnity (Picard, 1996).

The second one is about ambiguity aversion. There are several studies to characterize the ambiguity aversion. Gilboa and Schmeidler (1989) suggest the max-min expected utility and Ghirardato et al. (2004) use α-max-min expected utility model. Further, Klibanoff, Marinacci and Mukerji (2005) suggest the smooth model which explains that mean-preserving spreads in probabilities reduce the welfare of ambiguity averse individuals.

It is well known that risk aversion always increases the demand of self-insurance (Ehrlich and Becker, 1972), while the effect of risk aversion on demand of self-protection is ambiguous (Dionne and Eeckhoudt, 1985). The ambiguity aversion has also been studied in self-insurance and self-protection. Snow (2011) show that the insurance for both activities increases with greater ambiguity aversion since at optimum for self-insurance and self-protection of ambiguity neutral decision maker, a marginal increase in either activity leads to a mean preserving contraction in the distribution of expected utility, which is valuable to ambiguity averse decision maker with the same risk preference. On the other hand, Alary, Gollier and Treich (2013) find the condition that the demand of self- insurance increases and the demand of self-protection decreases. They explain that why a decision maker under subjective expected utility use a more pessimistic distribution of his belief. Further, Huang finds that a more ambiguity averse decision maker decreases the optimal effort when the cost of effort is non-monetary.

It is generally regarded that the ambiguity averse preference raises insurance demand since it is easy to think that ambiguity aversion reinforces risk aversion. However, Gollier (2011) shows that an ambiguity averse individual may purchase more ambiguous risky asset in a simple one-risky and one risk-free-asset portfolio model. That is, ambiguity aversion does not always reduce the risk exposure.

In this research, we follow the ambiguity model of Klibanoff, Marinacci and Mukerji (2005) since the effect of ambiguity aversion is separated from risk aversion effect on the insurance demand in their smooth model. We focus on the observation error of insurers and the incomplete insurance contract. We do not consider moral hazard and focus on the pure effect of randomness of indemnity and the distribution of indemnity.

III. Benchmark model

We first suppose a binary state model: loss and no loss state. The loss x occurs with probability p. We consider a risk-averse individual who has strictly increasing and concave utility function, u. She purchases insurance for loss. We suppose that the insurance is coinsurance that the coverage is decided as a percentage of loss. In particular, the indemnity I consists of the loss x and the observation error of insurers θ, so the indemnity I is denoted that $I = a(x+\theta)$, where a denotes the coverage and $0 \le a \le 1$. The observation error θ is a random variable and the distribution function of θ is represented by $f(\theta)$ on the support of $[\underline{\theta}, \overline{\theta}]$. The Ambiguity means that θ is unknown parameter and the expectation with respect to θ is denoted as $E(\theta)=\int_{\underline{\theta}}^{\bar{\theta}}\theta f(\theta)d\theta$. We also suppose that $E(\theta)=0$. Zero expectation of θ means that the randomness of indemnity disappears on average. With loading factor λ, the premium Q is denoted that $Q=(1+\lambda)apE(x+\theta)=(1+\lambda)apx$.The initial wealth is W. The utility function is increasing, concave and three times differentiable.

Without ambiguity, the problem to maximize the individual's expected utility is as follows.

$$\underset{a}{Max}\quad U=(1-p)u(W-Q)+pE_{\theta}[u(W-Q-x+a(x+\theta))] \qquad (1)$$
$$s.t.\quad Q=(1+\lambda)apx$$

For simplicity, let us denote $W-Q=W_0$ and $W-Q-x+a(x+\theta)=W_1$. Then the first order condition is:

$$U_a=-(1+\lambda)px(1-p)u'(W_0)+pE_{\theta}[(-(1+\lambda)px+x+\theta)u'(W_1))]=0 \qquad (2)$$

Under indemnity uncertainty, it may be optimal to purchase insurance when loss is sufficiently large.

$$\begin{aligned}U_{a|a=0}&=-(1+\lambda)px(1-p)u'(W)+pE_{\theta}[(-(1+\lambda)px+x+\theta)u'(W-x)] \qquad (3)\\&=-(1+\lambda)px(1-p)u'(W)+p(-(1+\lambda)px+x+\theta)u'(W-x)\end{aligned}$$

If $(1-(1+\lambda)p)u'(W-x)>(1+\lambda)(1-p)u'(W)$, then an individual purchases insurance. In addition, the optimal insurance satisfies following condition.

$$px[-(1+\lambda)(1-p)u'(W_0)+(1-(1+\lambda)p)E_{\theta}(u'(W_1))]+p\,\mathrm{cov}_{\theta}((-(1+\lambda)px+x+\theta),u'(W_1))=0$$
(4)

On the other hand, without loading, (2) is transformed into the following expression.

$$\begin{aligned}U_a&=-px(1-p)u'(W_0)+pE_{\theta}[(-px+x+\theta)u'(W_1))]\\&=-px(1-p)[u'(W_0)-E_{\theta}(u'(W_1))]+p\,\mathrm{cov}_{\theta}(\theta,u'(W_1)) \qquad (5)\end{aligned}$$

The risk from uncertain indemnity increases as coverage a increases in this setting. The covariance term of (5) indicates that the risk aversion effect due to the variability of indemnity remains even if an individual purchases full insurance. Since $\mathrm{cov}_{\theta}(\theta,u'(W_1))<0$, we first have that if $u'(W_0)\geq E_{\theta}(u'(W_1))$, then partial insurance is optimal by Jensen's inequality. That is, if $u'''(W)\leq 0$, then $a<1$ even when the premium is fair. Second, if $u'''(W)>0$ which means the individual is prudent, and the effect of prudence is not sufficiently greater than the covariance term, then she does not purchase full insurance.

As Lee (2012) pointed out, θ is regarded as a kind of additive background risk. The prudence implies that an individual purchases more insurance or saves more money to cope with background risk. However, if the effect to avoid the risk of indemnity uncertainty offsets the prudence effect, then full insurance is no longer optimal in spite of fair premium.

IV. Model under ambiguity aversion

4.1. Model with indemnity uncertainty

Now, we consider an ambiguity in observation error $\theta(w)$. Let us denote $\theta(w)$ as θ_w. As stated above, the true loss is x, while the insurers evaluate the loss as $(x+\theta)$. The distribution of θ depending on the prior belief is a function of w which has values 1,2,3,…,n. θ_w is ranked that $E_\theta(\theta_1) \le E_\theta(\theta_2) \le, ..., \le E_\theta(\theta_n)$ and $E_w E_\theta(\theta_w) = 0$. The ambiguity of θ_w is represented by a set of probability density function, $\prod = \{f_1, f_2, ..., f_n\}$. The second order probability distribution over the set of priors Π is denoted as $\{q_1, q_2, ..., q_n\}$, with $\sum_{w=1}^{n} q_w = 1$. However, the insurer is ambiguity neutral, so the insurance premium is still equal to $Q = (1+\lambda)apx$, since $E_w E_\theta(\theta_w) = 0$. Let us also denote that $u_1(a, \theta_w) = u(W - Q - x + a(x + \theta_w))$.

Following the work by Klibanoff, Marinacci and Mukerji (2005), let us assume that the ex-ante utility of an individual is evaluated by the certainty equivalent of the random variable $u_1(a, \theta_w)$ by using an increasing and concave valuation function ϕ.[1] We also define the ambiguity aversion as the concavity of ϕ based on the setting in Klibanoff, Marinacci and Mukerji (2005) as in Treich (2009), Gollier (2011), and Alary et al. (2013).[2] Then the ex-ante utility is:

$$\begin{aligned} U_1 &= (1-p)u(W-Q) + p\phi^{-1}[E_w(\phi(E_\theta u_1(a,\theta_w)))] \\ &= (1-p)u(W-Q) + p\phi^{-1}[\sum_{w=1}^{n} q_w(\phi(\int_{\underline{\theta}}^{\bar{\theta}} (u_1(a,\theta_w)) f_w(\theta) d\theta))] \qquad (6) \end{aligned}$$

The utility maximization problem of the individual is equal to following program.

$$\max_{a} \quad U_1 = (1-p)u(W-Q) + p\phi^{-1}[E_w(\phi(E_\theta u_1(a,\theta_w)))] \qquad (7)$$
$$s.t. \quad Q = (1+\lambda)apx$$

The first order condition is

$$\begin{aligned} &-(1-p)(1+\lambda)pxu'(W-Q) \\ &\quad + p\phi^{-1}{}'[E_w\phi(E_\theta u_1(a,\theta_w))]E_w[\phi'(E_\theta u_1(a,\theta_w))E_\theta\left\{u_1{}'(a,\theta)(-(1+\lambda)px + x + \theta_w)\right\}] = 0 \qquad (8) \end{aligned}$$

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[1] We also suppose that the function ϕ is three times differentiable as well.

[2] The function ϕ captures the individual's attitude toward ambiguity with $\phi' > 0$. Ambiguity neutrality is represented by the linearlity. The individual is ambiguity averse with $\phi'' < 0$, while she is ambiguity loving with $\phi'' > 0$. The concavity of ϕ means that an individual dislikes mean-preserving spread.

By comparing (2) and (8), we have following proposition 1.

Proposition 1. Suppose that ϕ exhibits non-increasing absolute ambiguity aversion. With indemnity uncertainty, the sufficient conditions for which an ambiguity averse individual purchases more insurance than an ambiguity neutral individual are as follows.

(1) The set of first distributions ($\theta(w_1),\theta(w_2),...,\theta(w_n)$) can be ranked according to first order stochastic dominance and the relative risk aversion is greater than or equal to 1.
(2) The set of first distributions ($\theta(w_1),\theta(w_2),...,\theta(w_n)$) can be ranked according to second order stochastic dominance, the utility function exhibits prudence, the relative risk aversion is greater than 1and the relative prudence is greater than or equal to 2.

Proof. Let $a*$ denote the optimal coverage of the ambiguity-neutral individual. Then $U_{a|a=a*}=0$ in (2). In order to determine whether the ambiguity averse individual purchase more insurance, it is enough to verify the sign of (8) at $a*$. Let us denote that $Q*=(1+\lambda)a*px$ for simplicity. Then we can rewrite (8) as follows:

$$-(1-p)(1+\lambda)pxu'(W-Q*)+p\frac{E_w[\phi'(E_\theta u_1(a*,\theta_w))E_\theta\left\{u_1'(a*,\theta_w)(-(1+\lambda)px+x+\theta_w)\right\}]}{\phi'\left\{\phi^{-1}[E_w\phi(E_\theta u_1(a*,\theta_w))]\right\}} \quad (9)$$

$$\begin{aligned}&=-(1-p)(1+\lambda)pxu'(W-Q*)\\&+p\frac{E_w[\phi'(E_\theta u_1(a*,\theta_w))]E_w[E_\theta\left\{u_1'(a*,\theta_w)(-(1+\lambda)px+x+\theta_w)\right\}]}{\phi'\left\{\phi^{-1}[E_w\phi(E_\theta u_1(a*,\theta_w))]\right\}}\\&+p\frac{\mathrm{cov}_w(\phi'(E_\theta u_1(a*,\theta_w)),E_\theta\left\{u_1'(a*,\theta_w)(-(1+\lambda)px+x+\theta_w)\right\})}{\phi'\left\{\phi^{-1}[E_w\phi(E_\theta u_1(a*,\theta_w))]\right\}}\end{aligned} \quad (10)$$

Under non-increasing absolute ambiguity aversion, the sum of the first and second term is greater than or equal to 0 at $a*$, since $E_w[\phi'(E_\theta u_1(a*,\theta_w))]\geq\phi'\left\{\phi^{-1}[E_w\phi(E_\theta u_1(a*,\theta_w))]\right\}$. As a result, the condition that the sign of the last term in (10) is positive is a sufficient condition to increase insurance demand. The following condition (11) indicates that $E_\theta u_1(a*,\theta_w)$ and $E_\theta\left\{u_1'(a*,\theta_w)(-(1+\lambda)px+x+\theta_w)\right\}$ are anti-comonotone in w.

$$\mathrm{cov}_w(\phi'(E_\theta u_1(a*,\theta_w)),E_\theta\left\{u_1'(a*,\theta_w)(-(1+\lambda)px+x+\theta_w)\right\})\geq 0 \quad (11)$$

Let us first suppose that $(\theta(w_1),\theta(w_2),...,\theta(w_n))$ are ranked according to FSD (First order stochastic dominance). Then following the assumption that the utility function u is increasing and concave, $E_\theta u_1(a*,\theta_w)$ is increasing in θ_w. Thus, if a function $f(\theta_w)=-u_1'(a*,\theta_w)(-(1+\lambda)px+x+\theta_w)$ is also increasing in θ_w, then the condition (11) is satisfied. This yields the following condition.

$$f'(\theta_w) = -u_1'(a^*, \theta_w) - u_1''(a^*, \theta_w)(-(1+\lambda)a^* px + a^* x + a^* \theta_w) \geq 0 \qquad (12)$$

Second, consider that $(\theta(w_1), \theta(w_2), ..., \theta(w_n))$ are ranked according to SSD (Second order stochastic dominance) and the utility function exhibits prudence ($u''' > 0$). In this case, the condition (11) is satisfied when a function f is increasing and concave. Therefore, we have the condition (13).

$$f''(\theta_w) = -a^* u_1''(a^*, \theta_w) - a^* u_1'''(a^*, \theta_w)(-(1+\lambda)a^* px + a^* x + a^* \theta_w) \leq 0 \qquad (13) //$$

In general, it is known that ambiguity aversion raises coverage. However, with the existence of indemnity uncertainty, the effect of ambiguity aversion is not clear. This is because more insurance reduces the risk from loss x while that increases the risk from indemnity. Proposition 1 shows that ambiguity aversion effect is closely correlated to utility function and prudence. In addition, if the ambiguity aversion is increasing function and the covariance term (10) is not sufficiently large, then the demand of insurance can decrease.

Meanwhile, the covariance term can be divided into two parts. By transforming (10), we have:

$$\begin{aligned}
&\operatorname{cov}_w(\phi'(E_\theta u_1(a^*, \theta_w)), E_\theta \left\{u_1'(a^*, \theta_w)(-(1+\lambda)px + x + \theta_w)\right\}) \\
&= \operatorname{cov}_w(\phi'(E_\theta u_1(a^*, \theta_w)), E_\theta \left\{u_1'(a^*, \theta_w)(-(1+\lambda)px + x)\right\}) \\
&\quad + \operatorname{cov}_w(\phi'(E_\theta u_1(a^*, \theta_w)), E_\theta \left\{u_1'(a^*, \theta_w)\theta_w\right\}) \qquad (14)
\end{aligned}$$

The first term in (14) indicates the effect of zero mean background risk added to wealth on insurance demand when the proposition 1 holds. An ambiguity averse individual purchases more insurance to cope with the background risk to wealth. This effect is precautionary effect. On the other hand, the second term is interpreted as a portfolio choice problem. As Gollier (2011) pointed out, the ambiguity aversion biases individual's beliefs about the prior by putting weight to worse case w. Hence, in this problem, the pessimism effect may influence on both terms in opposite way. The pessimism effect yields to increase precautionary motive in the first term, whereas it may yield to decrease the demand for uncertain asset. The condition to decrease the demand for uncertain asset in a portfolio choice problem is summarized in the following lemma.

Lemma 1. In a portfolio choice problem, the condition to decrease the demand for uncertain asset is as follows.

(1) The set of first distributions ($\theta(w_1), \theta(w_2), ..., \theta(w_n)$) can be ranked according to first order stochastic dominance and the relative risk aversion is less than or equal to 1.
(2) The set of first distributions ($\theta(w_1), \theta(w_2), ..., \theta(w_n)$) can be ranked according to second order stochastic dominance, the utility function exhibits prudence, the relative risk aversion is less than 1and the relative prudence is less than or equal to 2.

Proof. The proof is similar to proposition 1, so is omitted here.

4.2. Model with loss uncertainty

We now consider a variant of above model. In this subsection, we assume that loss is divided into observable and unobservable loss.[3] Observable random loss is x and occurs with probability p while unobservable loss θ follows the distribution $f(\theta)$. Both observable and unobservable loss are independent each other. The insurers pay the indemnity only for the observable loss x. The premium is still equal to $(1+\lambda)apx$, since the insurer is ambiguity neutral and $E_w E_\theta(\theta)=0$. Then the expected utility without ambiguity aversion is

$$\max_a \quad U_2 = (1-p)u(W-Q) + pE_\theta[u(W-Q-(x+\theta)+ax)] \qquad (15)$$
$$s.t. \quad Q=(1+\lambda)apx$$

The optimal coverage is determined at which

$$\begin{aligned} U_{2a} &= -(1+\lambda)px(1-p)u'(W-Q) + pE_\theta[(-(1+\lambda)px+x)u'(W-Q-(x+\theta)+ax))] \\ &= px[-(1+\lambda)(1-p)u'(W-Q) + (1-(1+\lambda)p)E_\theta[u'(W-Q-(x+\theta)+ax))] = 0 \qquad (16) \end{aligned}$$

According to Lee (2012), only the prudence affects insurance demand. In (16), let us consider that the loading is zero. Then, the individual purchases full insurance when she is prudent ($u'''(W)>0$). On the other hand, she may purchase partial insurance when she is imprudent ($u'''(W)\le 0$). The difference in results of above subsection comes from the interaction between coverage and the indemnity risk does not exist in (16). Since the indemnity only depends on the observable loss, the prudent individual purchases full (or more) insurance as precautionary motive.

Now, we consider ambiguity aversion regarding uncertain loss. Let us denote that $u_2(a,\theta_w) = u(W-Q-(x+\theta_w)+ax)$. Then, (16) is changed into as follows.

$$\max_a \quad U_3 = (1-p)u(W-Q) + p\phi^{-1}[E_w(\phi(E_\theta u_2(a,\theta_w)))] \qquad (17)$$
$$s.t. \quad Q=(1+\lambda)apx$$

The first order condition is

$$\begin{aligned} &-(1-p)(1+\lambda)pxu'(W-Q) \\ &+p\phi^{-1}{}'[E_w\phi(E_\theta u_2(a,\theta_w))]E_w[\phi'(E_\theta u_2(a,\theta_w))E_\theta\left\{u_2{}'(a,\theta_w)(-(1+\lambda)px+x)\right\}] = 0 \qquad (18) \end{aligned}$$

From (18), we have following lemma 2.

Lemma 2. Suppose that ϕ exhibits non-increasing absolute ambiguity aversion. With loss uncertainty, the sufficient conditions for which an ambiguity averse individual purchases more insurance than an ambiguity neutral individual are as follows.

[3] The loss can be regarded as insurable and uninsurable loss as well.

(1) The set of first distributions ($\theta(w_1),\theta(w_2),...,\theta(w_n)$) can be ranked according to first order stochastic dominance.

(2) The set of first distributions ($\theta(w_1),\theta(w_2),...,\theta(w_n)$) can be ranked according to second order stochastic dominance and the utility function exhibits prudence.

Proof. Let us denote the optimal coverage satisfying (16) as a^{**} and $(1+\lambda)a^{**}px$ as Q^{**}. Then (17) at a^{**} is :

$$-(1-p)(1+\lambda)pxu'(W-Q^{**})+p\frac{E_w[\phi'(E_\theta u_2(a^{**},\theta_w))E_\theta\left\{u_2'(a^{**},\theta_w)(-(1+\lambda)px+x)\right\}]}{\phi'\left\{\phi^{-1}[E_w\phi(E_\theta u_2(a^{**},\theta_w))]\right\}} \tag{19}$$

$$\begin{aligned}&=-(1-p)(1+\lambda)pxu'(W-Q^{**})\\&+p\frac{E_w[\phi'(E_\theta u_2(a^{**},\theta_w))]E_w[E_\theta\left\{u_2'(a^{**},\theta_w)(-(1+\lambda)px+x)\right\}]}{\phi'\left\{\phi^{-1}[E_w\phi(E_\theta u_2(a^{**},\theta_w))]\right\}}\\&+p\frac{\operatorname{cov}_w(\phi'(E_\theta u_2(a^{**},\theta_w)),E_\theta\left\{u_2'(a^{**},\theta_w)(-(1+\lambda)px+x)\right\})}{\phi'\left\{\phi^{-1}[E_w\phi(E_\theta u_2(a^{**},\theta_w))]\right\}}\end{aligned} \tag{20}$$

Similar to (10), under non-increasing absolute ambiguity aversion, the sum of the first and second term in (20) is greater than or equal to 0 at a^{**}. Since $-u$ is increasing in θ_w, insurance demand increases when $(\theta(w_1),\theta(w_2),...,\theta(w_n))$ follows FSD. In addition, u_2' is also increasing and concave in θ_w when u exhibits prudence. Thus, insurance demand also increases when $(\theta(w_1),\theta(w_2),...,\theta(w_n))$ follows SSD. //

The result of lemma 1 shows that the ambiguity aversion does not always increase insurance demand. In the second case in lemma 2, the prudence of utility function is still significant condition to increase demand under ambiguity aversion regarding uncertain loss. This lemma is in line with existing literatures which argue the conditions to increase the precautionary saving under ambiguity and background risk.

V. Comparative statics analysis

5.1. The effect of degree of ambiguity aversion on insurance demand

In this subsection, we investigate whether increase in ambiguity aversion leads to increase in insurance demand under uncertain indemnity. We still follow the smooth model of Klibanoff, Marinacci and Mukerji (2005), so an individual becomes more ambiguity averse than the other when her ambiguity function is obtained by transforming the other's ambiguity function using a strictly increasing and concave function. Thus, we suppose that there are two individuals with ambiguity function ϕ_1 and ϕ_2 respectively, and $\phi_2=g(\phi_1)$ where g is strictly increasing and concave. Then the utility maximization problem of the individual with ϕ_2 is similar to the program (10). The first order condition with ϕ_2 is

$$-(1-p)(1+\lambda)pxu'(W-Q^*)+p\frac{E_w[\phi_2'(E_\theta u_1(a^*,\theta_w))E_\theta\left\{u_1'(a^*,\theta_w)(-(1+\lambda)px+x+\theta_w)\right\}]}{\phi_2'\left\{\phi_2^{-1}[E_w\phi_2(E_\theta u_1(a^*,\theta_w))]\right\}}$$

$$\begin{aligned}&=-(1-p)(1+\lambda)pxu'(W-Q^*)\\&\quad+p\frac{E_w[\phi_2'(E_\theta u_1(a^*,\theta_w))]E_w[E_\theta\left\{u_1'(a^*,\theta_w)(-(1+\lambda)px+x+\theta_w)\right\}]}{\phi_2'\left\{\phi_2^{-1}[E_w\phi_2(E_\theta u_1(a^*,\theta_w))]\right\}}\\&\quad+p\frac{\mathrm{cov}_w(\phi_2'(E_\theta u_1(a^*,\theta_w)),E_\theta\left\{u_1'(a^*,\theta_w)(-(1+\lambda)px+x+\theta_w)\right\})}{\phi_2'\left\{\phi_2^{-1}[E_w\phi_2(E_\theta u_1(a^*,\theta_w))]\right\}}\end{aligned} \quad (21)$$

Increase in ambiguity aversion leads to increase in covariance term in (21) when proposition 1 holds. However, the effect on the second term in (21) is not clear. Under IAAA, the insurance demand may decrease when the covariance term does not offset the decrease caused by IAAA. Even under CAAA, it is not certain to increase in insurance demand when $\phi_2'\{U\}$ is sufficiently larger than $\phi_1'\{U\}$. Thus, we have following proposition 2.

Proposition 2. Let us suppose that $\phi_2 = g(\phi_1)$ where g is strictly concave and increasing function. If $\phi_2'\{U\} \le \phi_1'\{U\}$, then insurance demand increases.

Proof. See the text above.//

In general, it is easy to think that the increase in ambiguity aversion tends to increase in insurance demand. However, the model in subsection 4.2 shows clearly that more ambiguity averse individual is not always likely to purchase more insurance. The expression (19) implies that even when lemma 1 holds, insurance demand can decrease as $\phi_2'\{U\}$ is larger than $\phi_1'\{U\}$.

VI. Conclusion

We analyze the demand for insurance under ambiguity aversion when indemnity is uncertain. We find that the effect of ambiguity aversion on the insurance demand may be positive or negative in general. We propose a sufficient condition for the increase in insurance demand. It is shown that ambiguity aversion affects the demand in three ways. The first effect is captured by absolute ambiguity aversion (AAA). When AAA is non-increasing, ambiguity aversion affects positively insurance demand. Second, ambiguity aversion increases the precautionary motive, leading to the increase in insurance demand. Lastly, ambiguity aversion negatively affects the demand for uncertain assets.

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