Should Liability Insurance be Compulsory for Bicycle Accidents?*

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Abstract:
This study examines whether the introduction of compulsory bicycle liability insurance is socially desirable. We introduce both objective and subjective evaluations of the amount of liability. Then, we confirm that evaluation bias, which defines the difference between the objective and subjective evaluations of the amount of liability in bicycle accidents, is the key to deciding whether the introduction of compulsory bicycle liability insurance is socially desirable.

The main results of this study are as follows. First, if there is no evaluation bias, introducing compulsory bicycle liability insurance is socially desirable when the loading rate is low, the expected amount of liability that a victim cannot receive from bicycle rider is large, and the expected amount of liability is small. Second, if there is an evaluation bias and the accident probability distribution is a uniform distribution function, the magnitude of the loading rate determines whether introducing compulsory bicycle

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liability insurance is socially desirable. Third, when an evaluation bias is maximum, it enhances the social desirability for introducing compulsory bicycle liability insurance.

**Keywords:** Bicycle liability insurance, Voluntary insurance, Compulsory insurance

1. **Introduction**

   It has not been obligatory for bicycle riders to insure against liabilities that occur from bicycle accidents in Japan as well as other countries. Although insurance companies sell liability insurance for bicycle accidents, bicycle riders can choose if they want to insure themselves against such liability.

   However, a 2013 judgment in the Kobe District Court ordered a bicycle rider to compensate for a large amount of liability, which had a considerable impact on Japanese society. After this judgment was passed, neighboring local governments enacted ordinances that prohibited riding bicycles without bodily injury liability coverage. In other words, the compulsory bicycle liability insurance system was introduced by these local governments.

   Introducing the compulsory bicycle liability insurance system has not invited strong opposition to date. Furthermore, we know that compulsory bicycle liability insurance leads to decrease the number of victims who cannot receive full amount of compensation. However, it is not clear whether the legal obligation of all bicycle riders to take up liability insurance contracts is socially desirable because the total amount of loading contained in insurance premium, which is transaction costs of insurance contracts, becomes larger.

   Moreover, bicycle riders might evaluate the amount of liability subjectively, rather than objectively. Hence, they might have an evaluation bias. The evaluation bias defines the difference between the objective and subjective evaluations of the amount of liability in bicycle accidents. In other words, evaluation bias implies that subjective probability in low (high) amount of liability is larger (smaller) than objective probability. In Japan, it is likely that most bicycle riders under-evaluate the amount of liability in bicycle accidents. For example, Yamamoto *et al.* (2012) indicated that bicycle riders under-evaluated the amount of liability in bicycle accidents because they insured against such liability risks with an insignificant limit of liability. Also, we know that the discussion in under-evaluation might be categorized in prospect theory. According to Kahneman and Tversky (1979, p.283 Figure 4), we understand that the under-evaluation occurs frequently because a wide range of objective probabilities is under-evaluated.
In this situation, the introduction of compulsory bicycle liability insurance might be socially desirable because the effect of such evaluation bias is invalidated. Moreover, bicycle riders never voluntarily want to purchase bicycle liability insurance. From this viewpoint, we find that the evaluation bias is an important aspect for investigating the introduction of compulsory bicycle liability insurance.

The purpose of this study is to examine whether the introduction of compulsory bicycle liability insurance is socially desirable. We built an economic model and compared two situations in which bicycle liability insurance is voluntary and compulsory.

Previous studies such as Pauly (1974), Johnson (1977), and Dahlby (1981), discussed compulsory insurance in relation to the adverse selection problem. In the case of voluntary insurance, it is well known that low risk individuals have little incentive to purchase insurance when the average insurance premium rate is applied. On the other hand, high risk individuals want to purchase insurance and the insurance system is never maintained in the long term. The introduction of compulsory insurance provides that low risk individuals cannot withdraw from insurance system, and the adverse selection problem is solved.

In contrast, this study does not investigate the adverse selection problem. Instead, we focus on the bicycle riders’ evaluation of the amount of liability by introducing both objective and subjective evaluations in the amount of liability. In other words, how the evaluation bias affects the introduction of compulsory bicycle liability insurance is analyzed.

This article is organized as follows. Section 2 describes the background of bicycle accidents and bicycle liability insurance in Japan. The economic model is built in Section 3. In Section 4, we define the social loss for measuring the social desirability in voluntary and compulsory bicycle liability insurance. A comparison of voluntary and compulsory bicycle liability insurance is conducted in Section 5. Concluding remarks are presented in Section 6.

2. Background

Compared with European countries, bicycle riders in Japan usually tend not to comply with traffic rules. For example, the left-hand traffic rule in Japan mandates that all vehicles, including bicycles, must keep to the left. However, many bicycle riders do not follow this rule and we often see bicycles in the right-hand lane or on sidewalks. As a result, bicycle accidents occur frequently and bicycle accidents account for almost 20%
of the total traffic accidents. In addition, some bicycle accidents are accompanied by damages to other persons in accidents between bicycle and bicycle, and accidents between bicycle and pedestrian.

Figure 1: The number of bicycle accidents

According to Figure 1, the numbers of such bicycle accidents drastically increased from 1999 to 2008. Even after 2009, the ratio of these types of accidents to all types of bicycle

1 According to the webpage of Cabinet Office of Japan, although the number of bicycle accidents tends to decrease, the rate of bicycle accidents against all traffic accidents tends to increase (p. 6).
accidents has not decreased. Furthermore, court judgments that ordered a high amount of liability to bicycle riders have become common since 2000.²

Specifically, the judgment of the Kobe District Court, which was passed on July 4, 2013, had a considerable impact on Japanese society. In the case, an eleven year old boy riding on the bicycle at a high speed collided with a female pedestrian on the straight sloped road in Kobe city on September 22, 2008. She was severely injured and suffered severe residual disability, and was in a vegetative state. Her husband had an automobile insurance, including bodily injury indemnity coverage that could cover her losses owing to this accident, and the insurance company paid ¥ 60 million for benefits. Thereafter, she and the insurance company sued the mother of the boy for damages.³ The Kobe District Court ordered the mother to pay ¥ 35 million to the injured woman and ¥ 60 million to the insurance company. However, the mother could not pay such a high amount as she did not have enough money or assets. Furthermore, she did not have bicycle liability insurance. Finally, she went bankrupt, the injured woman could not receive the full amount of compensation, and the insurance company could not recover its costs from the mother of the boy.

It has not been obligatory for bicycle riders to insure against the liability incurred by bicycle accidents in Japan as well as other countries. Insurance companies sell liability insurance for bicycle accidents. The liability insurance coverages for bicycle accidents are very various. The simplest bicycle liability insurance is a liability insurance exclusively for bicycle accidents, but other kinds of bicycle liability insurance are also sold in Japan. Liability insurance endorsements for the accidents in daily life accompanied by automobile insurance or fire insurance, and group liability insurance for students are typical examples. In the end, we find that the liability insurance coverages for bicycle accidents are very diverse and the main coverages are the endorsements in various kinds of insurance and group insurance. Thus, it is difficult to measure the penetration of liability insurance coverages for bicycle accidents.

² For example, the judgment of the Tokyo District Court, September 30, 2013 ordered ¥ 68 million; the Tokyo District Court, September 14, 2015 ordered ¥ 40 million; judgment of Tokyo District Court, April 11, 2007 ordered ¥ 54 million; judgment of Tokyo District Court, June 4, 2008 ordered ¥ 93 million, and judgment of Kobe District Court, July 4, 2013 ordered ¥ 95 million.
³ In cases where a minor has inflicted damages on others, if the minor does not have sufficient intellectual capacity to appreciate his/her liability for his/her own act, the minor shall not be liable to compensate for that act (Civil Code of Japan, article 712). In cases where a person without capacity to assume liability is not liable in accordance with article 712, the person with the legal obligation to supervise the person without capacity to assume liability, who is the mother in the case of the judgment of Kobe District Court, July 4, 2013, shall be liable to compensate for damages (Civil Code of Japan, article 714(1)).
After the judgment of the Kobe District Court, a few neighboring local governments enacted ordinances that prohibit riding bicycles without bodily injury liability coverage. Specifically, the congress of the Hyogo prefecture, to which Kobe city belongs, was the first to enact such an ordinance. It came into force in October 2015.\textsuperscript{4} This was followed by the Osaka prefecture (which came into force in July 2016) and the Shiga prefecture (which came into force in October 2016). Furthermore, Kyoto city has enacted such an ordinance in March 2017, and the Kyoto prefecture is now proceeding with the enactment of such an ordinance.\textsuperscript{5}

3. The Model

Suppose that there are many risk averse bicycle riders. Their utility function, which is denoted by $u(\cdot)$, are identical and it is assumed that $u'>0,u''<0$ and $u(0)=0$. Without the loss of generality, the number of bicycle riders is normalized to one. $w>0$ represents the initial wealth of each bicycle rider and all bicycle riders have the same amount of initial wealth. Each bicycle rider has different accident probability and it is denoted by $\pi \in [0,1]$. $f(\pi)>0$ represents the accident probability distribution of $\pi$ and it also represents the number of bicycle riders whose accident probability is $\pi$. $F(\pi)$ represents the cumulative accident probability distribution. If the bicycle accident occurs, the bicycle riders might have a responsibility to compensate the amount of liability.

The amount of liability is denoted by $D \geq 0$. It is noticed that the case in $D=0$ is included. $D=0$ means that bicycle riders have no responsibility despite the accident occurred. The amount of liability follows the liability probability distribution $g(D) \geq 0$. However, bicycle riders might have different liability probability distribution because they cannot exactly know the form of true liability probability distribution. In other words, they might have “subjective” liability probability distribution and is represented by

\textsuperscript{4} Unfortunately, there is no English website about liability insurance for bicycle accidents. Instead, we show English website at Matsubara City, which belongs to Osaka Prefecture, as follows (accessed on 23 May 2017):

http://www.city.matsubara.osaka.jp.e.lo.hp.transer.com/index.cfm/6,60794,33,150,html

\textsuperscript{5} In details, see the following websites (Osaka Prefecture, Shiga Prefecture, Kyoto City, and Kyoto Prefecture):


In contrast, if all bicycle riders have perfect knowledge about the liability probability distribution, they have “objective” liability probability distribution, and then \( h(D) = g(D) \) is realized.

The premium of bicycle liability insurance is assumed to be computed not by accident probability of each bicycle rider but by average accident probability. The premium is denoted by \((1 + \theta) p > 0\), where \( p \) and \( \theta \geq 0 \) are net premium and the loading rate, respectively. Also, we assume that the amount of insurance in bicycle liability insurance is unlimited. Then, the amount of net premium can be computed as

\[
p = \int_{\hat{\pi}}^{1} \int_{0}^{\pi} \frac{\pi f(\pi) D g(D) d\pi dD}{\int_{\hat{\pi}}^{1} f(\pi) d\pi} = \frac{E[D] \int_{\hat{\pi}}^{1} \pi f(\pi) d\pi}{1 - F(\hat{\pi})},
\]

where \( E[\bullet] \) represents the operator of the expectation. Also, \( \hat{\pi} \) represents the minimum accident probability of the bicycle rider who wants to purchase bicycle liability insurance. In other words, all amounts of liability is compensated by bicycle liability insurance and bicycle riders do not have to compensate by their own wealth.

It is assumed that all bicycle riders have a limited liability. It implies that they do not need to compensate for an amount of liability beyond their own wealth. Thus, victims in bicycle accidents might not receive some portion of compensation when bicycle riders did not purchase bicycle liability insurance. In contrast, we do not need to consider the effect of limited liability when bicycle riders purchased bicycle liability insurance because the amount of insurance is unlimited.

4. Social Loss

In this study, we compare the situations in which voluntary and compulsory bicycle liability insurance are provided. For this comparison, we define “social loss,” which contains the expected amount of liability that the victim cannot receive from bicycle rider and loading premium of bicycle liability insurance. The situation whose social loss is smaller is evaluated as socially desirable.

First, consider the expected amount of liability that the victim cannot receive from bicycle rider. Denote \( \overline{D} \geq 0 \) as the expected amount of liability that the victim cannot receive from bicycle rider. The amount of liability that the victim cannot receive from bicycle rider is represented by \( D - w \) and the probability in \( D \geq w \) is represented by \( \int_{w}^{\infty} g(D) dD \). Then, we show
\[ \overline{D} = \int_0^\infty (D - w)g(D)\,dD. \] ---(2)

Also, we can confirm that \( \overline{D} < E[D] \).

Next consider the loading premium of bicycle liability insurance. The loading premium per one bicycle liability insurance is \( \theta p \). Then, the number of bicycle riders who purchase bicycle liability insurance must be computed. The expected utility that the bicycle rider who purchases bicycle liability insurance can be written as
\[ \pi u(w - (1 + \theta)p) + (1 - \pi)u(w - (1 + \theta)p) = u(w - (1 + \theta)p). \] ---(3)

In contrast, when the bicycle rider whose liability probability distribution is \( g(D) \), the expected utility for a rider who did not purchase bicycle liability insurance can be written as
\[ \pi \int_0^w u(w - D)g(D)\,dD + (1 - \pi)u(w). \] ---(4)

In equation (4), bicycle rider’s utility becomes zero when the accident occurs and \( D \geq w \) realizes because of the limited liability and \( u(0) = 0 \). By using equations (3) and (4), we derive the condition in which the bicycle rider purchases bicycle liability insurance as follows.
\[ u(w - (1 + \theta)p) \geq \pi \int_0^w u(w - D)g(D)\,dD + (1 - \pi)u(w) \]
\[ \Rightarrow \pi \geq \frac{u(w) - u(w - (1 + \theta)p)}{u(w) - \int_0^w u(w - D)g(D)\,dD}. \] ---(5)

Then, we can show
\[ \hat{\pi} = \frac{u(w) - u(w - (1 + \theta)p)}{u(w) - \int_0^w u(w - D)g(D)\,dD}. \] ---(6)

From equation (6), we find the bicycle riders whose accident probability is \( \pi \geq \hat{\pi} \) purchase bicycle liability insurance and vice versa. Then, the loading premium of bicycle liability insurance is \( \theta p(1 - F(\hat{\pi})) \).

However, whether the bicycle rider purchases bicycle liability insurance depends on not objective but subjective evaluation. Thus, it might be natural to consider that bicycle rider’s decision is based on subjective liability probability distribution. When the bicycle rider whose liability probability distribution is \( h(D) \), the expected utility for a rider who did not purchase bicycle liability insurance can be written as
\[ u(w - (1 + \theta)p) \geq \pi \int_0^w u(w - D)h(D)dD + (1 - \pi)u(w) \]

\[ \Rightarrow \pi \geq \frac{u(w) - u(w - (1 + \theta)p)}{u(w) - \int_0^w u(w - D)h(D)dD}. \quad \text{---(7)} \]

The right-hand side of equation (7) is defined as follows.

\[ \tilde{\pi} = \frac{u(w) - u(w - (1 + \theta)p)}{u(w) - \int_0^w u(w - D)h(D)dD}. \quad \text{---(8)} \]

From equation (8), we find the bicycle riders whose accident probability is \( \pi \geq \tilde{\pi} \) purchase bicycle liability insurance and vice versa. Then, the loading premium of bicycle liability insurance is \( \theta p(1 - F(\tilde{\pi})) \).

Now we assume that bicycle riders tend to under-evaluate the amount of liability. In the case of the under-evaluation of the amount of liability, they feel a smaller amount of liability has a higher probability. That under-evaluation can be represented by the following inequality.

\[ \int_0^w g(D)dD \leq \int_0^w h(D)dD. \quad \text{---(9)} \]

From equation (9), we find

\[ \int_0^w u(w - D)g(D)dD \leq \int_0^w u(w - D)h(D)dD. \quad \text{---(10)} \]

And then,

\[ \hat{\pi} \leq \tilde{\pi}. \quad \text{---(11)} \]

Furthermore, in order to measure the bicycle riders’ under-evaluation, the following variable \( \lambda \), which represents evaluation bias, is introduced.

\[ \lambda \equiv \frac{\tilde{\pi}}{\hat{\pi}} = \frac{u(w) - \int_0^w u(w - D)g(D)dD}{u(w) - \int_0^w u(w - D)h(D)dD} \geq 1. \quad \text{---(12)} \]

Also, from equation (12), we know

\[ \hat{\pi} = \lambda \tilde{\pi}. \quad \text{---(13)} \]

In the later discussion, we use \( h(D) \) as the bicycle rider’s liability probability distribution because \( h(D) = g(D) \), which represents the special case in which \( \lambda = 1 \) is realized, is also included.
5. Comparison

5.1. Compute the social loss

In this section, we compute social loss in the following three cases:

Case 1: no bicycle liability insurance
Case 2: voluntary bicycle liability insurance
Case 3: compulsory bicycle liability insurance

Let \( C_N^i \) and \( C_L^i \) for \( i \in \{1,2,3\} \) be the expected amount of liability that the victim cannot receive from bicycle rider and the loading premium of bicycle liability insurance, respectively. Also, social loss is represented by \( C^i = C_N^i + C_L^i \). Then, we can compute social loss in each case as follows.

Case 1:

\[
C_N^1 = \int_0^1 D \pi f(\pi) d\pi = D E[\pi], \tag{14}
\]

\[
C_L^1 = 0, \tag{15}
\]

\[
C^1 = C_N^1 + C_L^1 = D E[\pi]. \tag{16}
\]

Case 2:

\[
C_N^2 = \int_0^1 D \pi f(\pi) d\pi = D \int_0^\pi \pi f(\pi) d\pi, \tag{17}
\]

\[
C_L^2 = \theta p \int_0^1 f(\pi) d\pi = \theta p (1 - F(\bar{\pi})), \tag{18}
\]

\[
C^2 = C_N^2 + C_L^2 = D \int_0^\pi \pi f(\pi) d\pi + \theta p (1 - F(\bar{\pi}))
+ D \int_0^{\bar{\pi}} \pi f(\pi) d\pi + \theta p (1 - F(\lambda \bar{\pi})). \tag{19}
\]

Case 3:

\[
C_N^3 = 0, \tag{20}
\]

\[
C_L^3 = \theta p \int_0^1 f(\pi) d\pi = \theta p, \tag{21}
\]

\[
C^3 = C_N^3 + C_L^3 = \theta p. \tag{22}
\]
5.2. Compare the social loss

First, we compare the social loss in Case 1 and Case 3. The condition in which Case 3 is more desirable than Case 1 can be written as follows.

\[ C^1 > C^3 \Rightarrow \bar{D}E[\pi] > \theta p. \]

---(23)

All bicycle riders purchase bicycle liability insurance in Case 3, then \( \hat{\pi} = \bar{\pi} = 0 \) must be satisfied. By substituting \( \hat{\pi} = 0 \) to equation (1), \( p = E[\pi]E[D] \) is derived and then,

\[ C^1 > C^3 \Rightarrow \bar{D}E[\pi] > \theta p \Rightarrow \bar{D}E[\pi] > \theta E[\pi]E[D] \Rightarrow \bar{D} > \theta E[D]. \]

---(24)

Finally, we have

\[ \theta < \frac{\bar{D}}{E[D]} . \]

---(25)

Equation (25) tends to be satisfied when the loading rate is low, the expected amount of liability that the victim cannot receive from bicycle rider is large, and expected amount of liability is small.

Next compare the social loss in Case 2 and Case 3. Unlike the comparison in Case 1 and Case 3, we cannot derive simple result because the social loss function in Case 2, which is represented in equation (19), seems to be complex. In order to know the function form of \( C^2 \), By differentiating equation (19) with respect to \( \hat{\pi} \), we have

\[ \frac{\partial C^2}{\partial \hat{\pi}} = \frac{E[D] f(\hat{\pi})}{1 - F(\hat{\pi})} \left[ \int_{0}^{1} \frac{\pi f(\pi) d\pi - \hat{\pi} (1 - F(\hat{\pi}))}{1 - F(\hat{\pi})} \right]. \]

---(26)

From equation (1), \( \frac{\partial p}{\partial \hat{\pi}} \) can be computed as

\[ \frac{\partial p}{\partial \hat{\pi}} = \frac{E[D] f(\hat{\pi})}{1 - F(\hat{\pi})} \left[ \int_{0}^{1} \frac{\pi f(\pi) d\pi - \hat{\pi} (1 - F(\hat{\pi}))}{1 - F(\hat{\pi})} \right]. \]

---(27)

Equation (27) is always positive because

\[ \int_{0}^{1} \pi f(\pi) d\pi > \int_{0}^{1} \hat{\pi} f(\pi) d\pi = \hat{\pi} (1 - F(\hat{\pi})). \]

---(28)

Substituting equation (27) to equation (26), we have

\[ \frac{\partial C^2}{\partial \hat{\pi}} = \lambda f(\hat{\pi}) \left[ \frac{E[D] f(\hat{\pi})}{1 - F(\hat{\pi})} \left[ \int_{0}^{1} \frac{\pi f(\pi) d\pi - \hat{\pi} (1 - F(\hat{\pi}))}{1 - F(\hat{\pi})} \right] + \theta (1 - F(\hat{\pi})) E[D] f(\hat{\pi}) \right]. \]

---(29)

If the sign of equation (29) is always positive (negative), it means that \( C^2 \) is a monotone
increasing (decreasing) function of \( \hat{\pi} \). Then, if equation (29) is monotone increasing (decreasing), \( \hat{\pi} = 0 \) realizes (does not realize) minimum \( C^2 \) and we know introducing compulsory bicycle liability insurance is socially desirable (undesirable) because \( \hat{\pi} = 0 \) indicates that all bicycle riders purchase bicycle liability insurance and it reflects the case of compulsory bicycle liability insurance.\(^6\)

Generally, we cannot derive the determinate result in which Case 2 and Case 3 is socially desirable because the sign of equation (29) is indeterminate. Thus, in order to investigate the comparison in Case 2 and Case 3, some assumptions are introduced in the later subsections.

5.2. Case in no evaluation bias

In this subsection, we investigate the situation in which there is no evaluation bias, that is, \( \lambda = 1 \) is realized. Substituting \( \lambda = 1 \) to equation (29), the following equation is derived.

\[
\frac{\partial C^2}{\partial \hat{\pi}} = \hat{\pi} f(\hat{\pi})[D - \theta E[D]].
\] \( - - - (30) \)

From equation (30), we find whether \( C^2 \) is a monotone increasing or decreasing function of \( \hat{\pi} \) depends on the sign of \( D - \theta E[D] \). Thus, \( C^2 \) is a monotone increasing (decreasing) function of \( \hat{\pi} \) if equation (25) is satisfied (not satisfied). In a nutshell, the introduction of compulsory bicycle liability insurance is socially desirable directly depends on whether equation (25) is satisfied.

From the above discussion, the following proposition can be derived.

**Proposition 1:**

Suppose the situation in which there is no evaluation bias. In this situation, introducing compulsory bicycle liability insurance is socially desirable when loading rate is low, the expected amount of liability that the victim cannot receive from bicycle rider is large, and the expected amount of liability is small.

5.3. Case in uniform distribution function

In this subsection, we introduce the assumption in which \( f(\pi) \) is a uniform distribution function for investigating the situation in which there is an evaluation bias.

\(^6\) Theoretically, minimum social loss can be realized through the ban for selling voluntary bicycle liability insurance when equation (29) is a monotone decreasing function of \( \hat{\pi} \). However, regulator cannot impose such ban in actual insurance market.
that is, $\lambda > 1$ is realized.

In the case of uniform distribution function, the following equations are satisfied.

\[ f(\hat{\lambda}) = f(\hat{\lambda}), \quad \text{---(31)} \]
\[ \int \pi f(\pi) d\pi = \frac{1 - \hat{\pi}^2}{2}, \quad \text{---(32)} \]
\[ 1 - F(\hat{\pi}) = 1 - \hat{\pi}. \quad \text{---(33)} \]

Substituting equations (31) to (33) to equation (29), we have

\[ \frac{\partial C^2}{\partial \hat{\pi}} = \frac{f(\hat{\pi})}{2} \left[ 2\overline{D}\pi\hat{\lambda}^2 - \theta E[D](1 + 2\hat{\pi})\hat{\lambda} + \theta E[D] \right]. \quad \text{---(34)} \]

In order to confirm the sign of equation (34), the following function is defined.

\[ g(\lambda) = 2\overline{D}\pi\hat{\lambda}^2 - \theta E[D](1 + 2\hat{\pi})\hat{\lambda} + \theta E[D]. \quad \text{---(35)} \]

We find that equation (35) is a convex quadratic function of $\lambda$. Thus, equation (35) has one minimum value. First-order condition can be derived as

\[ \frac{\partial g(\lambda)}{\partial \lambda} = 4\overline{D}\pi\lambda - \theta E[D](1 + 2\hat{\pi}) = 0. \quad \text{---(36)} \]

Let $\lambda_{\min}$ be the $\lambda$ that minimizes equation (35). Then, we have

\[ \lambda_{\min} = \frac{\theta E[D](1 + 2\hat{\pi})}{4\hat{\pi}\overline{D}}. \quad \text{---(37)} \]

Substituting equation (6) to equation (37), we have

\[ \lambda_{\min} = \frac{\theta E[D]\left[3u(w) - 2u(w - (1 + \theta)p) - \int_w^w u(w - D)g(D) dD \right]}{4\overline{D}\left[u(w) - u(w - (1 + \theta)p) \right]} \]
\[ = \frac{\theta E[D]}{2\overline{D}} + \frac{\theta E[D]u(w) - \int_w^w u(w - D)g(D) dD}{4\overline{D}\left[u(w) - u(w - (1 + \theta)p) \right]}. \quad \text{---(38)} \]

From equation (38), $\lambda_{\min} \geq 0$ is surely satisfied and $\lambda_{\min} = 0$ is realized in the case of $\theta = 0$. Furthermore, $\theta \geq 2\overline{D}/E[D]$ is a sufficient condition to realize $\lambda_{\min} \geq 1$. From these characteristics, we find that $0 < \lambda_{\min} < 1$ is realized in the case of smaller $\theta$, while $\lambda_{\min} \geq 1$ is realized in the case of larger $\theta$.

For investigating the sign of equation (35), we distinguish two cases $0 < \hat{\lambda}_{\min} < 1$ and $\hat{\lambda}_{\min} \geq 1$.

First, we investigate the case of $0 < \hat{\lambda}_{\min} < 1$, that is equivalent to the case of smaller $\theta$. In this case, $g(\lambda)$ can be depicted in Figure 2. Thus, we find that $g(\lambda)$ is a monotone increasing function in $\lambda \geq 1$. This result implies that the evaluation bias always enhances
social desirability for introducing compulsory bicycle liability insurance. In other words, introducing compulsory bicycle liability insurance with evaluation bias might be socially desirable even when introducing compulsory bicycle liability insurance without evaluation bias is not socially desirable.

![Graph](image)

**Figure 2:** $g(\lambda)$ in the case of $0 < \lambda_{\text{min}} < 1$

Next investigate the case of $\lambda_{\text{min}} \geq 1$, that is equivalent to the case of larger $\theta$. In this case, $g(\lambda)$ can be depicted in Figure 3. $\tilde{\lambda}$ defines $\lambda$ that satisfies $\lambda \neq 1$ and $g(\lambda) = g(1)$. Then, $\tilde{\lambda}$ can be computed as

$$\tilde{\lambda} = \frac{\theta E[D] \left(2 + \frac{1}{\hat{\theta}} \right)}{2D} - 1.$$  \hspace{1cm} (39)

When $\lambda \in \left[1, \tilde{\lambda} \right]$, evaluation bias lowers social desirability for introducing compulsory bicycle liability insurance. In contrast, when $\lambda \in \left[\tilde{\lambda}, \frac{1}{\hat{\theta}} \right]$, evaluation bias always
enhances social desirability for introducing compulsory bicycle liability insurance. Also, from equation (39), we find that $\tilde{\lambda}$ is larger when $E[D]$ is larger and/or $\overline{D}$ is smaller.\footnote{How to affect the magnitude of $\theta$ to $\tilde{\lambda}$ is ambiguous because $\hat{\pi}$ is a decreasing function of $\theta$.}

Figure 3: $g(\lambda)$ in the case of $\lambda_{\text{min}} \geq 1$

From the above discussion, the following proposition can be derived.

**Proposition 2:**

Suppose the situation in which there is an evaluation bias and the accident probability distribution is a uniform distribution function. In this situation, the magnitude of the loading rate determines whether introducing compulsory bicycle liability insurance is socially desirable.

In the case of a smaller loading rate, evaluation bias always enhances social desirability for introducing compulsory bicycle liability insurance.

In the case of a larger loading rate, social desirability for introducing compulsory bicycle liability insurance is
bicycle liability insurance is enhanced (lowered) when the evaluation bias is larger (smaller).

5.4. Case in maximum evaluation bias

In this subsection, we investigate the situation in which the evaluation bias is maximum, that is represented by $\lambda = 1/\hat{\pi}$. This situation means that all bicycle riders never purchase bicycle liability insurance although some bicycle riders, whose accident probability contains $[\hat{\pi}, 1]$, purchase bicycle liability insurance in the case of no evaluation bias.

Substituting $\lambda = 1/\hat{\pi}$ to equation (29), we have

$$\frac{\partial C^2}{\partial \hat{\pi}} = \frac{1}{\hat{\pi}} f(1) \left( \overline{D} - \theta E[D] \right) \frac{\int_{\hat{\pi}}^1 \pi f(\pi) d\pi}{1 - F(\hat{\pi})}. \quad (40)$$

From equation (40), we find that $C^2$ is a monotone increasing function of $\hat{\pi}$ when the following inequality is satisfied.

$$\theta < \frac{1 - F(\hat{\pi})}{\int_{\hat{\pi}}^1 \pi f(\pi) d\pi} \overline{D} \cdot \frac{E[D]}{1 - F(\hat{\pi})}. \quad (41)$$

Also, from $\int_{\hat{\pi}}^1 \pi f(\pi) d\pi < \int_{\hat{\pi}}^1 f(\pi) d\pi = 1 - F(\hat{\pi})$, we know

$$\frac{1 - F(\hat{\pi})}{\int_{\hat{\pi}}^1 \pi f(\pi) d\pi} > 1. \quad (42)$$

From equation (42), we find that equation (41) is a sufficient condition to satisfy equation (25). Then we know that the maximum evaluation bias enhances the social desirability for introducing compulsory bicycle liability insurance. Also, we find that introducing compulsory bicycle liability insurance with maximum evaluation bias might be socially desirable even when introducing compulsory bicycle liability insurance without evaluation bias is not socially desirable.

From the above discussion, the following proposition can be derived.

**Proposition 3:**

Suppose the situation in which there is maximum evaluation bias. In this situation, maximum evaluation bias enhances the social desirability for introducing compulsory bicycle liability insurance.
6. Concluding Remarks

This study examined whether the introduction of compulsory bicycle liability insurance is socially desirable. We introduced both objective and subjective evaluations of the amount of liability. Then, we confirmed that evaluation bias, which defines the difference between the objective and subjective evaluations of the amount of liability in bicycle accidents, is a key factor in deciding whether the introduction of compulsory bicycle liability insurance is socially desirable.

The main results of this study are as follows. First, if there is no evaluation bias, introducing compulsory bicycle liability insurance is socially desirable when the loading rate is low, the expected amount of liability that the victim cannot receive from bicycle rider is large, and expected amount of liability is small. Second, if there is an evaluation bias and the accident probability distribution is a uniform distribution function, the magnitude of the loading rate determines whether introducing compulsory bicycle liability insurance is socially desirable. Third, when an evaluation bias is maximum, it enhances the social desirability for introducing compulsory bicycle liability insurance.

However, our model has some limitations. For example, all bicycle riders are assumed to be identical except for the accident probability. Actual bicycle riders have different amounts of initial wealth, different degrees of evaluation bias, etc. Especially, it is notable that different amounts of initial wealth seems to be a key factor because the bicycle riders whose amount of initial wealth are high (low) have a low (high) possibility to encounter the amount of liability beyond their own wealth. In the end, we prospect that the amount of social loss depends on not only the accident probability distribution function but also the wealth distribution function.

References


