Long-Term Care Models and Dependence Probability Tables by Acuity Level: New Empirical Evidence from Switzerland

Michel Fuino, Joël Wagner*

Abstract

Due to the demographic changes and population aging occurring in many countries, the financing of long-term care (LTC) poses a systemic threat. The scarcity of knowledge about the probability of an elderly person needing help with activities of daily living has hindered the development of insurance solutions that complement existing social systems. In this paper, we consider two models: a frailty level model that studies the evolution of a dependent person through mild, moderate and severe dependency states to death and a type of care model that distinguishes between care received at home and care received in an institution. We develop and interpret the expressions for the state- and time-dependent transition probabilities in a semi-Markov framework. Then, we empirically assess these probabilities using a novel longitudinal dataset covering all LTC needs in Switzerland over a 20-year period. As a key result, we are the first to derive dependence probability tables by acuity level, gender and age for the Swiss population. We discuss significant differences in the transition probabilities by gender, age and duration. Using sociodemographic covariates, we reveal the importance of household composition and geographical region of residence for selected transitions.

Key words long-term care · semi-Markov model · actuarial dependence tables · covariates

1 Introduction

One of the most dramatic challenges facing many high-income countries is population aging. Therefore, long-term care (LTC) delivered to elderly persons in need of assistance in activities of daily living (ADL, e.g., dressing, bathing, eating) is predicted to increase in the foreseeable future (United Nations, 2015). In many countries, over a 30-year horizon from the present, spending on formal LTC is expected to reach approximately 2% of GDP (Colombo et al., 2011; Rockinger and Wagner, 2016; Fuino and Wagner, 2017) while the value of informal care delivered by relatives remains important (Pickard et al., 2000; Karlsson et al., 2006; Brown and Finkelstein, 2009; Zhou-Richter et al., 2010). This stresses the relevance of proper financing and pricing of LTC. At present, countries employ various approaches to distribute these costs (see, e.g., Colombo, 2012, Costa-Font et al., 2015). Often, one part is taken over by state social systems, either through comprehensive universal schemes offering basic coverage to the

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entire population or means-tested schemes that subsidize individuals' expenses. Such systems are typically financed through levies from salaries or tax contributions. Another part of the LTC costs is borne by health or other private insurance plans. However, the availability of such insurance is often limited, even in the most developed LTC markets (e.g., the US, the UK, or France). Indeed, private insurers face difficulties in determining proper pricing, which often entails higher premiums and re-pricing (Carrns, 2015). Finally, in many countries, households cover more than one-third of the formal LTC costs. For example, in Switzerland, no attractive or affordable insurance offering exists, and Switzerland ranks among the countries with the highest out-of-pocket spending (Swiss Re, 2014). The problem in pricing LTC solutions essentially results from a lack of knowledge on individuals' health paths. Additionally, the effect of gender, age and other sociodemographic factors such as culture (Eugster et al., 2011; Gentili et al., 2016) is often not well understood.

The aim of our work aims is to reduce this gap by providing dependence tables that provide a basis for the pricing of LTC solutions. With respect to the main cost drivers, which are the frailty level, the type of care and the time spent in dependence, we describe the patterns of individual transitions through the dependency states. To do so, we use a comprehensive longitudinal dataset covering the total dependent population in Switzerland over a 20-year period. We detail the transition probabilities by gender and by age. Using covariates, we discuss the importance of household composition and region of residence in selected transitions.

The actuarial valuation of LTC products is commonly based on Markov processes (see, e.g., Haberman and Pitacco, 1999; Pritchard, 2006; Ameriks et al., 2011; Christiansen, 2012; Brown and Warshawsky, 2013; Ai et al., 2016). Thereby, the calculation of transition intensities between frailty states at different ages is often the focus (e.g., Levantesi and Menzietti, 2012; Fleischmann, 2015; Fong et al., 2015). This approach, however, only considers the previously visited state to be relevant information to determine the future state. Since the seminal work of Hoem (1972), several studies in the area of disability have identified two important factors: if the transition probability clearly depends on the previous state, it also depends on the time spent in that state. In their study on German LTC insurance, Czado and Rudolph (2002) extend a Markov-type model by introducing time-dependent transition intensities along frailty levels and types of care. Within a similar setup, Helms et al. (2005) estimate transition probabilities and calculate insurance premiums for LTC plans. To consider both factors, the *semi*-Markov model extends the Markovian approach and allows to choose the duration law (Janssen and Manca, 2001, 2007; Denuit and Robert, 2007). This semi-Markov framework has long been applied to understand individuals' patterns with respect to health status (see, e.g., Foucher et al., 2005, 2007, 2010; D'Amico et al., 2009). More recently, works on LTC have adopted the semi-Markov approach. When modeling reverse mortgages for the UK and US markets, Ji et al. (2012) consider LTC facilities among health-related reasons for terminating the mortgage. Based on a French LTC insurance portfolio, Biessy (2015b) estimates the semi-Markov parameters and discusses the transition intensities through four levels of dependency. Biessy (2016), using an illness-death model, studies the impact of pathologies on the evolution of LTC.

In this paper, we develop two semi-Markov models to address LTC pricing in Switzerland. Our work is most similar to Biessy (2015b); however, we consider two separate models, derive transition probabilities and apply our study to a larger empirical dataset. We address the question of the type of care received, basing our analyses on data on the total elderly population in need of

LTC. The first model is a five-state model in which we distinguish autonomy, three frailty levels along mild, moderate and severe dependency, and death. This model permits us to explain the evolution through the considered acuity levels. In the second model, our aim is to investigate the transitions between care received at home and care received in institutions because the costs associated with the two types of care are significantly different. To do so, we introduce a four-state model including autonomy, care at home, care in an institution, and death. We separately define age-dependent transition probabilities for men and women (cf. Fong et al., 2015). Our empirical analysis reveals that a Weibull duration law accurately models the time spent in the previous state. Furthermore, we introduce two covariates, namely household composition and linguistic region, using the Cox proportional hazard model (Cox, 1972). We formulate the likelihood for the semi-Markov model and find the solution by estimating the maximum likelihood. We provide an application using novel longitudinal data on the total old-age dependent population in Switzerland from 1995 to 2014. Our dataset provides complete information on the paths of 285 000 individuals, including dates of transitions, dependency states, gender, age, civil status and place of residence. This represents an extension of the studies mentioned above that solely use private insurance datasets with a more limited number of observations.

Our main results are two-dimensional transition probabilities defined by age and the elapsed time in the previous state for both men and women. To the best of our knowledge, we are the first to provide such a detailed study and to derive actuarial tables for Switzerland. We present significant results on the evolution of dependencies and the types of care received. Our key findings are as follows: First, we observe significant differences in transition probabilities between men and women and between individuals below and above 80 years of age. Further, the probabilities of staying in a dependency state decrease with age and duration, whereas the probabilities of leaving a state increase with duration. While the average total time spent in dependence is approximately three years, we find that elderly persons cared for at home enter institutional care after approximately one year. Second, for short durations, mildly dependent individuals have a higher mortality relative to moderately and severely dependent persons. This may be linked to the fact that mildly dependent elderly are more often cared for at home and that different pathologies are underlying their dependence (see, e.g., Monod-Zorzi et al., 2007; Biessy, 2016). Third, we find that women, given their lower mortality, spend more time than men in any of the dependency states (see, e.g., Fong, 2017). Finally, we also measure the impact of the considered covariates. On the one hand, we find that individuals living in a two-person household have a significantly higher relative risk to realize a given transition than those living in a single household. On the other hand, the region of residence affects the transition probabilities, with a lower relative risk in French- and Italian-language regions relative to the German-speaking region of Switzerland.

This article is organized as follows: Section 2 introduces our two models and the mathematical aspects of the semi-Markov framework. In Section 3, we provide an extended description of the dataset. In Section 4, after a brief presentation of the numerical implementation, we report the parameters of the model and present the dependence probability tables. Further, we discuss the relevance of covariates other than gender and age. We conclude in Section 5.

2 Model framework

In this paper, we consider a model framework conceptualizing the frailty levels and types of care as recorded in Switzerland. Typically, three frailty levels (mild, moderate, severe) are considered, and care given is differentiated between at-home and institutional care. After the introduction of the models (see Section 2.1), we develop on a semi-Markov-type model to calculate the hazard rates and transition probabilities. We separately discuss the probability of entering dependence in Section 2.3. The model framework developed in this section can be directly applied to statistical data to pose the actuarial bases for pricing insurance products.

2.1 Old-age dependency models

We consider two models: The first is linked to the transition between frailty levels and death, and the second focuses on the transitions between states with different types of care. Overall, six different states of dependency can be considered (Czado and Rudolph, 2002). They result from the combination of the three dependency levels (mild, moderate, severe) and the two types of care (at-home and institutional care).

Our first model focuses on frailty levels. The assessment of care needs is usually based on the number of limitations in ADL (e.g., dressing, bathing, eating; see also Section 3.1). This international measure is used to determine a dependent person's need for care and is considered a rather accurate proxy for the hours of care required. In this sense, serious cases require more care and entails higher costs. In many developed countries such as Switzerland, the state offers an allowance based on the patient's dependency level. This motivates our first model, which analyzes the possible evolution of a dependent person through three different frailty levels while recording the time spent in each state. Indeed, the model we consider uses five states. The first state is the autonomous state (0). Then, three dependency states are distinguished by their respective severity: mild (1), moderate (2) or severe (3). The last state considered is the death state (4). Only forward transitions are considered, i.e., a dependent person can neither recover nor decrease their acuity level. This is a reasonable assumption because in practice those probabilities are negligible and this hypothesis is frequently used (e.g., Foucher et al., 2010; Levantesi and Menzietti, 2012; Biessy, 2015a; Fong, 2017). The mortality of the autonomous population is

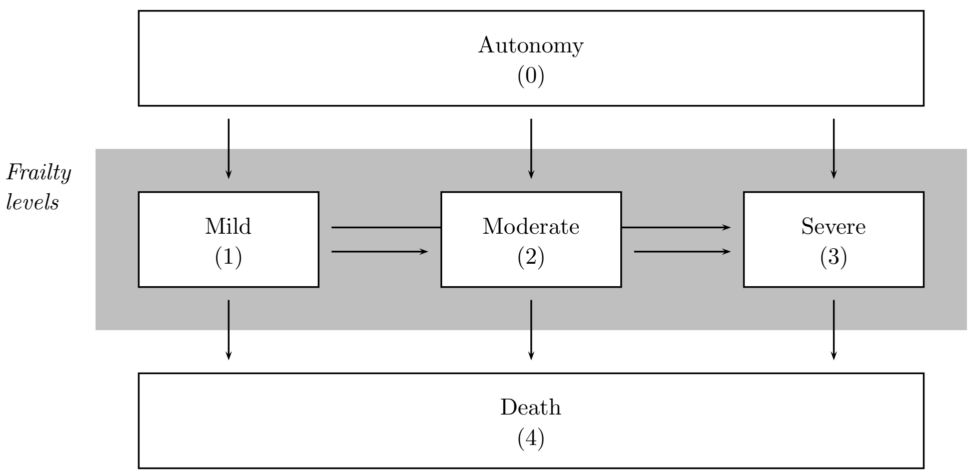


Figure 1: Illustration of the frailty level model.

outside the scope of our model because we are interested in the characteristics of the dependent population. Figure 1 provides a representation of the frailty level model. In our model, nine transitions are possible. The arrows describe the possible transitions. This frailty level model is best suited to studying the increasing dependency levels of the elderly. The focus is on the individual paths, i.e., the transitions and time spent in each state from autonomy to death.

Our second model focuses on the type of care. Depending on the country, different types of care facilities are available, and individuals may choose between receiving care at home and receiving care in an institution (Costa-Font and Courbage, 2012). Living in institutions produces higher costs than receiving care at home, especially due to the additional accommodation costs (e.g., laundry, feeding). This is highly relevant when pricing insurance solutions. In our study, we distinguish the type of care facilities based on whether accommodation is included: The at-home care type represents individuals receiving care in their own residence without needing lodging or meals, while the institutional care type includes lodging and meals. Based on this cost driver, we elaborate a second model that considers four states: autonomy (0); two types of care, care at home (a) and care in an institution (b); and death (4). Similar to the first model, we do not consider recovery or returns to care at home from institutional care. Figure 2 illustrates this second model and the possible transitions.

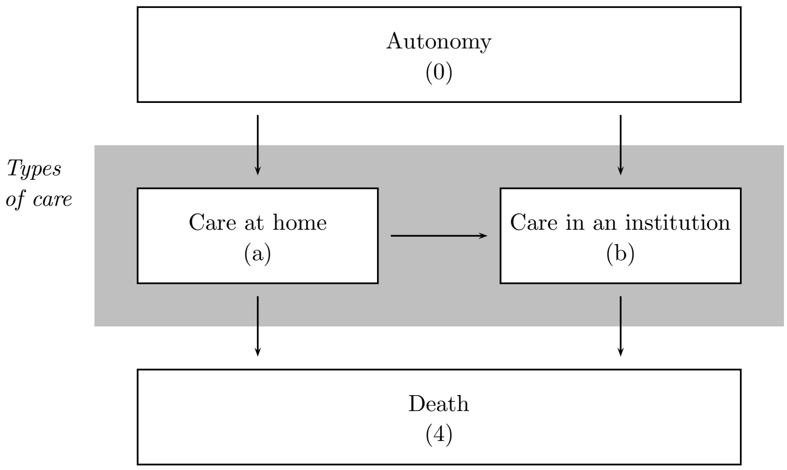


Figure 2: Illustration of the type of care model.

2.2 The Semi-Markov model framework

In the following, we introduce the general theoretical framework for a semi-Markov process first developed by Hoem (1972) (see, e.g., Janssen and Manca, 2007, for theoretical foundations and applications). In contrast to a Markov process, which depends solely on the previously visited state, a semi-Markov process incorporates a time variable; that is, the transition probability is both affected by the previously visited state and the time spent in it. Our objective is to calculate the transition probabilities between the dependency and death states in the two models described above, separately with respect to gender and age, breaking each age-gender group into a homogeneous model. The non-homogeneity is addressed by splitting the dataset by gender and by age when conducting our estimations (see Section 4.2).

Theoretical framework

In the sequel, we follow the notations proposed by Janssen and Manca (2001) and Saint-Pierre (2005). With the state space $I = \{1, 2, \ldots, m\}$, let us consider the states J_n, $n \in \mathbb{N}$. Let T_n denote the time of the $(n+1)$th transition going from state J_n to state J_{n+1}. Then,

$$X_{n+1} = T_{n+1} - T_n > 0, \tag{1}$$

is the sojourn time in state J_{n+1}. We assume that the transition probability for someone staying from some time s to time $t-s$ reduces to the same probability as someone staying from time 0 to t; i.e., when arriving in a given state, time is reset to zero. Thus, the time variable t is interpreted as a duration. The semi-Markov kernel $Q_{ij}(t)$ completely defines the process as follows:

$$Q_{ij}(t) = \Pr(J_{n+1} = j, X_{n+1} \leq t \mid J_n = i). \tag{2}$$

The function $Q_{ij}(t)$ represents the probability for the process to travel from state i to state j at the $(n+1)$th transition before a duration t. This expression is entirely defined by both the underlying embedded Markov chain and the duration law. The Markov chain describes the probability ϕ_{ij} to go from state i to state j disregarding the time spent in the states:

$$\phi_{ij} = \lim_{t \to +\infty} Q_{ij}(t) = \Pr(J_{n+1} = j \mid J_n = i). \tag{3}$$

The duration law $F_{ij}(t)$, characterizes the duration elapsed in state i before the transition to state j occurs, i.e.,

$$F_{ij}(t) = \Pr(X_{n+1} \leq t \mid J_n = i, J_{n+1} = j). \tag{4}$$

This distribution function describes the cumulative probability of the time spent in the previous state, knowing that the process traveled from state i to state j. Without loss of generality, the semi-Markov kernel $Q_{ij}(t)$ can be explicitly expressed as

$$Q_{ij}(t) = \phi_{ij} \, F_{ij}(t). \tag{5}$$

Furthermore, assuming that it exists, we define the density function $f_{ij}(t)$ of the duration law $F_{ij}(t)$ by

$$f_{ij}(t) = \frac{\partial F_{ij}(t)}{\partial t}. \tag{6}$$

By definition, the instantaneous transition probabilities $\lambda_{ij}(t)$ are obtained from (De Dominicis and Janssen, 1984):

$$\lambda_{ij}(t) = \frac{\phi_{ij} \, f_{ij}(t)}{\sum_{j=1}^{m} \phi_{ij} \, (1 - F_{ij}(t))} \qquad \text{if } \phi_{ij} \neq 0 \text{ and } F_{ij}(t) \neq 1, \tag{7}$$

and $\lambda_{ij}(t) = 0$ otherwise.

Transition probabilities

The above Equation (7) expresses the instantaneous probability that the process exits state i for state j in an infinitesimal time interval $]t, t + dt]$. Consequently, the transition probabilities solving the semi-Markov model are

$$p_{ij}(t) = \begin{cases} e^{-\int_0^t \sum_{k\neq i} \lambda_{ik}(\tau)\,\mathrm{d}\tau} + \int_0^t p_{ii}(\tau) \sum_{k\neq i} \lambda_{ik}(\tau)\, p_{kj}(t-\tau)\,\mathrm{d}\tau & \text{if } i = j, \\ \int_0^t p_{ii}(\tau) \sum_{k\neq i} \lambda_{ik}(\tau)\, p_{kj}(t-\tau)\,\mathrm{d}\tau & \text{if } i \neq j. \end{cases} \tag{8}$$

This actuarial formulation of the transition probabilities can be obtained from the Chapman-Kolmogorov equation (see Pitacco, 1995, who provides a comprehensive approach for pricing disability benefits). In the case in which $i = j$ in Equation (8), the expression of the semi-Markov probability can be decomposed into two parts. The first term considers the case of staying in state i, while the second term defines the case of having made at least one transition. This term accounts for all the possible paths that return to i after having left it. When $i \neq j$, the expression in Equation (8) simplifies because the process has left the initial state at least once and only the second term remains. In the case in which backward transitions are not possible, i.e., $p_{ij} = 0$ and $\lambda_{ij} = 0$ for $i > j$, we find the following from Equation (8):

$$p_{ij}(t) = \begin{cases} e^{-\int_0^t \sum_{k>i} \lambda_{ik}(\tau)\,\mathrm{d}\tau} & \text{if } i = j, \\ \int_0^t p_{ii}(\tau) \sum_{k>i} \lambda_{ik}(\tau)\, p_{kj}(t-\tau)\,\mathrm{d}\tau & \text{if } i < j. \end{cases} \tag{9}$$

We now apply the above general results to our models introduced in Section 2.1. From Equation (9), we can explicitly express the transition probabilities linked to the transitions from both models. For the frailty level model, we have the state space $I = \{1, 2, 3, 4\}$. We exclude the autonomous state (0) in I since we separately discuss the probability of losing autonomy in Section 2.3. The "staying" probabilities with $i = j$, where $i, j \in I$, are written as follows:

$$\begin{aligned} p_{11}(t) &= e^{-\int_0^t \lambda_{12}(\tau) + \lambda_{13}(\tau) + \lambda_{14}(\tau)\,\mathrm{d}\tau}, \\ p_{22}(t) &= e^{-\int_0^t \lambda_{23}(\tau) + \lambda_{24}(\tau)\,\mathrm{d}\tau}, \\ p_{33}(t) &= e^{-\int_0^t \lambda_{34}(\tau)\,\mathrm{d}\tau}, \\ p_{44}(t) &= 1. \end{aligned} \tag{10}$$

The probabilities $p_{11}(t)$, $p_{22}(t)$, $p_{33}(t)$ and $p_{44}(t)$ denote the probabilities of staying in the mild (1), moderate (2), severe (3) or death (4) states for a duration t. Note that state 4, representing death, is an absorbing state and leads to $p_{44} = 1$. The "leaving" probabilities when $i < j$ complement the definition of the semi-Markov chain:

$$\begin{aligned} p_{12}(t) &= \int_0^t p_{11}(\tau)\, \lambda_{12}(\tau)\, p_{22}(t-\tau)\,\mathrm{d}\tau, \\ p_{13}(t) &= \int_0^t p_{11}(\tau)\, \left[\lambda_{12}(\tau)\, p_{23}(t-\tau) + \lambda_{13}(\tau)\, p_{33}(t-\tau)\right]\mathrm{d}\tau, \\ p_{14}(t) &= \int_0^t p_{11}(\tau)\, \left[\lambda_{12}(\tau)\, p_{24}(t-\tau) + \lambda_{13}(\tau)\, p_{34}(t-\tau) + \lambda_{14}(\tau)\, p_{44}(t-\tau)\right]\mathrm{d}\tau, \\ p_{23}(t) &= \int_0^t p_{22}(\tau)\, \lambda_{23}(\tau)\, p_{33}(t-\tau)\mathrm{d}\tau, \\ p_{24}(t) &= \int_0^t p_{22}(\tau)\, \left[\lambda_{23}(\tau)\, p_{34}(t-\tau) + \lambda_{24}(\tau)\, p_{44}(t-\tau)\right]\mathrm{d}\tau, \\ p_{34}(t) &= \int_0^t p_{33}(\tau)\, \lambda_{34}(\tau)\, p_{44}(t-\tau)\,\mathrm{d}\tau. \end{aligned} \tag{11}$$

These expressions deserve a brief interpretation. The expression of the probability p_{12} considers the probability $p_{11}(\tau)$ of remaining for a time τ in state 1 and reaching state 2 through the direct transition from state 1 to state 2 after duration t because there are no intermediate states. The factor $p_{22}(t-\tau)$ expresses staying in state 2 during the remaining time $t-\tau$. The other transition probabilities follow the same reasoning. For example, the transition p_{14} considers all possible paths starting from the mild state (1) and ending with death (state 4). The first factor $p_{11}(\tau)$ in the expression of the integral uses the probability of staying for a time τ in state 1. The second factor (brackets $[\ldots]$) includes the possible transitions reaching state 4: This either materializes through an indirect transition via states 2 or 3 or a direct transition. For example, the first term $\lambda_{12}(\tau)p_{24}(t-\tau)$ in brackets considers the transition from 1 to 2 exactly after a duration τ, then considers all the possible ways to transit from state 2 to reach state 4 in the remaining time $t-\tau$. Here, the factor p_{24} must be calculated separately (cf. Equation 11), including the indirect path $2 \to 3 \to 4$ and the direct path $2 \to 4$. Similarly, for the type of care model with $I = \{a, b, 4\}$, we have the probabilities of staying in care at home (a) and care in an institution (b),

$$\begin{aligned} p_{aa}(t) &= e^{-\int_0^t \lambda_{ab}(\tau) + \lambda_{a4}(\tau)\,\mathrm{d}\tau}, \\ p_{bb}(t) &= e^{-\int_0^t \lambda_{b4}(\tau)\,\mathrm{d}\tau}, \end{aligned} \tag{12}$$

and, again, $p_{44}(t) = 1$. The leaving probabilities are as follows:

$$\begin{aligned} p_{ab}(t) &= \int_0^t p_{aa}(\tau)\,\lambda_{ab}(\tau)\,p_{bb}(t-\tau)\,\mathrm{d}\tau, \\ p_{a4}(t) &= \int_0^t p_{aa}(\tau)\,\left[\lambda_{ab}(\tau)\,p_{b4}(t-\tau) + \lambda_{a4}(\tau)\,p_{44}(t-\tau)\right]\mathrm{d}\tau, \\ p_{b4}(t) &= \int_0^t p_{bb}(\tau)\,\left[\lambda_{b4}(\tau)\,p_{44}(t-\tau)\right]\mathrm{d}\tau. \end{aligned} \tag{13}$$

The sets of equations (10) and (11), respectively (12) and (13), describe the semi-Markov probabilities of transitioning between the different dependency states and death for both models. The duration law introduced in Equation (4) and the probabilities of losing autonomy are specified in the following.

Duration law

In the semi-Markov model, the duration law $F_{ij}(t)$ introduced in Equation (4) plays an important role. This function is a stochastic representation of the time spent in the previous state that defines the probability distribution for the sojourn times. In other words, the duration law attributes a probability to each realization of the positive random variable X_n from Equation (1). In our application, we use a Weibull duration law because this distribution is well suited to our data: It is skewed to the right and it only requires the calibration of two parameters, thereby reducing the estimation errors (for empirical evidence, see Figures 5, 6 and 7 in Section 3.5). The Weibull duration law is expressed as follows:

$$F_{ij}(t) = 1 - e^{-(t/\theta_{ij})^{\sigma_{ij}}}, \tag{14}$$

where $\sigma_{ij} > 0$ represents the shape parameter and $\theta_{ij} > 0$ the scale parameter for a transition from i to j occurring after a duration $t \geq 0$ in state i. The corresponding density function $f_{ij}(t)$, see Equation (6), yields the following:

$$f_{ij}(t) = \frac{\sigma_{ij}}{\theta_{ij}} \left(\frac{t}{\theta_{ij}}\right)^{\sigma_{ij}-1} e^{-(t/\theta_{ij})^{\sigma_{ij}}}. \tag{15}$$

Duration law with covariates

We consider an extension of the model framework introduced above and include covariates other than gender and age that represent specific characteristics related to the dependent persons such as the type of household or the linguistic region of residence. In doing so, we refer to the Cox proportional hazard rate approach, a log-linear regression approach that is widely used in survival analysis (Cox, 1972). This method has the advantage of providing relevant results that can be easily interpreted if the risk is a proportion. Indeed, by coding covariates as binary variables, the coefficients are interpreted as relative risk, which means an increase or decrease in risk of the related variable of interest with respect to a baseline. In our paper, we assume that the risk can be expressed as a proportion. For each covariate, the estimated coefficient is independent of the duration. Based on Equation (4), the expression of the duration law with covariates becomes

$$F_{ij}(t, \mathbf{z}) = 1 - \left(e^{-(t/\theta_{ij})^{\sigma_{ij}}}\right)^{e^{\boldsymbol{\beta}_{ij}\mathbf{z}_{ij}}}, \tag{16}$$

where $\mathbf{z}_{ij}$ represents the vector containing the binary values (0 or 1) of the covariates for a transition from i to j and $\boldsymbol{\beta}_{ij}$ is the regression coefficient, for a transition from i to j. In this approach, the covariates have no influence on the embedded Markov chain probabilities but affect the shape and scale parameters of the Weibull duration law. The corresponding respective density function $f_{ij}(t, \mathbf{z})$ with covariates is

$$f_{ij}(t, \mathbf{z}) = \frac{\sigma_{ij}}{\theta_{ij}} \left(\frac{t}{\theta_{ij}}\right)^{\sigma_{ij}-1} \left(e^{-(t/\theta_{ij})^{\sigma_{ij}}}\right)^{e^{\boldsymbol{\beta}_{ij}\,\mathbf{z}_{ij}}} e^{\boldsymbol{\beta}_{ij}\,\mathbf{z}_{ij}}. \tag{17}$$

2.3 Probability of losing autonomy

A common challenge in the estimation of transition probabilities for LTC concerns the "entry" probability, that is, the probability of losing autonomy and entering one of the acuity states. This challenge arises because datasets focusing on LTC only contain information about the dependent population while disregarding others. The lack of knowledge about the total population impedes the estimation of the probability of losing autonomy and leaves an important gap to fill. Moreover, as we only consider the population aged over 65 years, the semi-Markov model is not appropriate for these transitions. In this context, the time spent in the previous state has a lower bound at the person's age of 65 years.

We propose to enrich an LTC dataset with data describing the total population. The estimation of the transition probabilities $p_{0j}(x)$ for a given age x reduces, on the one side, to the estimation of prevalence rates $\pi(x)$ and, on the other side, to the estimation of the Markov probabilities $\phi_{0j}(x)$. The prevalence rates $\pi(x)$ represent for a given age x the ratio of the population entering one of the three acuity states over the total population. The estimated transition probabilities $p_{0j}(x)$ from autonomy (0) to any acuity state $j \in I$ correspond to the product of the prevalence rate multiplied by the Markov probability:

$$p_{0j}(x) = \pi(x)\phi_{0j}(x). \tag{18}$$

3 Dataset and descriptive statistics

In this section, we introduce the dataset used for our analysis (see Section 3.1) and describe the major characteristics that are relevant to the model and the interpretation of the results. In Section 3.2, we report the descriptive statistics in Table 1 and discuss the prevalence rates. Then, we present statistics on the evolution through dependency, including the number of dependent persons and the durations in the different dependency states (see Section 3.3). In Section 3.4, we apply the data to both dependency models and analyze the paths followed. Finally, we argue for the choice of the duration law with empirical evidence in Section 3.5.

3.1 Description of available data

The Swiss old-age care system provides LTC benefits for non-autonomous persons aged over 65. The first pillar of the Old-Age Social Insurance (OASI) law regulates benefits. They cover the dependence of an elderly person suffering from limitations in ADL such as dressing, bathing, and eating, which require different levels of assistance and personal supervision, as fully described by the Swiss Federal Social Insurance Office (2015, FSIO). The amount of the allowance depends both on the acuity of the dependence and the canton of residence. There are cantonal differences in the allowance paid since each canton can select the amount. The Swiss system distinguishes three levels of acuity. *Mild* acuity characterizes persons needing regular assistance in at least two ADL or permanent personal supervision. *Moderate* acuity defines dependents needing assistance in at least two ADL and permanent personal supervision, while *severe* acuity identifies insured persons in need of regular assistance with all the daily living activities and further entails permanent personal supervision.

In Switzerland, two offices make statistics on old-age care available. The Swiss Federal Statistical Office (FSO) publishes yearly statistics on a broad range of topics including the population census[1] and aggregate figures on elder care.[2] The Swiss Central Compensation Office (CCO)[3] specializes in the benefits paid under the old-age insurance scheme concerning both pension benefits and disability benefits. The CCO provides more detailed information upon motivated request. For the purposes of our study, we combine the two sources to create a unique, novel dataset. First, we use the FSO census of the resident population. This dataset covers a period of 20 years from 1995 to 2014 and reports the annual number of individuals living in Switzerland by age and gender. Second, we consider a detailed dataset that reports information on elderly persons receiving old-age care benefits under the OASI law. This dataset was purpose-built for our study by the CCO and contains, for each beneficiary, information on gender, year of birth, civil status, canton of residence, level of dependency and type of care received.[4] For the

[1] www.bfs.admin.ch/bfs/de/home/statistiken/bevoelkerung/stand-entwicklung/bevoelkerung.html

[2] www.bfs.admin.ch/bfs/de/home/statistiken/soziale-sicherheit/sozialversicherungen/ahv.html

[3] www.zas.admin.ch

[4] The original data were compiled by the CCO in 2016 and contain information for the period from 1995 to 2015. However, at that time, the data for the year 2015 were still provisional because the figures are typically adjusted following updates from the cantonal instances. Therefore, we remove the 2015 data and cover the years from 1995 to 2014.

dependency states, the related start and end dates are reported at monthly precision. Each time that a change in the individual status appears, e.g., a change in the acuity level, death or departure from Switzerland, a variable records the reason, and an updated entry appears in the data (except for death). The level of dependency variable reports on the acuity level (mild, moderate or severe), while the type of care variable indicates whether the individual receives care at home or in an institution (see Section 2.1). Civil status is defined in terms of nine categories.[5] We consolidate these categories into two groups: the *two-person households* containing the "married" and the "registered partner" categories and the *single-person households* group containing all other categories. Furthermore, we cluster the 26 cantons into the three main linguistic regions, the *German-*, *French-* and *Italian*-speaking parts of Switzerland that we will interpret as a simple proxy for cultural differences.[6]

Before reshaping the data into (1) a cross-sectional dataset for calculating yearly statistics on prevalence rates and (2) a longitudinal dataset to be employed in the semi-Markov model (see Sections 4.2 to 4.4) and deriving transition probabilities, some data cleaning is required. For a limited number of beneficiaries, the payment stream discontinues, and we are unable to identify whether the introduced gaps are due to seasonal movements (e.g., living in an institution during winter and at home without being registered for care during summer) or to missing entries. We remove entries presenting such discontinuities. By doing so, we not only eliminate the incomplete entry but also disregard the history of this individual to avoid potential outliers. We also remove beneficiaries who leave Switzerland because such persons can no longer be tracked.

3.2 Cross-sectional old-age care statistics

Table 1 reports descriptive statistics for the period from 1995 to 2014 on the population registered for old-age care benefits, the total population aged 65+ and the prevalence rates. The row labeled "CCO data" reports the aggregated number of elderly persons derived from the CCO data at the end of each year, while the "FSO data" row reports the numbers published by the FSO in its brief statistics starting in the year 1999. The cross-sectional view constructed from the detailed CCO data overestimates the FSO numbers by 10.71%.[7] Therefore, the CCO data and the prevalence rates presented in the table are corrected by this average factor to better reflect published values. We also account for this correction in the estimates of the probabilities of losing autonomy (see Section 4.1). Overall, we observe that the population registered for old-age care benefits and the prevalence rates increase continually over the period considered (see also Fuino and Wagner, 2017). When considering the distributions, we note that the average individual receiving old-age care benefits is a woman over 80 years of age living alone in the German-speaking region with a moderate or severe state of dependency being cared for in an institution. Indeed, females represent more than 65% of the dependent persons contained in the dataset. The share of beneficiaries living in single-person households decreases from 70.5% to

[5] The reported civil status categories in the CCO data are single, married, widower, divorced, separated by judicial decision, registered partner, dissolved partnership between persons of the same sex, dissolved partnership due to the death of one partner and separated by judicial decision for persons of the same sex.

[6] Three large linguistic regions are distinguished in Switzerland. These regions are (1) the *German*-speaking region comprising the cantons of Aargau, Appenzell Innerrhoden, Appenzell Ausserrhoden, Bern, Basel-Landschaft, Basel-Stadt, Glarus, Graubünden, Luzern, Nidwalden, Obwalden, St. Gallen, Schaffhausen, Solothurn, Schwyz, Thurgau, Uri, Zug, and Zürich; (2) the *French*-speaking region comprising the cantons of Fribourg, Genève, Jura, Neuchâtel, Vaud, and Valais; and (3) the *Italian*-speaking region formed by the canton of Ticino.

[7] Differences in the numbers may arise from the exact registration dates of the acuity levels, how up to date the sources are, the processes for aggregation used, and the cleaning of incomplete entries.

Year		1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
Population registered for old-age care benefits																					
CCO data	th.	33.5	34.4	35.1	36.6	37.2	38.7	40.3	40.8	41.2	42.4	43.0	43.4	43.7	44.0	44.5	45.5	54.3	56.0	57.8	58.9
FSO data	th.	n.a.	n.a.	n.a.	n.a.	37.8	38.4	39.2	41.4	42.6	42.6	44.1	44.6	45.6	45.7	45.8	44.5	51.1	53.8	55.8	57.4
Distribution by age classes																					
65–79	%	35.0	35.6	36.3	36.2	36.5	36.2	35.4	35.3	34.9	34.4	34.4	34.3	34.5	34.6	35.0	34.9	34.4	34.7	35.0	35.3
80+	%	65.0	64.4	63.7	63.8	63.5	63.8	64.6	64.7	65.1	65.6	65.6	65.7	65.5	65.4	65.0	65.1	65.6	65.3	65.0	64.7
Distribution by gender																					
Male	%	29.0	29.1	29.4	29.6	29.8	30.1	30.2	30.2	30.3	30.3	30.5	30.6	31.0	31.3	31.8	32.2	32.5	33.0	33.3	33.5
Female	%	71.0	70.9	70.6	70.4	70.2	69.9	69.8	69.8	69.7	69.7	69.5	69.4	69.0	68.7	68.2	67.8	67.5	67.0	66.7	66.5
Mean age by gender																					
Male	yr.	78.7	78.7	78.7	78.9	78.9	79.0	79.1	79.2	79.3	79.5	79.6	79.6	79.6	79.5	79.5	79.5	79.8	79.8	79.9	79.9
Female	yr.	83.6	83.6	83.6	83.7	83.7	83.7	83.9	83.9	83.9	84.0	84.0	84.0	84.0	83.9	83.9	83.8	83.7	83.7	83.6	83.6
Type of household																					
Single person	%	70.5	69.5	68.5	67.8	66.7	66.1	65.8	65.3	65.3	65.1	64.7	64.4	63.5	62.8	61.7	60.8	60.0	59.1	58.5	58.1
Two persons	%	29.5	30.5	31.5	32.2	33.3	33.9	34.2	34.7	34.7	34.9	35.3	35.6	36.5	37.2	38.3	39.2	40.0	40.9	41.5	41.9
Linguistic regions																					
German	%	68.7	68.4	68.2	68.2	68.0	67.2	66.8	66.7	66.8	66.7	66.7	66.8	66.7	65.9	65.0	64.8	64.6	64.2	64.3	64.5
French	%	25.3	25.4	25.2	24.8	24.5	25.0	25.4	25.3	25.0	25.0	24.9	24.6	24.7	25.4	26.1	26.2	27.2	27.5	27.5	27.2
Italian	%	6.0	6.2	6.6	7.0	7.5	7.8	7.8	8.0	8.2	8.3	8.4	8.6	8.6	8.7	8.9	9.0	8.2	8.3	8.4	8.3
Distribution by frailty levels																					
Mild	%	5.4	5.6	5.7	5.6	5.7	5.7	5.7	5.7	5.8	5.8	5.9	6.0	6.4	6.8	7.1	7.3	19.0	21.6	23.2	24.1
Moderate	%	31.1	32.4	34.0	35.1	36.5	37.1	38.0	38.9	39.8	40.4	41.4	41.9	42.6	43.2	44.2	45.8	41.2	40.9	41.1	41.0
Severe	%	63.5	62.0	60.3	59.3	57.8	57.2	56.3	55.4	54.4	53.8	52.7	52.1	51.0	50.0	48.5	46.9	39.8	37.5	35.7	34.9
Distribution by type of care																					
At home	%	–	–	–	–	–	–	0.0	0.0	0.0	0.4	0.9	1.8	3.1	4.1	5.2	6.0	18.4	21.6	23.8	25.0
In institution	%	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	99.6	99.1	98.2	96.9	95.9	94.8	94.0	81.6	78.4	76.2	75.0
Care at home: distribution by frailty																					
Mild	%	–	–	–	–	–	–	0.0	0.0	0.0	0.2	0.4	0.8	1.4	2.0	2.7	3.1	15.5	18.4	20.5	21.5
Moderate	%	–	–	–	–	–	–	0.0	0.0	0.0	0.1	0.3	0.6	1.1	1.4	1.7	2.0	2.0	2.2	2.3	2.4
Severe	%	–	–	–	–	–	–	0.0	0.0	0.0	0.1	0.2	0.4	0.6	0.7	0.8	0.9	0.9	1.0	1.0	1.1
Total	%	–	–	–	–	–	–	0.0	0.0	0.0	0.4	0.9	1.8	3.1	4.1	5.2	6.0	18.4	21.6	23.8	25.0
Care in an institution: distribution by frailty																					
Mild	%	5.4	5.5	5.6	5.6	5.7	5.7	5.7	5.7	5.8	5.7	5.5	5.2	5.0	4.8	4.5	4.2	3.4	3.1	2.8	2.6
Moderate	%	31.1	32.3	34.0	35.1	36.5	37.1	38.0	38.9	39.7	40.2	41.1	41.2	41.5	41.8	42.6	43.8	39.3	38.7	38.8	38.6
Severe	%	63.5	62.0	60.4	59.3	57.8	57.2	56.3	55.4	54.5	53.7	52.5	51.6	50.4	49.3	47.7	46.0	38.9	36.6	34.6	33.8
Total	%	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	99.6	99.1	98.2	96.9	95.9	94.8	94.0	81.6	78.4	76.2	75.0
Population																					
65+	th.	1044.3	1055.1	1066.9	1079.8	1094.3	1109.2	1131.1	1142.5	1156.7	1174.3	1192.5	1216.7	1245.2	1276.4	1308.7	1334.3	1365.2	1398.6	1432.7	1465.6
Distribution by age classes																					
65–79	%	73.2	73.4	73.7	73.9	74.1	73.7	72.9	72.6	72.4	72.1	71.8	71.6	71.6	71.6	71.6	71.8	72.0	72.1	72.2	72.1
80+	%	26.8	26.6	26.3	26.1	25.9	26.3	27.1	27.4	27.6	27.9	28.2	28.4	28.4	28.4	28.4	28.2	28.0	27.9	27.8	27.9
Prevalence rates: number of old-age care beneficiaries divided by the population																					
65+	%	3.2	3.3	3.3	3.4	3.4	3.5	3.6	3.6	3.6	3.6	3.6	3.6	3.5	3.4	3.4	3.4	4.0	4.0	4.0	4.0
Distribution by age classes																					
65–79	%	1.5	1.6	1.6	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.9	1.9	1.9	2.0
80+	%	7.8	7.9	8.0	8.3	8.3	8.5	8.5	8.4	8.4	8.5	8.4	8.3	8.1	7.9	7.8	7.9	9.3	9.4	9.4	9.3

Notes: "n.a" stands for not available data, "0.0" characterizes an entry below 0.1 rounding, and "–" are zero values.

Table 1: Descriptive cross-sectional statistics on old-age care benefits and calculation of prevalence rates for the years from 1995 to 2014.

58.1% over the period considered. Finally, elderly persons being cared for at home have received benefits only since 2001; they represent one-quarter of all dependent registered individuals in 2014. This significant proportion is a consequence of the state incentivizing at-home care versus institutional care by recognizing the role of informal care through the attribution of a specific allowance to the relatives. This is particularly observable in the jump in the share of dependent persons with mild severity living at home (recognized since 2011). Overall, the share of persons cared for at home increases from 6.0% in 2010 to 18.4% in 2011. However, the difference in the figures between the two types of care deserves further explanation. In fact, the number of persons receiving care at home may be strongly underestimated in our statistics. As noted by Weaver (2012), elderly persons living at home are sometimes unaware of the benefits they are entitled to or may forget to request them despite being eligible. This is less the case for elderly being cared in an institution since the institutions manage most administrative tasks. Finally, the total population of elderly persons in Switzerland is presented at the bottom of Table 1. The Swiss census of the population recorded 1 044 thousand individuals aged 65+ in 1995, and this number had grown by 40% by 2014. Further, examining at the distribution of the old-age population, persons over 80 years of age represent approximately one-quarter of the elderly population but show significantly higher prevalence rates than those aged between 65 and 79 years.

Independent of the acuity level, prevalence rates grow with age (see also Fuino and Wagner, 2017). Figure 3 illustrates the 1995 to 2014 average rates as a function of age. The steepest increase, showing exponential behavior, is observed for the severe state. At age 90, on average, 8.0% of the population is in the severe state, 5.4% in the moderate state and less than 1% is in the mild state. This reveals that approximately 14% of the 90-year-old population has significant difficulties in performing ADL.

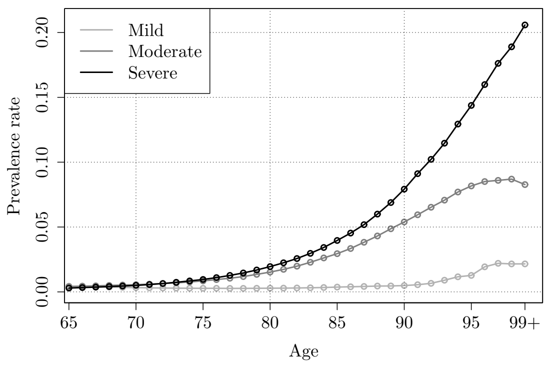


Figure 3: Prevalence rates by age averaged over the period from 1995 to 2014.

3.3 Flow statistics through dependency

Disregarding autonomy and death, we denote six states stemming from the combinations of the three dependency levels (mild, moderate, severe) and the two types of care (at home and institutional). Over the period from 1995 to 2014, the CCO dataset records 284 482 individuals who entered one of these states. We can rely on 269 891 and 220 277 uncensored transitions in the frailty level and the type of care models, respectively (see Section 3.4, Table 2). These numbers can be compared with the data available in other LTC studies, e.g., Biessy (2016) where a total of 20 988 individuals are recorded in France, of whom approximately 16 000 are uncensored. Furthermore, recall that our transitions reflect the total population of Switzerland, where other studies mostly focus on a private insurance dataset (e.g., D'Amico et al., 2009). The dependent persons evolve through different states. In Figure 4, we report statistics on the number of elderly persons entering and leaving each of the states and on the time spent therein. On the one hand, the statistics reveal three main paths to enter dependency: 21 368 autonomous individuals firstly enter the mild state and receive care at home, 134 142 enter the moderate state and receive care in an institution, and 128 214 enter the severe state and receive care in an institution. On the other hand, two main transitions are observed with 5 940 individuals going from a mild level of dependency and being cared for at home to a moderate level and being cared for in an institution, and 49 407 evolving from a moderate to severe dependency level and cared for in an institution. Finally, death appears in our statistics mainly for persons in the moderate and severe states cared for in an institution at 63 691 and 145 625 individuals, respectively.

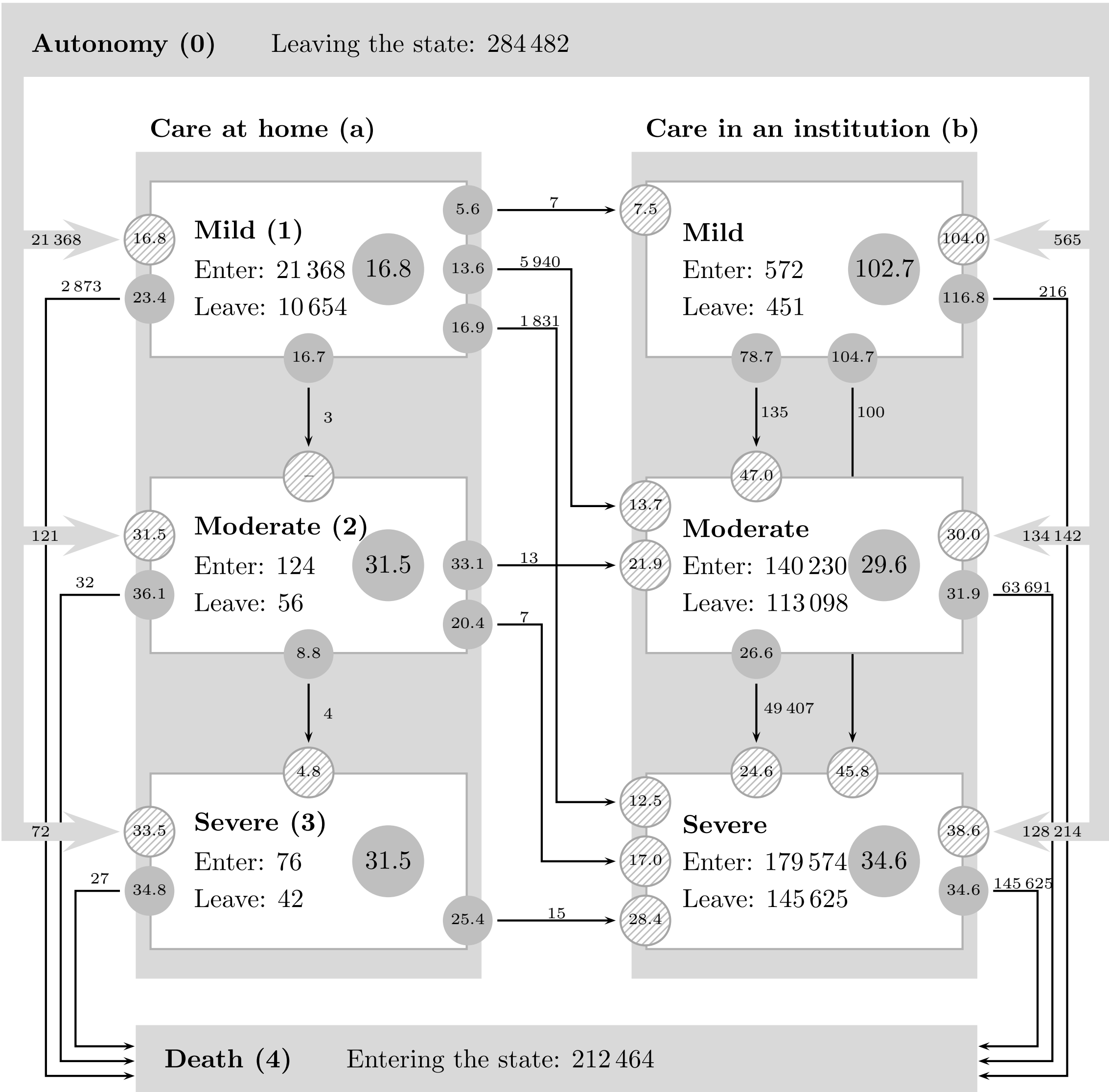


Figure 4: Illustration of the transition from the autonomous state to the different states of dependency until death distinguished by the type of care received.

We next consider the average time spent in each state before the transition to another state. To value the duration in each state, we calculate the difference between the entry and leaving date of a state. In Figure 4, the average number of months spent in each state is presented in the larger bubble. We find that elderly persons receiving care at home stay on average 16.8 months in a mild and 31.5 months in a moderate or a severe state of dependency. Beneficiaries receiving care in an institution spend on average 102.7 months in the mild state, 29.6 months in the moderate state and 34.6 months in the severe state. The duration of staying in a given state depends on which state the individual comes from and which state the individual leaves for. Detailed numbers are reported in Figure 4 for each dependency state. Bubbles at the start of an arrow represent the mean time spent in the departure state for a particular transition, while hatched bubbles at the end of an arrow report the mean (future) time in the state of arrival. For example, the 5 940 persons with mild dependency cared for at home that transited to a moderate state of dependency being cared for in an institution spend 13.6 months in the departure state and remain 13.7 months in the arrival state. The time before dying also varies with the individual's dependency state. Regarding moderately dependent persons cared for in an institution, we observe that more than half (63 691 out of 113 098) remain on average 31.9 months before dying, while 49 407 enter the severe state after 26.6 months. Severely dependent persons die on average after three years (34.6 months).

3.4 Transitions and paths in the dependency models

Matching our data to both frailty level and type of care models introduced in Section 2.1, we summarize the flow statistics for each in Table 2. Therein, for each transition, the number of observations and the time spent in the states are reported. In doing so, only transitions that are not right-censored remain (see the discussion below). For readability, we use the notations 0 to 4, or 0, a, b and 4, respectively, when referring to the dependency states (see Figures 1 and 2 for the definitions). After fitting the CCO data to the framework of the frailty level model, a total of 269 891 transitions are available for analysis. The transitions leaving state 2 and those

Model	Transition $i \to j$	Number in state i	Duration in i	Duration in j	Number in state j
Frailty level	$1 \to 2$	6 078	15.0	14.8	2 919
	$1 \to 3$	1 931	21.4	15.3	984
	$1 \to 4$	3 089	29.9	–	–
	$2 \to 3$	49 418	26.6	24.6	39 931
	$2 \to 4$	63 723	31.9	–	–
	$3 \to 4$	145 652	34.6	–	–
	Total	269 891	–	–	–
Type of care	$a \to b$	7 813	14.4	15.0	3 129
	$a \to 4$	2 932	23.6	–	–
	$b \to 4$	209 532	39.0	–	–
	Total	220 277	–	–	–

Table 2: Descriptive statistics on the number of transitions and duration (in months) in the different states in the frailty level and type of care models.

from state 3 to state 4 are the most numerous and thus most suitable for providing most reliable results (see Section 4). We will focus on these transitions when presenting selected results of the model estimates (Figure 10) and in the covariates analysis in Section 4.4. In the type of care model, we identify 220 277 transitions.

Table 3 describes the average total duration for the complete paths in the frailty level model for all individuals (both genders and all ages). We call a path *complete* if we can observe the full journey of an individual entering dependency and ending with death. In this approach, we consider all paths where we observe both the moment that a person entered dependency and death. In our dataset, we count a total of 212 464 complete paths (compare with, e.g., Biessy, 2015b, where 31 731 trajectories underlie the study). A major part of the mildly dependent persons (3 089) remains on average 29.9 months in a mild state of dependency (1) before death (4). This duration is very close to the 1 759 mildly dependent individuals who transit to the moderate dependency state and then die (the path $1 \rightarrow 2 \rightarrow 4$ has an average duration of 29.2 months). When a person in mild dependency enters a severe state of dependency, the duration increases and takes values above 38 months. Persons entering dependency by the moderate state (2) are distinguished into two groups. People remain on average 32.4 months if dying directly. If the path includes the severe state (3), the time spent in dependence increases to 51.6 months. The factors explaining this longer duration may be linked to the specific pathologies of these individuals, e.g., dementia versus cancer (Biessy, 2016). Unfortunately, this information is not available in our data (see also the concluding remarks in Section 5). Finally, the most important number of paths corresponds to the 104 737 elderly persons directly becoming severely dependent. They remain on average 38.6 months in that state before death. Distinguishing between the types of care received, we observe 2 932 persons benefiting from care at home for approximately 23.6 months. This duration compares to the 27.3 months of the individuals receiving care at home (a) who later moved on to an institution (b) before death (4). The largest number of persons observed on a path is the 206 403 individuals who directly received care in an institution for approximately 39.6 months before death. Overall, we conclude that most persons spent approximately three years in dependency states before death, and they are mainly cared

Model	Path	Number	Duration
Frailty level	$1 \rightarrow 4$	3 089	29.9
	$1 \rightarrow 2 \rightarrow 4$	1 759	29.2
	$1 \rightarrow 3 \rightarrow 4$	984	38.1
	$1 \rightarrow 2 \rightarrow 3 \rightarrow 4$	539	38.7
	$2 \rightarrow 4$	61 964	32.4
	$2 \rightarrow 3 \rightarrow 4$	39 392	51.6
	$3 \rightarrow 4$	104 737	38.6
Type of care	a $\rightarrow$ 4	2 932	23.6
	a $\rightarrow$ b $\rightarrow$ 4	3 129	27.3
	b $\rightarrow$ 4	206 403	39.4
Total number of complete paths		212 464	–

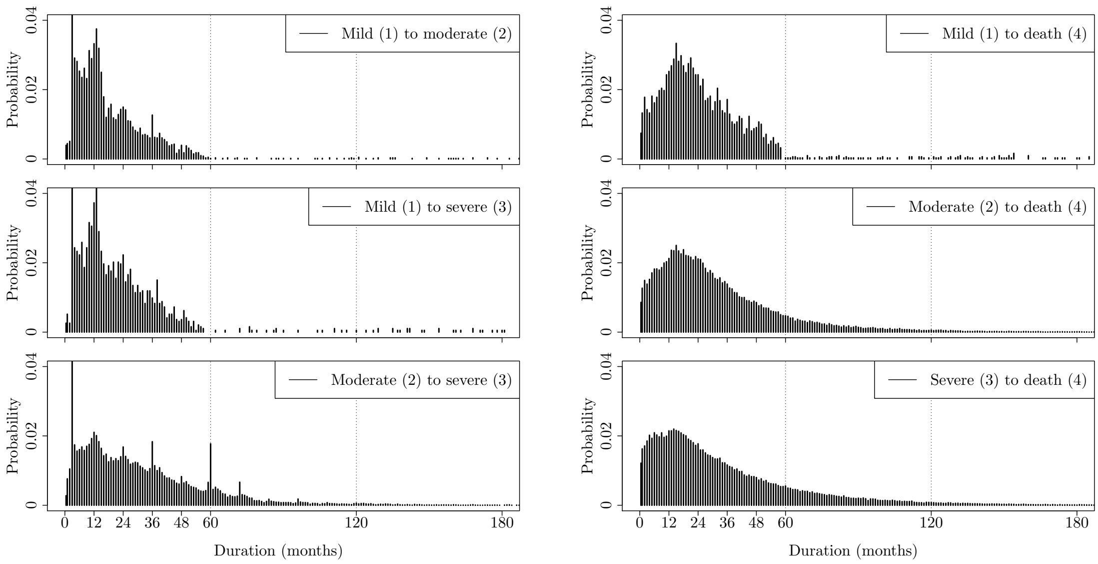
Table 3: Descriptive statistics on the number and duration (in months) for all complete paths to death in the frailty level model.

for in institutions. These findings compare to the situation in the US, where only twelve percent of men and twenty percent of women will spend more than three years in dependence (Brown and Finkelstein, 2008).

Here, we should emphasize certain limitations of the data used to represent the *total* LTC needs in Switzerland. In fact, both left and right censoring affect our data. Left censoring characterizes data for which the starting date of dependency is unknown or lies before the beginning of the observation period in 1995. Right censoring defines data for which the end date of dependency is unknown, i.e., the individual remains alive and in a state of dependency at the end of the observation period. When censoring is not informative, meaning that the censoring is not the consequence of a particular event (e.g., a change in the law), the inclusion of censored data reduces the precision of the estimation. In our case, the dataset is not affected by informative censoring, and given the large sample size, we remove censored data. Further, we observe only a very small number of backward transitions, which justifies their exclusion from our models (cf. Section 2.1). Overall, the original dataset is reduced by approximately ten percent, essentially due to the removal of censored data. The final dataset used in our analysis covers all the completely defined transitions in the period 1995–2014.

3.5 Empirical evidence for the choice of the duration law

In Equation (14), we proposed a Weibull law to describe the distribution of the time spent in the different states of dependency. In the following, we provide empirical evidence that supports our choice of duration law in the semi-Markov model. We present the empirical density of the elapsed number of months spent in the respective current state *before* transiting to the next state for each transition in both models. Figures 5 and 6 report these empirical duration densities in the frailty level and the type of care model, respectively, considering data for both genders and all ages. For example, the duration density reported in the first graph of Figure 5a) illustrates the probability density of the sojourn times in the mild frailty state (1) before transiting to



(a) Transitions among frailty levels.

(b) Transitions from frailty to death.

Figure 5: Empirical duration density (months before transition) in the frailty level model.

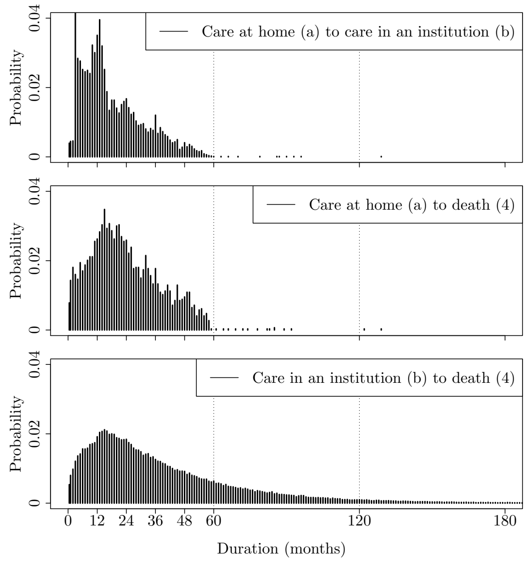
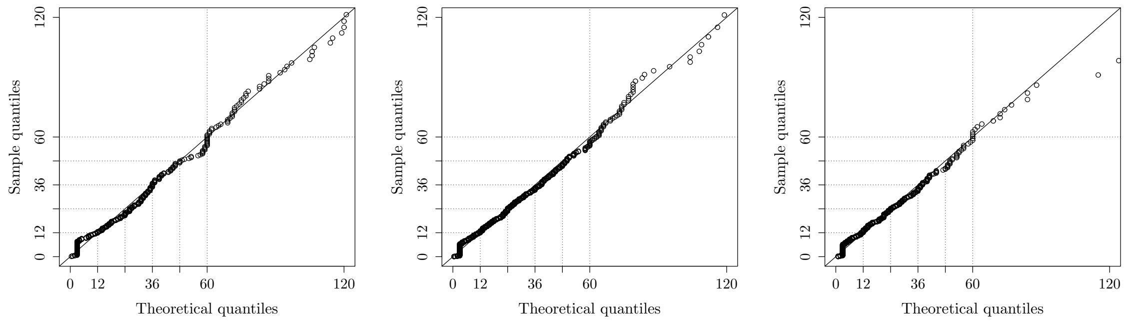


Figure 6: Empirical duration density (months before transition) in the type of care model.

the moderate frailty state (2) for all individuals independent of their gender and age. Note that in the application used to calculate the dependence tables (see Section 4), we estimate the Weibull parameters separately for each data subset by gender and by age. The individuals' ages considered in the study of the transitions from state i to state j refer to the ages when entering dependency state i, as noted below in Section 4.2.



(a) Age 70 (b) Age 80 (c) Age 90

Figure 7: Q-Q plots of the duration (in months) before the transition from moderate (2) to severe (3) in the frailty level model for males at the ages of 70, 80 and 90 years.

The majority of the transitions occur within the first 60 months, leading to a right-skewed empirical distribution. This is typically the shape one observes from the Weibull distribution. The Weibull distribution offers the important advantage of requiring the calibration of only two parameters. Alternative distribution laws are possible (see, e.g., Foucher et al., 2005), but for our analysis, we continue to employ this simple framework. We statistically support the choice of this duration law using the quantile-quantile plots reported in Figure 7. The graphs (a–c) illustrate the goodness-of-fit of the duration law for the transition from moderate (2) to severe (3) in the frailty level model for men at the ages of 70, 80 and 90 years. We observe that the number of data points is large and that the fit is rather appropriate for durations below 60 months. This is why we will present results in Section 4 for durations up to five years.

4 Application of the model and presentation of results

In this section, we present and discuss the results produced by applying the empirical data. First, we calculate the transition probabilities from autonomy to any dependency state for both LTC models (see Section 4.1). Then, we report parameter estimates of the semi-Markov model and the first numerical results for selected ages (see Section 4.2). For one of the transitions, we provide detailed parameter results for all ages and show the differences between males and females (Figure 10). In Section 4.3, we provide the main results of our paper: the transition probabilities in both models. In the last part (see Section 4.4), we consider the impact of the type of household and the linguistic region covariates.

4.1 Estimation of the probability of losing autonomy

Following the methodology described in Section 2.3 and the available prevalence data presented in Section 3.2 (Figure 3) and Section 3.4, we determine the probability of losing autonomy and entering one of the dependency states with the help of Equation (18). We report numerical values for selected ages in Table 4. The variables p_{01}, p_{02} and p_{03} denote the transition probabilities from autonomy to the three frailty levels in the frailty level model, while $p_{0\mathrm{a}}$ and $p_{0\mathrm{b}}$ are the probabilities of entering a type of care in our second model. The sum of the probabilities yields the same number in both models and is reported in row $\sum_j p_{0j}$.[8] After age 80, we observe that the increase in the total transition probabilities from autonomy becomes much more important. For example, for women, $\sum_j p_{0j}$ increases by 2.62% and 11.43% between the ages of 70 and 80 and 80 and 90 years, respectively. This outcome is not surprising because major degenerative illnesses implying dependence appear primarily at higher ages (see, e.g., Kaeser, 2012).

By comparing the probabilities for both genders, we note that their numbers are similar at the ages of 70 and 80 years. In total, approximately 1.42% of men and 1.34% of women become dependent at age of 70 and between 3.44%, and 3.96% do so at age 80. A difference can be observed at age 90: The probability for males is approximately 10%, while that for females reaches 15%. This higher probability can be explained by the substantial number of women surviving to higher ages compared to men. In fact, male mortality is much higher at older ages. As mentioned earlier, we disregard the mortality of autonomous individuals and only focus on the dependent population and their transitions. We observe that the transition probabilities

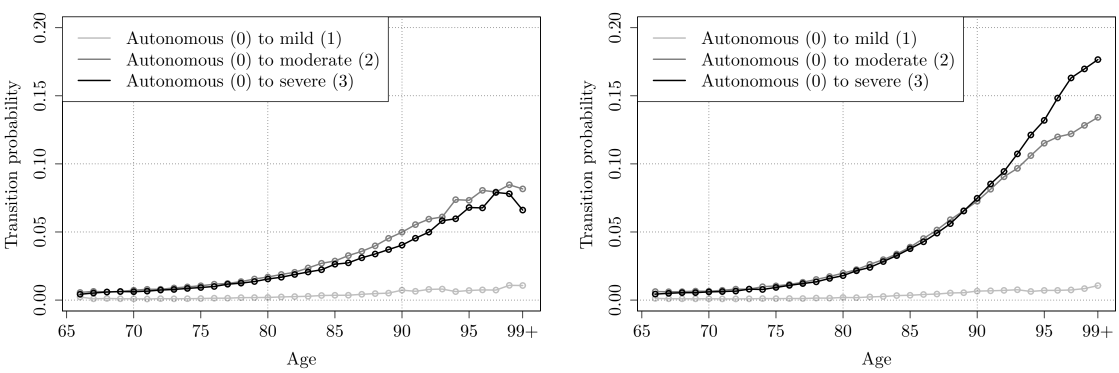
Model	Transition $0 \to j$	Age	Male 70	Male 80	Male 90	Female 70	Female 80	Female 90
Frailty level	$0 \to 1$	p_{01}	0.0009	0.0019	0.0073	0.0010	0.0019	0.0066
	$\to 2$	p_{02}	0.0072	0.0170	0.0498	0.0065	0.0198	0.0726
	$\to 3$	p_{03}	0.0061	0.0155	0.0403	0.0060	0.0179	0.0747
Type of care	$0 \to \mathrm{a}$	$p_{0\mathrm{a}}$	0.0007	0.0020	0.0074	0.0009	0.0020	0.0068
	$\to \mathrm{b}$	$p_{0\mathrm{b}}$	0.0135	0.0324	0.0900	0.0126	0.0376	0.1471
		$\sum_j p_{0j}$	0.0142	0.0344	0.0974	0.0134	0.0396	0.1539

Table 4: Probability of losing autonomy by gender at the ages of 70, 80 and 90 years.

[8] Note that the probability $1 - \sum_j p_{0j}$, with $j \in \{1, 2, 3\}$ or $j \in \{\mathrm{a}, \mathrm{b}\}$, does not yield the probability p_{00} of staying autonomous since the mortality p_{04} of autonomous individuals is also included.

from autonomy to any state of dependency are positively correlated with the age. For example, an 80-years-old man has a 1.55% probability p_{03} of entering the severe dependency state (3), while at 90 years of age this probability is 4.03%. In the following, we analyze the impact of age in greater detail.

The graphs in Figure 8 report the values by age for males (Fig. 8a) and females (Fig. 8b) in the frailty level model. For both genders, the probability of losing autonomy increases with age, and the analysis of the results reveals that two transitions prevail: the probabilities p_{02} and p_{03} of entering in a moderate (2) or severe (3) state of dependency are significantly higher than that entering mild dependency (1). The corresponding probabilities depict exponential shapes as age increases. For both genders, the values until age 80 are similar (see the discussion above). The shape of the transition probability p_{01} from autonomy to mild dependency remains flat and close to zero for both genders. Beyond these similarities, important differences can be observed after age 80 and even more strongly after age 90. For example, the women's transition probability p_{03} from autonomy to severe dependency significantly outpaces the others. Furthermore, the maximum transition probability to dependency for men is 8.5% at age 98 years and exceeds 17% for women of the same age.

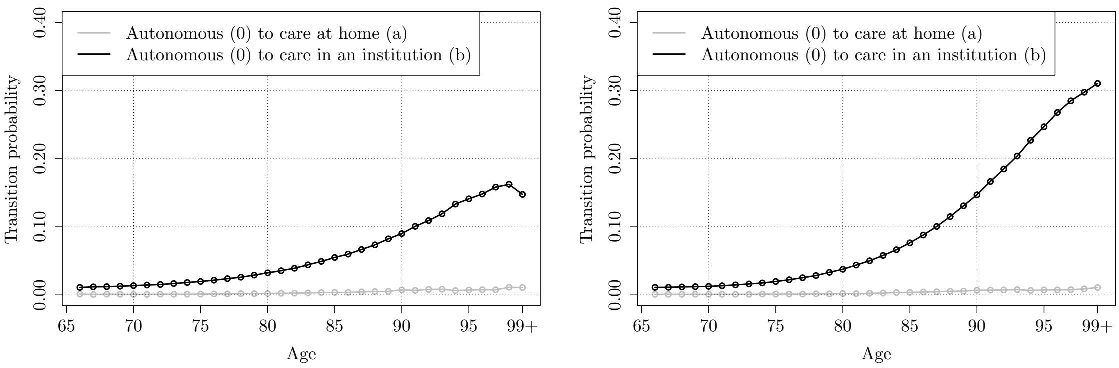


(a) Male

(b) Female

Figure 8: Transition probabilities from autonomy in the frailty level model by gender and age.

Figure 9 presents the results for the type of care model. Again, the two graphs (a) and (b) illustrate the transition probabilities for males and females by age. Our results show that the probability of receiving care in an institution exceeds that of receiving care at home. They also underline the dependence in age and women's higher probability of receiving care than men. From ages 70 to 95, the probability p_{0b} of receiving care in an institution grows from 1% to 14% for men and from 1% to 25% for women. The transition to care at home (a) remains at much lower levels across ages. On the one hand, this can be explained by the fact that benefits for care at home were only allowed from 2001 onward (i.e., only during 14 years of our 20-year observation period). On the other hand, as mentioned above, this may be due to an underestimation in our statistics stemming from an unawareness of the availability of this allowance (see Section 3.1).



(a) Male

(b) Female

Figure 9: Transition probabilities from autonomy in the type of care model by gender and age.

4.2 Parameter estimation of the semi-Markov model

Maximum likelihood estimation

The calibration of the semi-Markov model requires the estimation of different parameters. They are the Markov transitions ϕ_{ij} and the two parameters of the Weibull duration law, the shape σ_{ij} and scale θ_{ij} parameters. To explain our estimation of the parameters, we offer some technical remarks. Maximum likelihood estimation (MLE) is a method that calibrates parameters such that the likelihood of the observations is maximized. In our model, for each gender and age, we calibrate numerous parameters for each transition (ij) from state i to state j: the Markov probabilities ϕ_{ij}, the parameters σ_{ij} and θ_{ij} of the Weibull distribution and the coefficients β_{ij} for the covariates. For this purpose, we divide the sample into subsets by gender and by age and perform MLE, where each set of data contains the transitions realized, the times spent in the previous state and the values of the covariates. Thereby, for a transition (ij), the age in years refers to the individual's age when entering state i.[9]

Based on Equation (5), $C^h_{(ij)}$ defines the marginal contribution to the likelihood of each individual h of a certain gender and age for the transition (ij). It is calculated as follows:

$$C^h_{(ij)} = \phi^h_{ij} f^h_{ij}(t). \tag{19}$$

The contribution to the likelihood represents information contained in the data that is relevant for the parameter calibration. In the extended framework with the duration law applied to the covariates, the likelihood contribution is calculated for each individual by gender, by age *and* by group of covariates (i.e., by type of household and by linguistic region). The likelihood function $\mathcal{L}$ aggregates the individual contributions $C^h_{(ij)}$ for all h and over all transitions (ij):

$$\mathcal{L} = \prod_h \prod_{(ij)} C^h_{(ij)}. \tag{20}$$

[9]In other words, for a given transition, we refer to this (constant) entrance age through the duration t in state i and for the transition (ij). Since t takes values beyond one year, the actual individual's age changes. However, in our approach, we do not take this change into account, and our results always refer to the age at entry to the state. This assumption, although it introduces a deviation, allows for a smooth solution with the duration t.

For computational reasons, we use the log-likelihood function ℓ given by the logarithm of the likelihood function $\mathcal{L}$:

$$\ell = \log \mathcal{L} = \sum_h \sum_{(ij)} \log C^h_{(ij)}. \tag{21}$$

For each contribution of the individuals' gender and age, the above problem yields a homogeneous semi-Markov model. This feature allows us to apply the R package "semi-Markov"(Król and Saint-Pierre, 2015) to estimate the model parameters (see Tables 5 and 6).

Calibration of the frailty level model

In Table 5, we present the parameter estimates for the transitions (ij), denoted $i \to j$, in the frailty level model. For each transition, we report the estimates of the Markov probabilities ϕ_{ij} and of the Weibull shape (σ_{ij}) and scale (θ_{ij}) parameters. We support the precision of the estimates by reporting the standard deviation. We also calculate the expected staying time $\mathbb{E}(X)$ in the state before the transition[10] and report the number of underlying observations N. The results are presented for both genders at the ages of 70, 80 and 90. We note that the number N of observations strongly depends on the transition. This number is important since it drives the reliability of the estimates. For the transitions mild (1) to moderate (2), mild (1) to severe (3), and mild (1) to death (4), fewer than 100 data points are available for both genders. The situation is different for the other transitions, where we count between 500 and 6 000 points at the ages of 80 and 90.

Considering the estimates, we first discuss the Markov probabilities ϕ_{ij}. These probabilities introduced in Equation (3) correspond to the total transition probabilities disregarding the time spent in the states. For example, at age 70, 47.6% of the men in the mild state will enter the moderate state, whereas 19.8% and 32.6% will join the severe and death states, respectively. At the same age, 59.6% of the mildly dependent women enter the moderate state, 19.0% enter the severe state, and 21.4% die. For the transitions leaving the mild state (1), we observe that the share of individuals entering a more severe frailty state (2 or 3) is higher than those dying (4). This holds for both genders and even at higher ages. Regarding the transition from moderate (2) to severe (3), the Markov probabilities ϕ_{ij} decrease with the age of the person. This decrease is of course complemented by the increasing probability of the transition from moderate (2) to death (4).

After the Markov probabilities, we focus on the estimates of the Weibull duration law. In most of the reported cases, the shape parameter σ_{ij} yields similar values close to 1. The situation is different for the scale parameter θ_{ij} because we observe a high sensitivity with respect to the transition and gender considered. A specific trend appears when comparing the changes with the ages. In all of the transitions from the moderate and the severe states ($2 \to 3$, $2 \to 4$ and $3 \to 4$), an increase in the entrance age comes with a decrease in θ_{ij}. Since the shape parameter σ_{ij} is close to 1, the scale parameter approximates the expected duration $\mathbb{E}(X)$. In this case, smaller values of θ_{ij} correspond to a reduction in the expected duration. For example, for a 70-years-old man and the transition from moderate (2) to severe (3), the scale parameter θ_{ij} is 35.936 and decreases to 23.002 and 17.822 at the ages of 80 and 90, respectively. In comparison, the corresponding expected durations $\mathbb{E}(X)$ are approximately 36, 23 and 18 months. Finally,

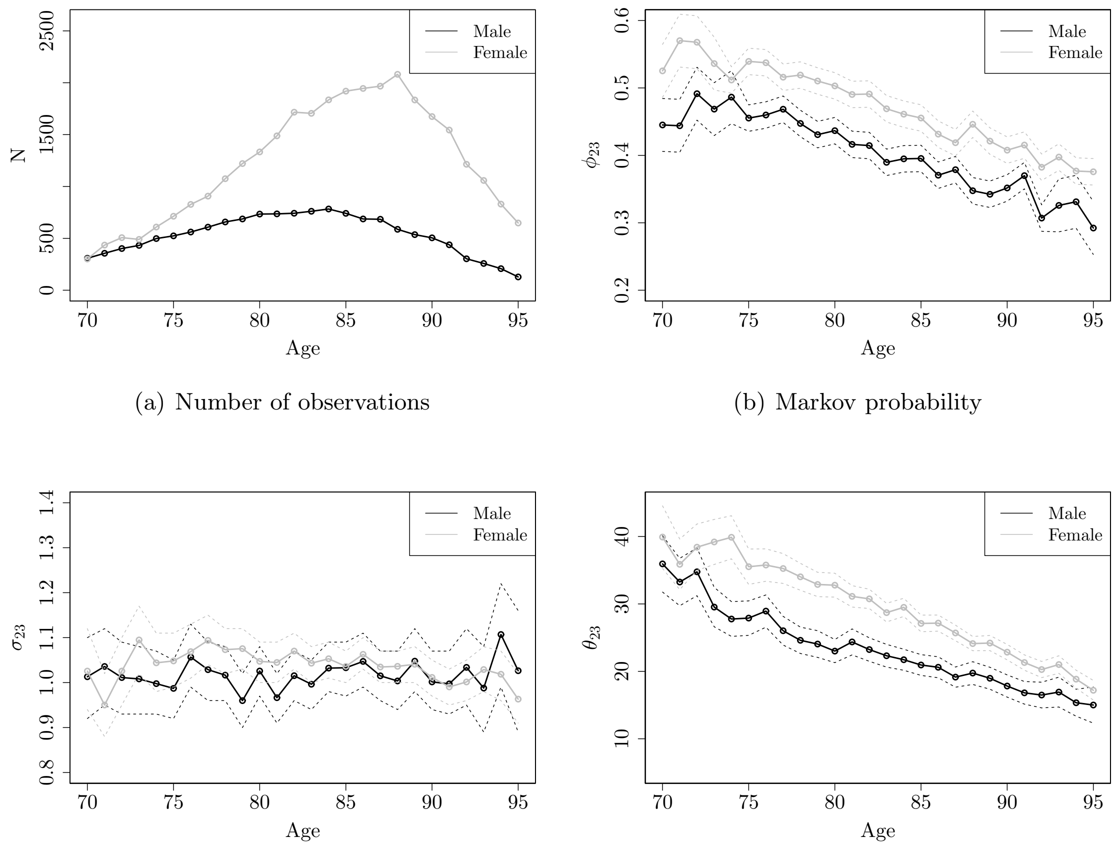
[10] With the notation X, we omit the index n in X_n (see Equation 1) since the order of the transitions is not the focus of our study.

		Male						Female					
	Age	70		80		90		70		80		90	
Transitions 1 → 2	ϕ_{ij}	0.476	(0.054)	0.564	(0.036)	0.495	(0.035)	0.596	(0.054)	0.619	(0.031)	0.491	(0.026)
	σ_{ij}	0.856	(0.092)	1.051	(0.078)	1.170	(0.089)	0.852	(0.087)	1.143	(0.070)	1.145	(0.065)
	θ_{ij}	14.514	(2.818)	13.209	(1.302)	14.198	(1.286)	19.276	(3.400)	14.312	(1.062)	14.579	(1.012)
	$\mathbb{E}(X)$	15.720		12.952		13.446		20.943		13.644		13.894	
	N	41		105		100		50		156		178	
1 → 3	ϕ_{ij}	0.198	(0.043)	0.124	(0.024)	0.168	(0.026)	0.190	(0.043)	0.171	(0.024)	0.198	(0.021)
	σ_{ij}	0.922	(0.170)	1.124	(0.180)	1.116	(0.153)	0.771	(0.146)	1.362	(0.170)	1.379	(0.128)
	θ_{ij}	34.098	(9.497)	14.341	(2.820)	15.323	(2.498)	38.564	(13.258)	19.924	(2.338)	18.199	(1.639)
	$\mathbb{E}(X)$	35.410		13.743		14.716		44.907		18.242		16.628	
	N	17		23		34		16		43		72	
1 → 4	ϕ_{ij}	0.326	(0.051)	0.312	(0.034)	0.337	(0.033)	0.214	(0.045)	0.210	(0.026)	0.311	(0.024)
	σ_{ij}	0.959	(0.138)	1.168	(0.114)	1.718	(0.170)	0.690	(0.128)	1.735	(0.184)	1.677	(0.126)
	θ_{ij}	56.451	(11.757)	24.087	(2.851)	27.383	(2.026)	68.368	(24.658)	29.351	(2.447)	28.364	(1.675)
	$\mathbb{E}(X)$	57.506		22.823		24.415		87.707		26.154		25.332	
	N	28		58		68		18		53		113	
2 → 3	ϕ_{ij}	0.445	(0.019)	0.437	(0.012)	0.352	(0.013)	0.525	(0.021)	0.503	(0.009)	0.408	(0.008)
	σ_{ij}	1.013	(0.046)	1.026	(0.029)	1.002	(0.034)	1.026	(0.046)	1.047	(0.023)	1.012	(0.019)
	θ_{ij}	35.936	(2.130)	23.002	(0.875)	17.822	(0.839)	39.911	(2.368)	32.777	(0.903)	22.845	(0.583)
	$\mathbb{E}(X)$	35.744		22.764		17.811		39.500		32.180		22.736	
	N	308		734		505		301		1333		1674	
2 → 4	ϕ_{ij}	0.555	(0.000)	0.563	(0.000)	0.648	(0.000)	0.475	(0.000)	0.497	(0.000)	0.592	(0.000)
	σ_{ij}	1.242	(0.049)	1.284	(0.032)	1.351	(0.034)	1.109	(0.052)	1.364	(0.029)	1.402	(0.022)
	θ_{ij}	47.847	(2.072)	33.680	(0.898)	25.196	(0.644)	52.733	(3.041)	42.754	(0.910)	30.849	(0.470)
	$\mathbb{E}(X)$	44.630		31.187		23.102		50.747		39.137		28.109	
	N	384		947		931		272		1317		2432	
3 → 4	ϕ_{ij}	1.000	–	1.000	–	1.000	–	1.000	–	1.000	–	1.000	–
	σ_{ij}	1.131	(0.033)	1.176	(0.020)	1.194	(0.023)	1.180	(0.037)	1.244	(0.017)	1.251	(0.013)
	θ_{ij}	51.809	(1.810)	34.019	(0.676)	23.009	(0.495)	69.401	(2.445)	47.718	(0.715)	32.041	(0.353)
	$\mathbb{E}(X)$	49.558		32.174		21.673		65.574		44.490		29.839	
	N	712		2031		1681		638		3192		5844	

Table 5: Parameter estimates and standard deviation (in brackets) for the Markov probabilities, the Weibull duration law with expected duration (in months) and the number of underlying observations for the transitions in the frailty level model. The results are represented by gender at the ages of 70, 80 and 90 years.

the small standard deviations of the parameters for the transitions with more than 300 underlying observations confirm the quality of our estimation.

In Figure 10, we illustrate the above estimates for the transition from moderate (2) to severe (3) in the frailty level model through the ages of 70 to 95 for both genders. We present (a) the number of observations N, (b) the Markov probabilities ϕ_{ij}, (c) and (d) the Weibull duration law shape and scale parameters σ_{ij} and θ_{ij}. In graphs (b) to (d), the 95%-confidence interval is given. For any transition, the number of observations of women always exceeds that of men. We also observe a significant difference in the Markov probabilities when comparing the genders, and the same holds for the shape parameter of the Weibull law. This finding supports the decision to separately consider males and females throughout our study. The scale parameter takes values close to one for both genders at all ages. We note that the values for men and women cannot be distinguished. Furthermore, at ages above 90, the estimates become more erratic, as a result of the lower number of observations. This limited number of data points drove our decision to present results only between ages 70 and 95.



(a) Number of observations

(b) Markov probability

(c) Scale parameter of the Weibull law

(d) Shape parameter of the Weibull law

Note: Dashed lines indicate the confidence interval at the 95% level.

Figure 10: Estimates of the number of observations, the Markov probabilities, the shape and scale parameters of the Weibull law by gender and age for the transition from moderate (2) to severe (3).

Calibration of the type of care model

The estimates for the type of care model are shown in Table 6. The majority of our data cover the transition from care in an institution (b) to death (4). At age 90, we observe 2 504 men and 7 985 women for this transition. The difference between the two figures underlines the higher proportion of women living in institutions at advanced ages. For the other two transitions (a $\rightarrow$ b, a $\rightarrow$ 4), the number of data points is below 300. The Markov probabilities ϕ_{ij} are decreasing with age for the elderly moving from care at home (a) to care in an institution (b). This is the case for both genders. We note that 77.3% of the 70-year-old men receiving care at home transition to institutional care (the remaining 22.7% die). In comparison, for an 80- and 90-year-old, this percentage is 69.2 and 66.7, respectively. This is in line with the increasing mortality.

Regarding the duration law, we find a similar situation to the frailty level model. The shape parameter σ_{ij} is approximately one, inducing that the value of the scale parameter θ_{ij} is close to the expected duration $E(X)$. Our estimates reveal that persons receiving care at home change to care in an institution after approximately one year. The values of the expected duration vary between 11.357 and 14.731 months for the reported ages. For the elderly being cared for at

		Male						Female					
Transitions	Age	70		80		90		70		80		90	
a → b	ϕ_{ij}	0.773	(0.052)	0.692	(0.034)	0.667	(0.033)	0.829	(0.045)	0.795	(0.026)	0.686	(0.024)
	σ_{ij}	1.270	(0.136)	1.063	(0.072)	1.166	(0.078)	1.149	(0.117)	1.184	(0.065)	1.188	(0.058)
	θ_{ij}	12.236	(1.429)	13.410	(1.184)	14.613	(1.146)	14.770	(1.788)	15.605	(0.991)	15.522	(0.873)
	$\mathbb{E}(X)$	11.357		13.094		13.853		14.061		14.731		14.641	
	N	51		128		134		58		198		251	
a → 4	ϕ_{ij}	0.227	(0.003)	0.308	(0.001)	0.333	(0.001)	0.171	(0.002)	0.205	(0.001)	0.314	(0.001)
	σ_{ij}	1.870	(0.418)	1.400	(0.152)	1.709	(0.171)	1.014	(0.226)	1.892	(0.214)	1.696	(0.127)
	θ_{ij}	20.815	(2.976)	22.427	(2.221)	27.547	(2.065)	20.449	(6.126)	28.394	(2.203)	28.588	(1.654)
	$\mathbb{E}(X)$	18.480		20.441		24.570		20.333		25.199		25.512	
	N	15		57		67		12		51		115	
b → 4	ϕ_{ij}	1.000	–	1.000	–	1.000	–	1.000	–	1.000	–	1.000	–
	σ_{ij}	1.240	(0.028)	1.305	(0.018)	1.332	(0.020)	1.234	(0.031)	1.368	(0.015)	1.188	(0.012)
	θ_{ij}	58.640	(1.441)	38.556	(0.565)	27.437	(0.434)	73.675	(1.973)	53.527	(0.598)	36.332	(0.312)
	$\mathbb{E}(X)$	54.719		35.579		25.221		68.819		48.971		33.222	
	N	1197		3037		2504		1014		4753		7985	

Table 6: Parameter estimates and standard deviation (in brackets) for the Markov probabilities, the Weibull duration law with expected duration (in months) and the number of underlying observations for the transitions in the type of care model. Results are represented by gender at the ages of 70, 80 and 90 years.

home, the duration before death increases with the age. A 70-year-old male remains on average 18.480 months at home before dying, 20.441 months if he is 80 years old and 24.570 months at age 90. An explanation for this increase may be linked to the pathology of the person. For example, individuals affected by cancer exhibit lower expected lifetimes than those affected by mental diseases (see, e.g., Kaeser, 2012). The latter are usually diagnosed at higher ages (typically above 80 years), justifying the trend that we observe.

4.3 Transition probabilities

Using the parameter estimates derived in Section 4.2, the calculation of the transition probabilities requires the evaluation of the $p_{ij}(t)$ expressions given in Equations (10) and (11) and Equations (12) and, (13) for the two dependency models. We evaluate the time integrals contained in these expressions using numerical integration. To do so, we apply a trapezoidal rule with 1 000 steps per month. We first compute the staying probabilities $p_{ii}(t)$ (Equations 10 and 12). Next, in the frailty level model, the leaving probabilities are calculated in the following order: $p_{34}(t)$, $p_{23}(t)$, $p_{24}(t)$, $p_{12}(t)$, $p_{13}(t)$ and $p_{14}(t)$. In the type of care model, these probabilities are evaluated in the following order: $p_{\text{b4}}(t)$, $p_{\text{a4}}(t)$ and $p_{\text{ab}}(t)$. For illustration, we provide numerical results for durations t up to 60 months spent in the states.

Transition probabilities in the frailty level model

Table 7 presents an excerpt from the actuarial dependence table for the states of the frailty level model. The dependence table is an important consideration for the pricing of LTC insurance products since it represents the technical basis for premium calculations. In our case, the table corresponds to a dependence table for the 1995–2014 period that, in contrast to a cohort table, assigns the same transition probability to persons of the same age regardless of the year of birth. This approach ensures that the values for each transition are supported by sufficient data. For both genders and at the ages of 70, 80 and 90, we report the transition probabilities for the durations $t \in \{3, 6, 12, 18, 24, 36, 48, 60\}$ in months. The numerical values

	Male								Female							
	3	6	12	18	24	36	48	60	3	6	12	18	24	36	48	60
Age 70																
p_{11}	0.8522	0.7496	0.5984	0.4899	0.4079	0.2932	0.2180	0.1658	0.8415	0.7391	0.5907	0.4849	0.4054	0.2950	0.2238	0.1755
p_{12}	0.0260	0.0442	0.0719	0.0927	0.1093	0.1336	0.1502	0.1617	0.0301	0.0518	0.0862	0.1137	0.1366	0.1730	0.2009	0.2227
p_{13}	0.0265	0.0443	0.0699	0.0881	0.1020	0.1227	0.1383	0.1509	0.0391	0.0645	0.1006	0.1257	0.1441	0.1693	0.1858	0.1976
p_{14}	0.0953	0.1620	0.2598	0.3293	0.3808	0.4505	0.4936	0.5216	0.0893	0.1446	0.2225	0.2757	0.3139	0.3626	0.3895	0.4042
p_{22}	0.9479	0.8925	0.7840	0.6833	0.5920	0.4384	0.3198	0.2304	0.9450	0.8892	0.7836	0.6881	0.6028	0.4603	0.3497	0.2647
p_{23}	0.0110	0.0220	0.0436	0.0646	0.0849	0.1229	0.1575	0.1880	0.0158	0.0317	0.0630	0.0931	0.1217	0.1742	0.2204	0.2598
p_{24}	0.0411	0.0856	0.1724	0.2521	0.3231	0.4387	0.5227	0.5815	0.0393	0.0791	0.1534	0.2189	0.2756	0.3655	0.4299	0.4755
p_{33}	0.9609	0.9163	0.8259	0.7389	0.6577	0.5155	0.3996	0.3071	0.9757	0.9459	0.8815	0.8159	0.7515	0.6307	0.5235	0.4308
p_{34}	0.0391	0.0837	0.1741	0.2611	0.3423	0.4845	0.6004	0.6929	0.0243	0.0541	0.1185	0.1841	0.2485	0.3693	0.4765	0.5692
Age 80																
p_{11}	0.8470	0.7058	0.4833	0.3285	0.2227	0.1025	0.0475	0.0221	0.8880	0.7653	0.5468	0.3773	0.2527	0.1038	0.0378	0.0121
p_{12}	0.0114	0.0226	0.0426	0.0592	0.0726	0.0914	0.1026	0.1087	0.0180	0.0379	0.0751	0.1057	0.1295	0.1601	0.1753	0.1821
p_{13}	0.0166	0.0312	0.0531	0.0673	0.0765	0.0862	0.0901	0.0917	0.0231	0.0475	0.0887	0.1188	0.1399	0.1640	0.1744	0.1785
p_{14}	0.1250	0.2404	0.4210	0.5450	0.6282	0.7198	0.7598	0.7775	0.0709	0.1493	0.2894	0.3982	0.4779	0.5721	0.6125	0.6272
p_{22}	0.9245	0.8445	0.6932	0.5608	0.4485	0.2792	0.1686	0.0992	0.9474	0.8888	0.7712	0.6604	0.5598	0.3922	0.2674	0.1781
p_{23}	0.0077	0.0155	0.0311	0.0462	0.0606	0.0873	0.1105	0.1294	0.0109	0.0225	0.0461	0.0697	0.0928	0.1368	0.1764	0.2102
p_{24}	0.0679	0.1400	0.2757	0.3931	0.4908	0.6335	0.7210	0.7714	0.0417	0.0887	0.1827	0.2699	0.3474	0.4710	0.5563	0.6117
p_{33}	0.9441	0.8781	0.7455	0.6230	0.5150	0.3434	0.2234	0.1425	0.9685	0.9270	0.8357	0.7428	0.6536	0.4945	0.3652	0.2645
p_{34}	0.0559	0.1219	0.2545	0.3770	0.4850	0.6566	0.7766	0.8575	0.0315	0.0730	0.1643	0.2572	0.3464	0.5055	0.6348	0.7355
Age 90																
p_{11}	0.8933	0.7749	0.5606	0.3901	0.2620	0.1059	0.0369	0.0111	0.9030	0.7903	0.5790	0.4066	0.2757	0.1148	0.0420	0.0135
p_{12}	0.0031	0.0071	0.0156	0.0242	0.0320	0.0445	0.0525	0.0568	0.0061	0.0132	0.0278	0.0414	0.0531	0.0707	0.0812	0.0867
p_{13}	0.0050	0.0103	0.0190	0.0255	0.0303	0.0366	0.0405	0.0429	0.0099	0.0208	0.0404	0.0561	0.0681	0.0832	0.0905	0.0933
p_{14}	0.0986	0.2078	0.4048	0.5602	0.6757	0.8129	0.8701	0.8892	0.0810	0.1757	0.3527	0.4959	0.6031	0.7313	0.7863	0.8064
p_{22}	0.9101	0.8127	0.6286	0.4717	0.3456	0.1749	0.0832	0.0377	0.9288	0.8504	0.6960	0.5561	0.4357	0.2547	0.1413	0.0753
p_{23}	0.0026	0.0052	0.0106	0.0161	0.0217	0.0328	0.0434	0.0527	0.0059	0.0119	0.0240	0.0361	0.0482	0.0713	0.0923	0.1100
p_{24}	0.0873	0.1821	0.3608	0.5122	0.6327	0.7922	0.8734	0.9096	0.0653	0.1378	0.2800	0.4078	0.5162	0.6740	0.7664	0.8147
p_{33}	0.9159	0.8179	0.6314	0.4743	0.3494	0.1815	0.0902	0.0433	0.9496	0.8842	0.7462	0.6150	0.4982	0.3145	0.1905	0.1117
p_{34}	0.0841	0.1821	0.3686	0.5257	0.6506	0.8185	0.9098	0.9567	0.0504	0.1158	0.2538	0.3850	0.5018	0.6855	0.8095	0.8883

Table 7: Dependence table by gender for selected ages (70, 80, 90 years) and durations (3 to 60 months) in the frailty level model.

correspond to the transition probabilities $p_{ij}(t)$ for the transition $i \to j$ and the duration t. For each state i, we consider the staying probability $p_{ii}(t)$ and the leaving probabilities $p_{ij}(t)$. The staying probabilities are highlighted, and we have $p_{ii}(t) = 1 - \sum_j p_{ij}(t)$, where the index j takes values in $\{1, 2, 3, 4\}$.

In addition to the numerical values reported in Table 7, we illustrate the transition probabilities for male and female in Figures 11 and 12, respectively. In both figures, we present the transition probabilities affecting the mild (1), moderate (2) and severe (3) states. The corresponding graphs for each dependency level i are displayed in rows. In a given row, we present the graphs related to ages 70, 80 and 90. The graphs related to a state i list the probabilities $p_{ij}(t)$ for the given i and all possible $j \geq i$ after a duration $t \in [0, 60]$ in months.

For a mildly dependent person, the probability $p_{11}(t)$ of remaining in the mild state decreases both with the duration t and the person's age. We observe an important reduction in the probability p_{11} during the first 24 months. A *70-year-old man* has a 59.84% probability of staying in the mild dependency state after one year. This probability becomes 40.79% after two years and continues to decrease over time. When not remaining in the mild dependency state, he can either become moderately dependent with probability p_{12}, severely dependent with probability p_{13} or die with probability p_{14}. After 12 months, the probabilities are $p_{12} = 7.19\%$, $p_{13} = 6.99\%$, and $p_{14} = 25.98\%$. These three probabilities are 10.93%, 10.20% and 38.08% after 24 months and 13.36%, 12.27% and 45.05% after 36 months. We observe that these three transition probabilities increase with the time spent in the mild state. Age is also a relevant factor. The probabilities p_{11}, p_{12} and p_{13} decrease with a person's age because the mortality p_{14} meaning-

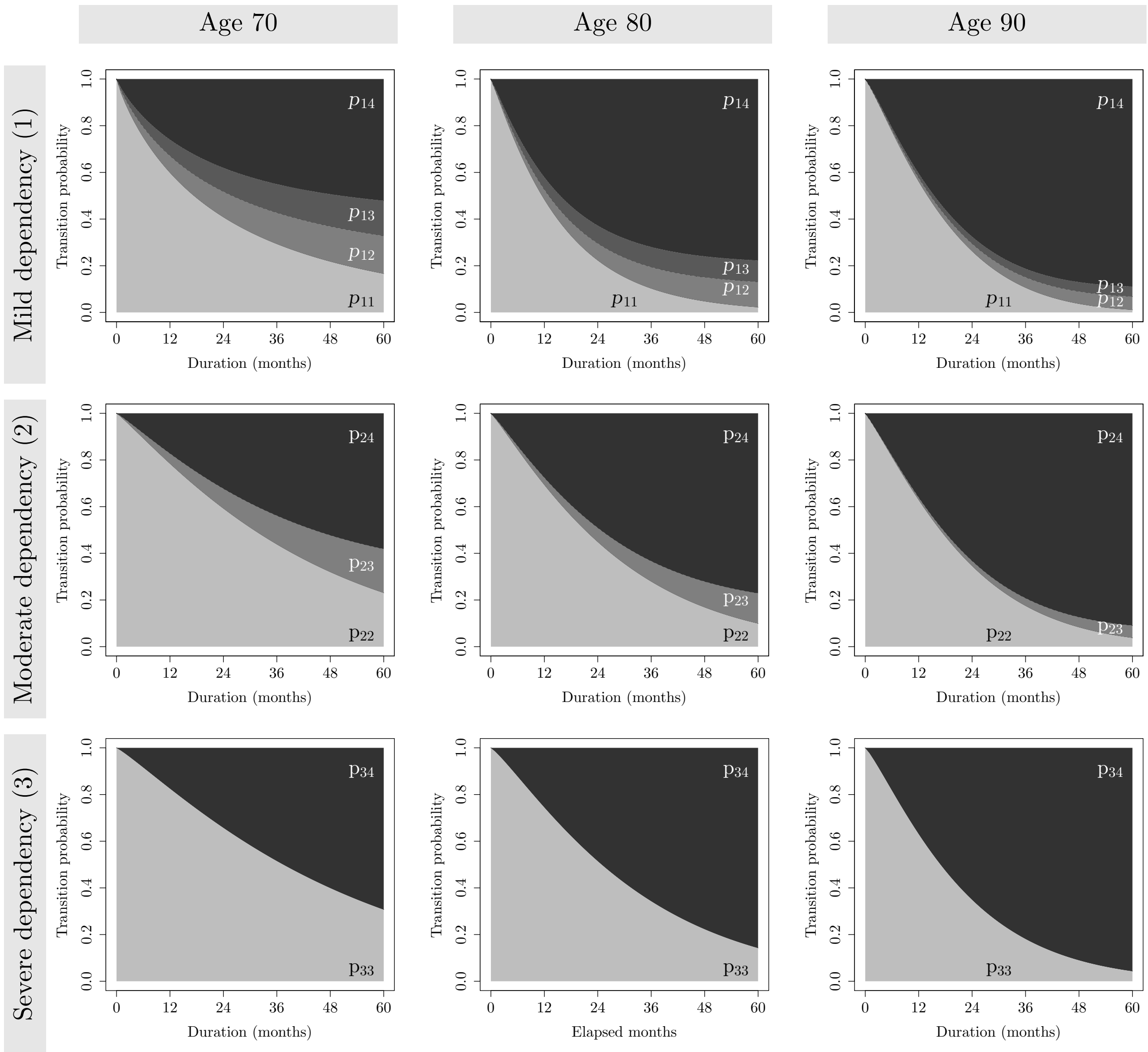


Figure 11: Transition probabilities for males at the ages of 70, 80 and 90 in the frailty state model.

fully increases. For example, after a 36-months duration in the mild state of dependency, a 70-year-old man has a 45.05% probability of dying. This probability increases to 71.98% for an 80-year-old man and to 81.29% for a 90-year-old man.

Analyzing the results for moderately and severely dependent males aged 70 years, we observe the same trends as described above. On the one hand, the probabilities of remaining in the moderate state p_{22} or of remaining in the severe state p_{33} are both decreasing with age and duration. After a 12-month duration, the values of p_{22} are 78.40%, 69.32% and 62.86% for a 70-, 80- and 90-year-old man,respectively. For p_{33}, they are 82.59%, 74.55% and 63.14%, respectively. On the other hand, the leaving probability p_{23} and the death probabilities p_{24} and p_{34} increase with the duration. For a 70-year-old moderately dependent male, the probability p_{23} of entering the severe state is 4.36% after 12 months, 8.49% after 24 months and 12.29% after 36 months. After the same durations, the death probabilities p_{24} and p_{34} are 17.24%, 32.31%, and 43.87% and 17.41%, 34.23%, and 48.45%, respectively. Finally, for short durations, we observe that mildly dependent individuals have a higher mortality p_{14} relative to the moderately and severely dependent persons (p_{24} and p_{34}). At a first glance, this may appear counterintuitive since limitations in ADL are typically linked to poorer health. However, two effects may be concealed behind this observation. First, mildly dependent persons are more often cared for at home (cf. Figure 4) with no permanent assistance and no professional care infrastructure. Second, pathologies such as cancer may entail a very high mortality but express only few limitations in ADL. Other pathologies including cognitive diseases entail important ADL limitations without having a specific impact on the mortality (see also, e.g., Biessy, 2016).

For elderly females, the three main trends discussed above for males hold. First, the staying probabilities $p_{ii}(t)$ are decreasing with the time t spent in state i. Second, the leaving probabilities $p_{ij}(t)$, $i \neq j$, are increasing with the duration t. Third, given the increasing mortality $p_{i4}(t)$ with the age, the sum $\sum_{j \neq 4} p_{ij}(t)$ of all the other probabilities, i.e., the probabilities of staying and leaving for another frailty state decrease with age. This can be observed from in Figure 12. Further, from the right-hand side of Table 7, we find, for example, that a mildly dependent *70-year-old woman* has a $p_{11} = 59.07\%$ probability of remaining mildly dependent after 12 months, a probability that decreases to 40.54% after 24 months and to 29.50% after 36 months. The leaving probability $p_{12} = 8.62\%$ for entering the moderate dependency state

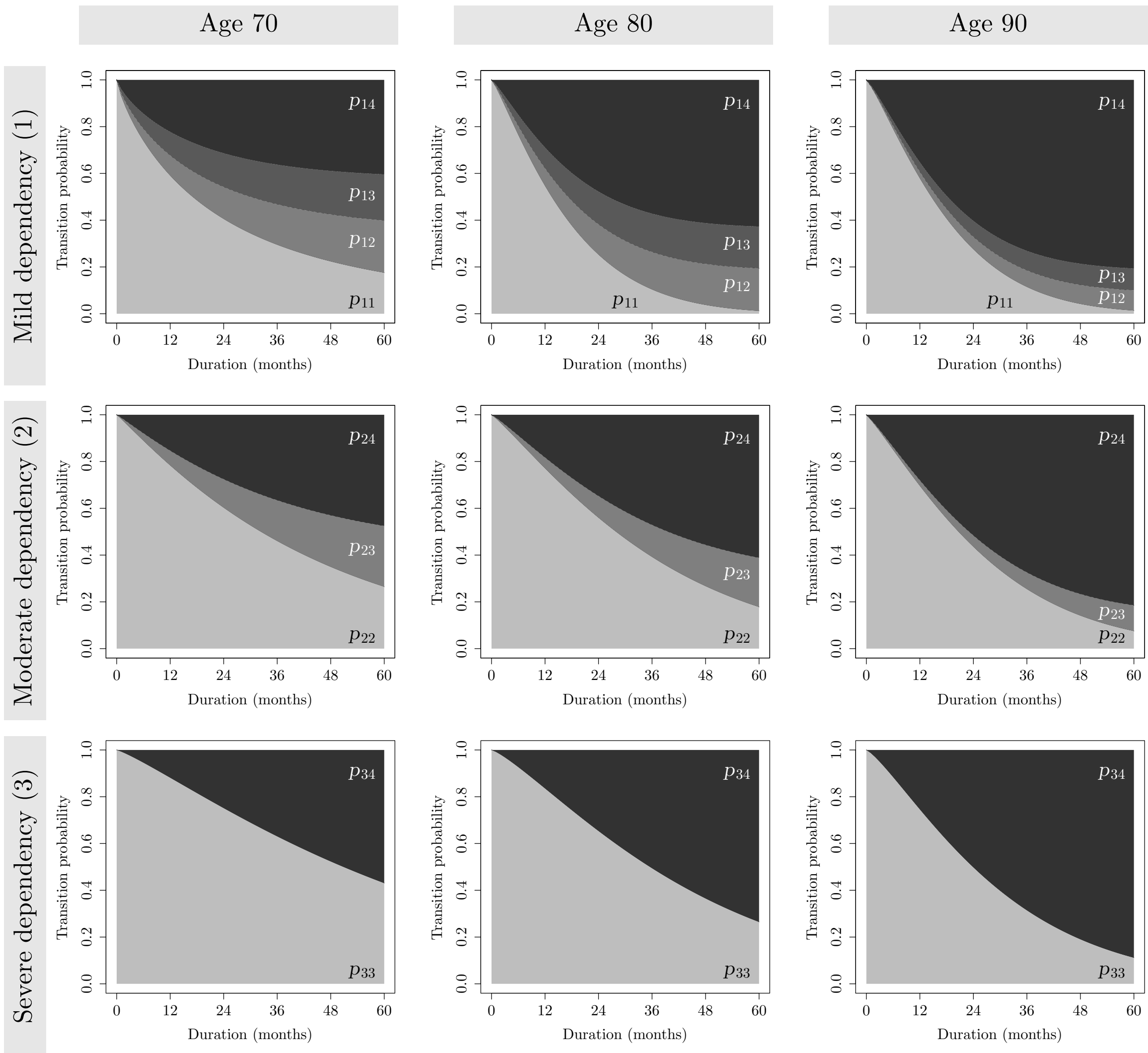
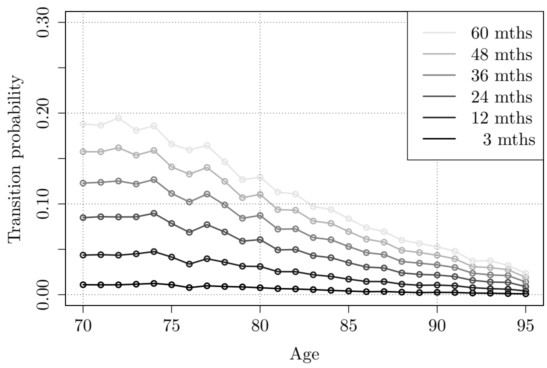


Figure 12: Transition probabilities for females at the ages of 70, 80 and 90 in the frailty state model.

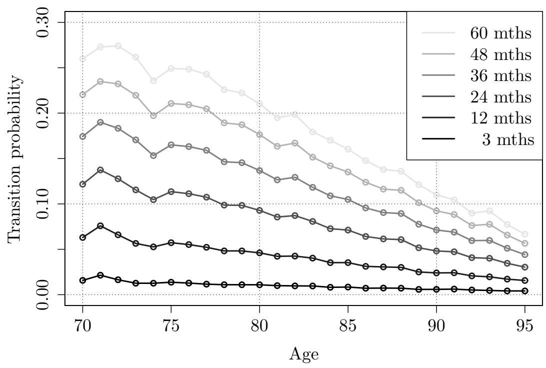
after 12 months becomes 13.66% after 24 months. Finally, the probability of dying increases from p_{14} = 22.25% after 12 months to 31.39% after 24 months for a 70-year-old woman. For $t = 12$ months, p_{14} increases to 28.94% at age 80 and to 35.27% at age 90. In these cases, the complementary probability, i.e., the sum of p_{11}, p_{12} and p_{13}, decreases.

By comparing the male transition probabilities with the female ones, we observe gender differences. At the three reported ages and for any duration in any dependency state, women show higher values than men for all the probabilities of leaving for another frailty state (p_{12}, p_{13}, p_{23}), while their mortality (p_{14}, p_{24}, p_{34}) is lower. Further, at ages 80 and 90, women show higher values for the staying probabilities (p_{11}, p_{22}, p_{33}) than men. This may be explained by the significantly lower female mortality at higher ages.

Figure 13 details the above probabilities in the example of the moderate (2) to severe (3) transition through ages from 70 to 95 for both genders. The graphs show the transition probabilities for the durations of 3, 12, 24, 36, 48 and 60 months. Recall that the ages presented in the graphs correspond to the entrance ages in state 2. Thus, along a given curve on the graphs, the actual age is obtained by summing the entrance age (reported on the x-axis) and the time spent in the state. For short durations, e.g., for individuals having spent three months in the moderate state (2), the transition probabilities to the severe state (3) are quasi-independent of age and stay close to zero for both genders. For longer durations, e.g., greater than 12 months, the transition probability is much higher at lower ages. In fact, both genders show a decreasing transition probability p_{23} with increasing entrance age. This effect becomes more important for longer durations. For example, the transition probability p_{23} for a 70-year-old man having spent 60 months in the moderate state (2) is approximately 19%, while for a 90-year-old man it is 5%. In fact, for the latter man, the mortality p_{24} is significantly higher (cf. Figure 11). Our results also allow us to identify the combined effect of an individual's (entrance) age and the duration on the transition probability. For example, a man who entered state 2 at age 70 attains an effective age of 75 years after a 60-month duration and bears a 19% transition probability p_{23}. This compares to a nearly zero transition probability for a 75-year-old man entering state 2. This example illustrates the important additional effect of the duration beyond the sole consideration of (entrance) age.



(a) Male



(b) Female

Figure 13: Illustration of the transition probability $p_{23}(t)$ for selected durations t and at the ages from 70 to 95 years for both genders.

The derivation of the dependence tables for the frailty level model has identified three important variables, the gender, the (entrance) age and the duration. We discover that women, compared to men, stay longer in the dependence states given their lower mortality. This result is consistent with Mathers (1996), Mathers et al. (2001) and Fong (2017), who find that elderly females live more years in dependence. As discussed above, the combined effect of the age and the duration impacts the transition probabilities. Finally, we argue that the type of care received and the specific pathologies inducing dependency may be key factors for explaining of the transition probabilities. In the following section, we focus on the influence of the type of care received by studying the transition probabilities in the type of care model.

Transition probabilities in the type of care model

Table 8 summarizes the transition probabilities in the states of the type of care model for males and females at ages 70, 80 and 90. The tables are constructed analogously to those for the frailty level model (cf. Table 7) and report the probabilities for durations between 3 and 60 months. Figures 14 and 15 graphically illustrate these results.

	Male								Female							
	3	6	12	18	24	36	48	60	3	6	12	18	24	36	48	60
Age 70																
p_{aa}	0.8747	0.7218	0.4503	0.2571	0.1351	0.0291	0.0046	0.0006	0.8545	0.7092	0.4726	0.3074	0.1974	0.0804	0.0332	0.0143
p_{ab}	0.0442	0.0983	0.1947	0.2626	0.3048	0.3417	0.3513	0.3534	0.0579	0.1197	0.2275	0.3089	0.3669	0.4330	0.4612	0.4723
p_{a4}	0.0812	0.1799	0.3550	0.4803	0.5601	0.6292	0.6441	0.6461	0.0876	0.1711	0.2999	0.3837	0.4358	0.4866	0.5056	0.5134
p_{bb}	0.9752	0.9425	0.8695	0.7936	0.7187	0.5792	0.4583	0.3574	0.9810	0.9558	0.8990	0.8390	0.7785	0.6616	0.5547	0.4602
p_{b4}	0.0248	0.0575	0.1305	0.2064	0.2813	0.4208	0.5417	0.6426	0.0190	0.0442	0.1010	0.1610	0.2215	0.3384	0.4453	0.5398
Age 80																
p_{aa}	0.8547	0.7153	0.4876	0.3240	0.2107	0.0841	0.0312	0.0109	0.8919	0.7703	0.5505	0.3776	0.2494	0.0967	0.0319	0.0091
p_{ab}	0.0228	0.0452	0.0846	0.1163	0.1408	0.1732	0.1903	0.1986	0.0341	0.0739	0.1497	0.2126	0.2609	0.3212	0.3493	0.3608
p_{a4}	0.1226	0.2395	0.4277	0.5597	0.6485	0.7427	0.7784	0.7905	0.0740	0.1558	0.2997	0.4098	0.4896	0.5821	0.6188	0.6301
p_{bb}	0.9650	0.9156	0.8042	0.6908	0.5836	0.4008	0.2642	0.1684	0.9808	0.9511	0.8787	0.7984	0.7162	0.5592	0.4225	0.3107
p_{b4}	0.0350	0.0844	0.1958	0.3092	0.4164	0.5992	0.7358	0.8316	0.0192	0.0489	0.1213	0.2016	0.2838	0.4408	0.5775	0.6893
Age 90																
p_{aa}	0.8952	0.7774	0.5629	0.3919	0.2633	0.1068	0.0374	0.0113	0.9024	0.7890	0.5781	0.4072	0.2775	0.1170	0.0433	0.0141
p_{ab}	0.0063	0.0142	0.0311	0.0476	0.0625	0.0855	0.0996	0.1068	0.0133	0.0299	0.0639	0.0951	0.1215	0.1588	0.1789	0.1882
p_{a4}	0.0985	0.2084	0.4060	0.5605	0.6742	0.8076	0.8630	0.8819	0.0842	0.1812	0.3580	0.4976	0.6010	0.7242	0.7778	0.7978
p_{bb}	0.9489	0.8763	0.7173	0.5653	0.4331	0.2379	0.1217	0.0587	0.9679	0.9190	0.8036	0.6829	0.5677	0.3725	0.2310	0.1366
p_{b4}	0.0511	0.1237	0.2827	0.4347	0.5669	0.7621	0.8783	0.9413	0.0321	0.0810	0.1964	0.3171	0.4323	0.6275	0.7690	0.8634

Table 8: Dependence table by gender for selected ages (70, 80, 90 years) and durations (3 to 60 months) in the type of care model.

Different conjectures can be drawn from the results. For a dependent elderly person receiving care at home, the probability $p_{\mathrm{aa}}(t)$ of remaining in this type of care is decreasing with the duration t and increasing with age. We observe that a 70-year-old man has a $p_{\mathrm{aa}} = 45.03\%$ probability of still being cared for at home after 12 months. This value decreases to 13.51% after 24 months and 2.91% after 36 months. At age 80, these probabilities are 48.76%, 21.07% and 8.41%. The probability p_{ab} of entering a care institution after having been cared for home increases from 19.47% (after 12 months) to 30.48% (after 24 months). Both death probabilities p_{a4} and p_{b4} increase with the duration and age. After 36 months, we observe a 62.92% probability of dying for a 70-year-old man receiving care at home (a). This probability becomes 74.27% and 80.76% at the ages of 80 and 90 years, respectively. The corresponding mortality p_{b4} is lower for elderly persons living in an institution (b): a 70-year-old man has a 42.08% probability of dying after 36 months; at ages 80 and 90, the mortality is 59.92% and 76.21%, respectively. Regarding the male to female comparison, we can draw the same conclusion as in the frailty level model, i.e., the trends observed for men also hold for women. By further contrasting genders, we find lower death probabilities p_{a4} and p_{b4} for women.

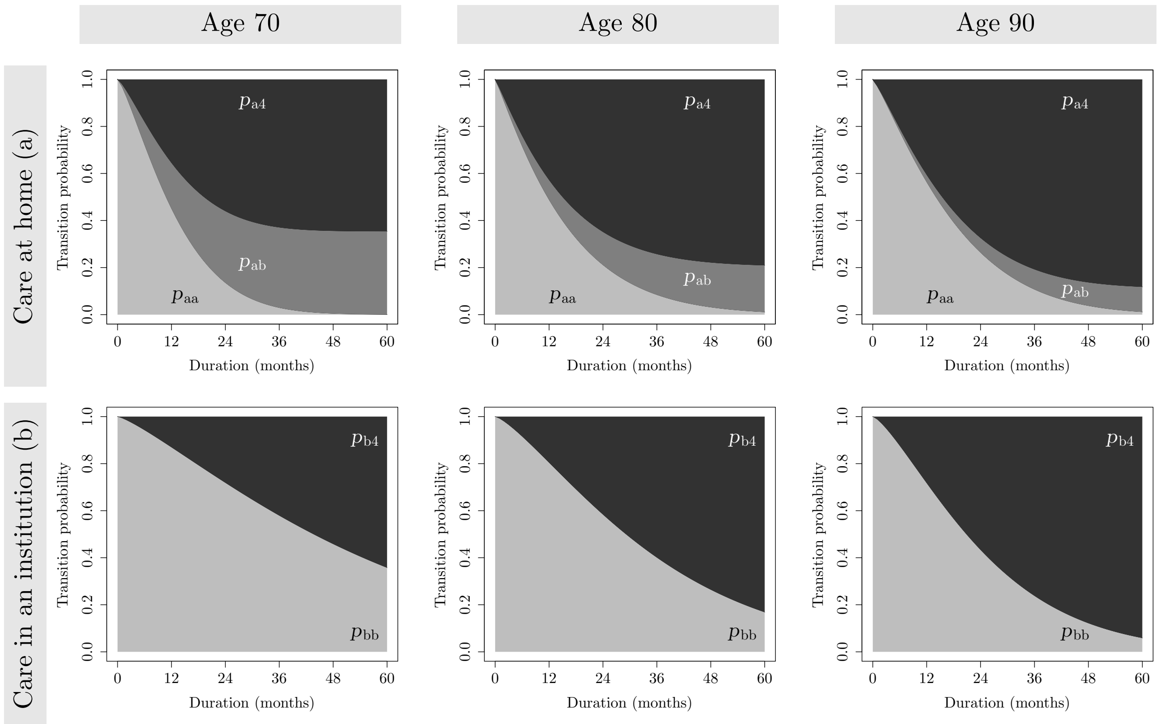


Figure 14: Transition probabilities for males at the ages of 70, 80, 90 in the type of care model.

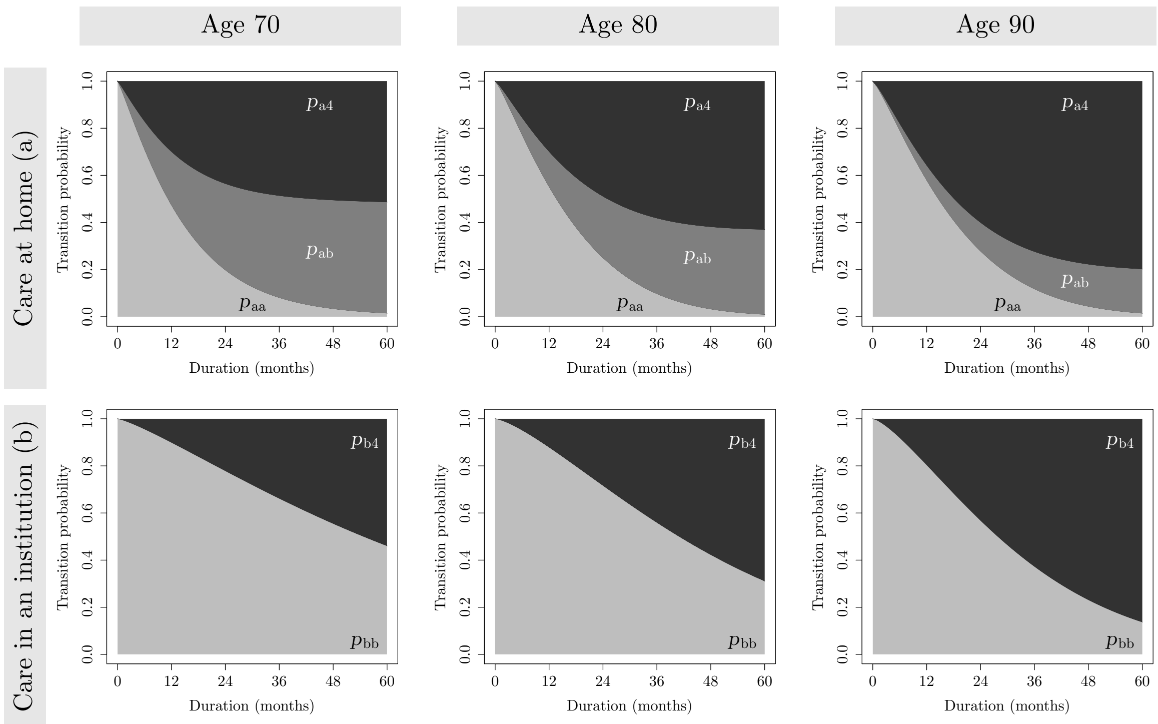


Figure 15: Transition probabilities for females at the ages of 70, 80, 90 in the type of care model.

The results from the type of care model are most relevant for the development of insurance products. In fact, the costs differ significantly between care at home and care in an institution. Similar to our findings in the frailty level model, we identify that the gender, the age and the duration are three relevant variables for calculating transition probabilities. In particular, an important share of elderly persons cared for at home enters an institution after one year. At ages 70, 80 and 90, we conclude that elderly persons living in an institution have lower death probabilities than do those living at home. This supports the hypothesis regarding the importance of the type of care made above (see the discussion of the results of the frailty level model). Institutions offer 24-hour supervision and more specialized infrastructure. Finally, an open point remains concerning the effect of the underlying pathologies of the dependent persons on the transition probabilities.

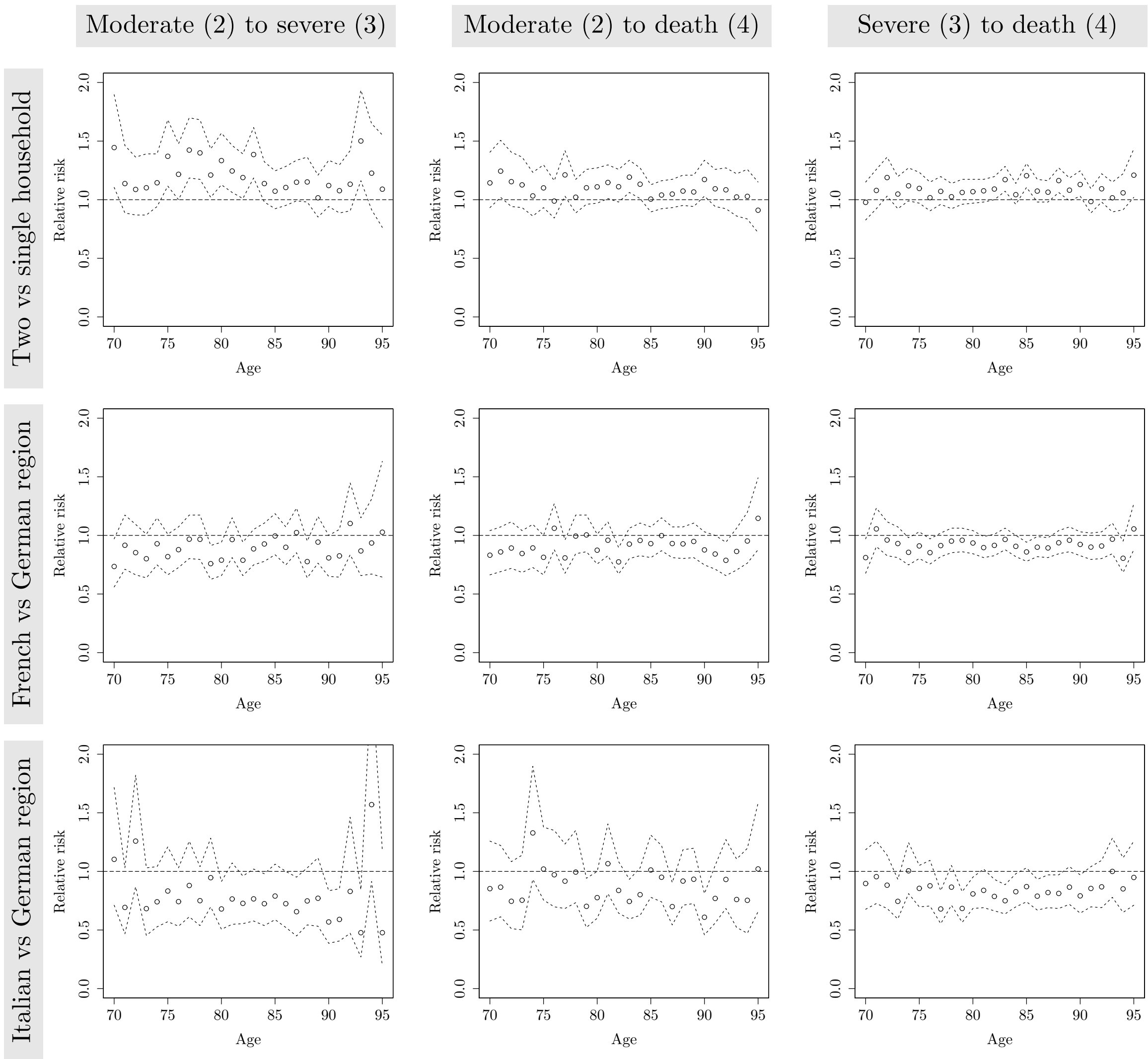
4.4 Role of culture and type of household on transition probabilities

In this part, considering the frailty level model, we present results from an extension that includes covariates (see Section 2.2). Three covariates are discussed with respect to the type of household and the linguistic region of residence. The covariate on the type of household expresses the effect on the transition probabilities for persons elderly living in two-person households compared to those living in a single household. Two other covariates are derived from the three linguistic regions of Switzerland. We compare the effect of living in the French- and Italian-speaking regions to living in the German-speaking region.

In Figures 16 and 17, we present the relative risk compared to the baseline for males and females by age for the three transitions of moderate (2) to severe (3), moderate (2) to death (4) and severe (3) to death (4) in the frailty level model. The relative risk is given by $\exp(\boldsymbol{\beta}_{ij})$, where $\boldsymbol{\beta}_{ij}$ is the regression coefficient for the transition from state i to state j, as introduced in Equation (16). The elements of the vector $\boldsymbol{\beta}_{ij}$ are the respective coefficients for the three covariates. Recall that, by our definition, the covariates leave the Markov probabilities ϕ_{ij} unchanged but affect the parameters of the duration law. We do not report here the new shape and scale parameters σ_{ij} and θ_{ij} of the Weibull distribution, but we focus on comparing the impacts of the covariates. In the graphs, the 95% confidence intervals are displayed.

For both genders and at all ages, we observe that individuals living in a two-person household have a higher relative risk to realize a given transition than those living in a single household. For example, for the transition from moderate (2) to severe (3), the relative risk varies in a range from 1.0 to 1.5 for males aged between 70 and 95 years. This means that an individual living in a two-person household has a 0 to 50% higher likelihood of realizing this transition than someone from a single household. Specifically, for an 80-year-old man living in a two-person household, the relative risk is 1.3. We interpret this number as a 30% higher likelihood of realizing the transition relative to living in a single household. Note that only for certain ages is the relative risk significantly different from one (cf. the confidence bounds). Similar observations can be made for the other two transitions, i.e., dependent elderly individuals living in a two-person households display a higher relative mortality compared to those living alone. This result is counterintuitive, especially when contrasted with the findings of other studies showing that persons living as couples live longer than singles (see, e.g., Elwert and Christakis, 2008; Sanders and Melenberg, 2016).

The other graphs in Figure 16 show that the region of residence affects the transition probabilities. For example, for the transition from moderate (2) to severe (3), an 80-year-old man living in the French-speaking region has a relative risk value of approximately 0.80 compared to his German-speaking counterparts. Indeed, persons living in the French or Italian linguistic regions

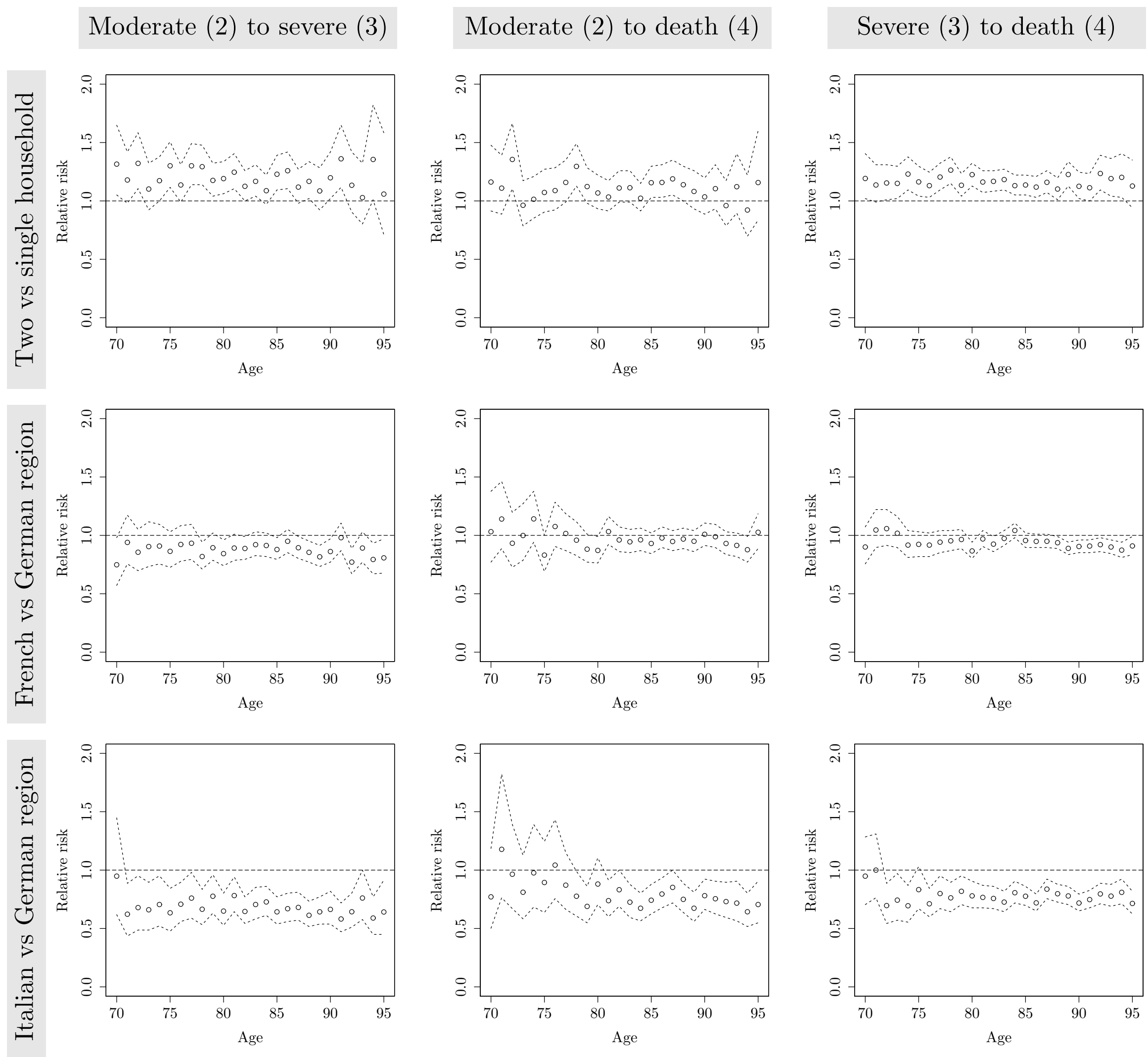


Note: Dashed lines indicate the confidence interval at the 95% level.

Figure 16: Relative risk for males for the three covariates by age.

present a relative risk globally lower than one. This result implies that the inhabitants of those two regions have lower transition probabilities compared to the inhabitants of the German-language region. The reasons for these outcomes might be linked to cultural differences, as discussed, e.g., by Gentili et al. (2016) in the context of LTC and by Eugster et al. (2011) with regard to the demand for social insurance. However, we also need to nuance the importance of the cultural effect. In fact, our data are based on elderly persons registered for receiving an LTC allowance (see Section 3.1). Assessing the care needs of a patient also depends on subjective factors, e.g., in determining of the dependency level, a doctor often has to rely on the patient's explanations. Depending both on the doctor and the patient, following the local usages, similar cases can lead to different assessments of LTC needs.

The above conclusions also hold for women, who exhibit relative risk values varying in comparable ranges (see the graphs in Figure 17). One relevant distinction concerns the statistical significance of the obtained results. The results for males are statistically significant only for a few ages, that is, where the boundaries of the confidence intervals do not cross one. For females, we observe statistically significant differences across most ages for the three covariates and the



Note: Dashed lines indicate the confidence interval at the 95% level.

Figure 17: Relative risk for females for the three covariates by age.

three considered transitions. Furthermore, the confidence intervals are narrower, indicating better precision in our estimates.

5 Conclusion

Due to limited data availability, most of the literature on LTC cannot account for the duration effect on the transition probabilities between different states of dependency. In this article, we develop dependence probability tables based on two models focusing on the frailty levels and the types of care received. In both models, we examine the paths followed by elderly persons from autonomy to death. In the frailty level model, we distinguish the three states of dependency, mild, moderate and severe, while in the type of care model, we concentrate on the types of care received, i.e., at home and in an institution. Our approach relies on the semi-Markov framework, and we derive analytical expressions for the transition probabilities. The proposed solution allows for a straightforward interpretation since it only depends on the estimation of the hazard rates. We reinforce the existing literature on LTC and insurance pricing (compare with the work of Biessy, 2015b) by applying this framework to two models and a unique longitudinal dataset that contains observations on the total population's LTC needs recorded over a 20-year period in Switzerland.

From the descriptive statistics on the paths followed and on the time spent by dependent persons in the considered states, we find that the average duration spent in LTC dependence is approximately three years. Then, we provide actuarial dependence tables by acuity level for both genders and selected ages. Our results show that transition probabilities depend on the individual's gender, age and duration in the previous state. In both models, we find that women spend more time than men in all of the dependency states (Fong, 2017). From the analyses in the type of care model, we learn that a major part of the dependents cared for at home switch to institutional care after one year. We conclude that receiving institutional care, compared to home-based care, is associated with lower death probabilities due to the specialized services offered. This argument, together with different underlying pathologies may explain why, for short durations, mildly dependent individuals have a higher mortality than the moderately and severely dependent persons.

To deepen our understanding of the effects stemming from sociodemographic variables, we show that individuals living in two-person households exhibit higher risk for certain transitions compared to those living in single households. Further, we find significant differences among the linguistic regions in Switzerland, with people from the French- and Italian-speaking parts of the country exhibiting a lower risk for certain transitions compared to the dependent population in the German-speaking region. These results showcase the importance of recognizing underlying cultural differences when considering LTC needs.

Finally, we identify two main directions for further research. First, the inclusion of data on the dependents' pathologies could help to improve the interpretation of the durations in the different acuity states (Biessy, 2016). Moreover, further socioeconomic factors such as former occupation or profession, the level of education, previous income and wealth may prove to be significant drivers (e.g., Szanton et al., 2010; Van den Bosch et al., 2013). Second, our work lays the basis for further development of LTC pricing and valuation that may lead to an assessment and further development of the social systems and insurance solutions offered. The methodology and our findings are directly relevant for academics and insurance practice, including beyond Switzerland.

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