# Insurance Demand, Default Risk and Diversification

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**Abstract**

We reconsider the insurance demand model of Doherty and Schlesinger (1990) that describes the optimal level of insurance coverage, if the policy's indemnification is exposed to a risk of non-performance. We modify the model by allowing for a multiple risk-sharing agreement called co-insurance, in which a consortium of several insurers share the risk and premium of one policy. We analyze, how co-insurance affects the policyholder's utility and optimal quantity of insurance coverage. Particularly, we point out the monotonicity of the optimal quantity, both when additional co-insurers can be used to diversify the default risk and when default correlation among the co-insurers rises. The results indicate that the optimal demand for insurance coverage under default risk in a classic, normative utility-based insurance demand model can be well explained by the policyholder's degree of prudence and a corresponding precautionary investment motive.

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## Introduction

Since its publication five decades ago, the classic insurance demand model with the well-known theorem of Mossin (1968) has formed the basis for numerous research articles (see, e.g., Ehrlich and Becker, 1972; Mayers and Smith, 1983; Doherty and Schlesinger, 1983). Regarding default risk, Doherty and Schlesinger (1990) were among the first to analyze how the optimal demand for an insurance policy is disturbed by a partial or total nonperformance of the insurer. Their results illustrate that under an actuarially fair premium, a risk-averse policyholder will demand more or less coverage as compared to the case in which a full indemnification is guaranteed. Mahul and Wright (2007) further specify these results by showing that, when sold at a fair price, the optimal unreliable insurance coverage depends on a so-called trigger recovery rate.[1] That is, if the actual recovery rate exceeds (falls below) this trigger rate, the expected utility of the policyholder is maximized by over-insurance (under-insurance). Further recent research on optimal insurance demand under default risk is provided by Mahul and Wright (2004), Bernard and Ludkovski (2012), and Peter and Ying (2016).

While all these papers examine the demand for (re-)insurance under default risk, they disregard that policyholders could rely on risk management measures to mitigate the intrinsic nonperformance risk of (re-)insurance policies. In a practitioner-oriented study, Ehrlich et al. (2010) point out that primary insurers may diversify the counterparty risk within their reinsurance portfolio. More specifically, instead of purchasing coverage from a single reinsurer, the risk can be transferred to several companies. This also holds true for the primary market, on which policyholders can share their risk with more than one insurer. In the recent academic literature, these so-called co-(re-)insurance agreements have been studied by several scholars (Fragnelli and Marina, 2003; Boyer and Nyce, 2013; Malamud et al., 2016; Boonen et al., 2016). Their results regarding the optimal coverage level are mainly driven by varying cost structures, pricing strategies, and risk preferences among the (re-)insurers. An analysis of the implicit diversification of the counterparty risk through such co-(re-)insurance, however, is missing. Hence, by answering the question how diversification through a co-(re-)insurance agreement impacts the policyholders optimal level of insurance coverage under default risk, the paper at hand bridges the gap between these two prominent literature streams.

Firstly, we will formulate the model, where we rely on a simple two-state model (loss or no loss), which is in accordance of Doherty and Schlesinger (1990) extended by the state of a default. Within this model, we suppose that the policyholder can freely choose the quantity of insurance

[1]This trigger rate only depends on the probability of loss and the probability of default (Mahul and Wright, 2007).

coverage in a co-insurance policy that is shared by $n$ co-insurers. For the number of co-insurers that fail to indemnify their share in the loss we introduce the mixed beta binomial framework. Secondly, the model is used to deduce the policyholder's decision-making on the optimal quantity of insurance coverage. Thereby, we restate the results of Mahul and Wright (2004) for the case of multiple co-insurance, that is, we point out, when over- and under-insurance, respectively, is optimal from the policyholder's perspective. Thirdly, we research the effect of a changing default correlation and an increasing number of co-insurers on both the policyholder's utility as well as on the optimal quantity of insurance coverage. In a conclusive discussion our analytical results are underlaid with an economic interpretation which points out the meaning of prudence on the policyholder's normative demand under default risk.

## Model Framework

We assume that the policyholder's non-random initial wealth amounts to $w$ and is exposed to some insurable binary risk $L$, for which we have $L = l < w$ with probability $p$ and $L = 0$, else. The policyholder can buy insurance coverage for this risk, where the quantity of the coverage is represented by the decision-variable $e_I$ and associated with the insurance premium $c(e_I, n)$. If $e_I = l$ holds, full coverage has been purchased; if, for instance, $e_I/l = 0.5$ is taken up, the policyholder has decided for partial insurance coverage indemnifying just 50 percent of an occurred loss. In general, $e_I < l$ corresponds to under-insurance, whereas $e_I > l$ implies over-insurance.

The insurance policy is co-insured by $n$ homogeneous insurance companies each holding a constant share of $1/n$ in the co-insurance policy. This means that insurer $i$ obtains the pro-rata premium amount $c(e_I, n)/n$ and must therefore indemnify the amount $e_I/n$ in the case of an occurred loss. The solvency state of insurer $i$, that is the insurer's ability to cover its enitre share in loss, is described by $\chi_i$ taking two values: for $\chi_i = 0$ the insurer is said to be solvent and pays its share entirely; if $\chi_i = 1$ holds, the insurer is insolvent and the policyholder is indemnified only partially. More precisely, in the state of insolvency an indemnification payment by insurer $i$ is scaled down by $(1 - \tau)$, where $0 \le \tau \le 1$. Thus, for $\tau = 1$, the policyholder would loose his or her full receivables on co-insurer $i$. For $\tau < 1$, the policyholder will experience at least a partial indemnification. In this context, $(1 - \tau)$ can be interpreted as the insurers' individual recovery rate being the part of obligation which is indemnified by a failed co-insurer in spite of its insolvency. For the further examination, we assume that the insurers' solvency states are stochastically independent of the loss state. If it is furthermore supposed that default events among the co-insurers were independent, the random number of failed insurers $F$ can be expressed by a Binomial distribution with density

$$\mathbb{P}[F = k] = \binom{n}{k} q^k (1-q)^{n-k}, \tag{1}$$

where $q$ denotes the identical default probability $P[\chi_i = 1] = q$ for any of the $i = 1, \dots, n$ co-insurers. While the identity on the default probability is a prosecution of assumed homogeneity, the assumption on independence of the default events appears to be too restrictive taking into account possible matters of contagion risk among financial institutes that have been displayed by recent studies on systemic risk (cf., e.g., **?**Eling and Pankoke, 2012; Baluch et al., 2011). We therefore generalize Equation (1) and introduce the possibility of having a positive correlation factor $\theta > 0$ on the default events of any pair $(\chi_i, \chi_j), i \neq j$, by relying for the joint probability of $\chi_1, \dots, \chi_n$ on a mixed beta binomial framework. In this framework, the density for the number of failed insurers $F_\theta$ is given by (cf., e.g., Moraux, 2010)

$$\mathbb{P}[F_\theta = k] = \int_0^1 \binom{n}{k} x^k (1-x)^{n-k} \psi(x; \alpha, \beta)\, dx \tag{2}$$

where $\psi$ is the density of a Beta distribution with parameters $\alpha$ and $\beta$ both being strictly positive. If $B$ is the beta function, then $\psi$ can be written as

$$\psi(x; \alpha, \beta) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1} (1-x)^{\beta-1}. \tag{3}$$

By comparing Equations (1) and (2), one observes as distinction that the generalized mixed framework assumes a prior distribution on the co-insurers' default probability. Thus, other than in the correlation-free framework represented by the former equation, the marginal default probability itself is a random variable $Q$ having a Beta distribution. By conditioning on $Q$, the number of failed insurers $F_\theta$ is then again described by a Binomial distribution. Given this randomization of the default probability, a positive, pairwise default correlation has been introduced, as one can verify (cf. Moraux, 2010) that

$$\theta = cor[\chi_i, \chi_j] = \frac{1}{1 + \alpha + \beta} > 0 \tag{4}$$

holds true. The mixed beta binomial framework is a common approach in credit risk modeling to deal with default correlation (cf. Moraux, 2010; Frey and McNeil, 2003). Moraux (2010) interprets the approach as a situation with a portfolio of homogeneous borrowers all having a known and identical credit rating which is associated with a marginal expected default probability being equal to $\tilde{q} := \mathbb{E}[\chi_i] = \mathbb{E}[Q] = \frac{\alpha}{\alpha+\beta}$. By using the re-parametrization of Moraux (2010) we can write

$$d_{k,n}(\theta,\tilde{q}) = \mathbb{P}[F_\theta = k] = \int_0^1 \binom{n}{k} x^k (1-x)^{n-k} \psi\left(x; \tilde{q}\frac{1-\theta}{\theta}, (1-\tilde{q})\frac{1-\theta}{\theta}\right) dx \tag{5}$$

amending the correlation-free framework of Equation (1) by the factor $\theta$ controlling for the default correlation. If the effect of a changing default correlation is aimed to be researched, one can vary parameter $\theta$; if it is supposed that the co-insurers' credit ratings change – where in our framework the change must be the same for all co-insurers due to the assumption on homogeneity – one adjusts parameter $\tilde{q}$. Regarding the variation of $\theta$, the two marginal cases $\theta = 0$ and $\theta = 1$ must be considered separately, since the Beta distribution is not defined for these values. Firstly, for $\theta \to 0$, the Beta distribution converges to an one-point distribution with all mass in $\tilde{q}$. Consequently, the model of (5) appropriately approaches the correlation-free model of Equation (1) with fixed default probability $\tilde{q}$. Secondly, if $\theta \to 1$, the distribution of $F_\theta$ converges to the two-point distribution with $\mathbb{P}[F_1 = 0] = 1 - \tilde{q}$ and $\mathbb{P}[F_1 = n] = \tilde{q}$, respectively, meaning that either no or all co-insurers fail.[2]

In the proposed setting of co-insurance with a homogeneous group of co-insurers, for which we assume the identical participation rate $1/n$ (A1), the identical and known recovery rate $1-\tau$ (A2) as well as an identical credit rating expressed by parameter $\tilde{q}$ (A3) and for which we also assume an independence between loss occurrence and the insurers' insolvency state (A4), we see, given the purchased insurance quantity $e_I$, that the expected indemnification from the coinsurance policy with $n$ co-insurers reads

$$\mathbb{E}[I_n] = \mathbb{E}\left[\mathbb{1}_{\{L=l\}} \frac{e_I}{n} \sum_{i=1}^{n} (1 - \chi_i \tau)\right] = e_I p (1 - \tilde{q}\tau), \tag{6}$$

which does not depend on the number of co-insurers nor on the degree of default correlation. By taking into account that $F_\theta|Q$ is a Binomial distribution with $n$ trials and success probability $Q$, the indemnification's variance turns out to be

$$\mathbb{V}[I_n] = \mathbb{E}\left[\mathbb{V}[I_n|Q]\right] + \mathbb{V}\left[\mathbb{E}[I_n|Q]\right] = e_I^2(1-\tau\tilde{q})^2 p(1-p) + \tau^2 e_I^2 p\tilde{q}(1-\tilde{q})(1/n + \theta(1-1/n)). \tag{7}$$

The variance's decomposition into two summands gives information on the sources of risk and how it is affected by changing $n$. The first term corresponds to the primary loss risk. Obviously, this risk must not depend on the chosen number of co-insurers. The second term comprises the

[2] Both convergence claims are verified by Lemma 2 in the appendix.

secondary default risk that evolves from purchasing insurance coverage. Basically, two cases can be distinguished: if there is no default correlation, i.e. $\theta = 0$, the second term vanishes as $n \to \infty$ and the default risk is thus extinguished. In contrast, a positive value for $\theta$ means that a residual default risk will always remain, even if the number of co-insurers were theoretically boundless, that is, the default risk is not entirely diversiable, if some default correlation is at hand. Moreover, if we assume that the co-insurers' individual default risk described by $\tilde{q}$ were unchanged just as the number of co-insurers, an increasing default correlation $\theta$ would in turn increase the indemnification's variance as well.

In the model of Doherty and Schlesinger (1990), the policyholder's ex-post wealth can take three different states: no loss, loss with no default and loss with default. Under multiple co-insurance, the random wealth $W = w - c(e_I, n) - L + I_n$ can overall take $n + 2$ different states, since the number of failing co-insurers is a number between 0 and $n$. Table 1 provides an overview on these different states. Regarding decision-making on the optimal quantity of insurance coverage, we assume that the policyholder is a risk-averse utility maximizer having a Neumann-Morgenstern utility function $u$ that is at least twice differentiable with $u' > 0$ and $u'' < 0$.[3] The policyholder will evaluate its utility by relying on the expected utility function

$$U_n(e_I; \theta, \tilde{q}) = (1-p)u\left(W_{NL}\right) + p\sum_{k=0}^{n} d_{k,n}(\theta, \tilde{q})u\left(W_{L,k}\right), \tag{8}$$

where the first summand of the right-hand side corresponds to the state of no loss and the second part corresponds to the state of loss by summing over all possible counts of failed insurers. The model can be traced back to the work of Doherty and Schlesinger (1990), who work as mentioned above with $n = 1$, which we term subsequently as *single-insurer policy*. The model of Doherty and Schlesinger (1990) allowing for contract non-performance is in turn a generalization of the model of Mossin (1968), which we denote as *default-free setting*.

For the cost functions $c$ we agree upon a pricing principle that sets the premium for the insurance policy equal to the expected payoffs adjusted by a proportional cost loading that covers the costs and profits of the co-insurers. We therefore write

$$\pi_I := p(1 - \tilde{q}\tau)(1 + \lambda), \tag{9}$$

[3]Some of our results require that $u(w - c(e_I, n) - l + e_I(1 - x))$ is continuous in $x \in [0, 1]$. This will be in fact implicitly given, if we assume by default that $u$ is differentiable.

| State | Probability | Final Wealth |
|---|---|---|
| loss $\wedge$ $n$ defaults | $pd_{n,n}$ | $W_{L,n} := w - c(e_I, n) - l + e_I(1-\tau)$ |
| $\vdots$ | $\vdots$ | $\vdots$ |
| loss $\wedge$ $k$ defaults | $pd_{k,n}$ | $W_{L,k} := w - c(e_I, n) - l + e_I(1-\frac{k\tau}{n})$ |
| $\vdots$ | $\vdots$ | $\vdots$ |
| loss $\wedge$ 0 defaults | $pd_{0,n}$ | $W_{L,0} := w - c(e_I, n) - l + e_I$ |
| no loss | $1-p$ | $W_{NL} := w - c(e_I, n)$ |

Table 1: Final wealth and corresponding probabilities.

which is the price for one unit of the insurance coverage with $\lambda$ being the corresponding cost-loading. Thus, the policyholder's costs for purchasing insurance coverage then amounts to $c(e_I, n) = e_I\pi_I$. In this definition of the co-insurance policy's premium principle we implicitly assume that the cost-loading is proportionally shared by the co-insurers, hence, the price for co-insurance is particularly not a function of the number of co-insurers. This is a simple but in our point of view not irrelevant premium principle: to our knowledge, it is by all means common practice that the total premium is proportionally shared in co-insurance agreements. Nonetheless, the principle is probably not indefinitely applicable: the larger $n$ becomes, the smaller is the absolute amount of premium that is assigned to each co-insurer; more precisely, $\frac{1}{n}c(e_I)$, which is the amount of premium for each co-insurers decreases towards zero as $n$ becomes large. Since each co-insurer possesses fixed running costs, $n$ is probably limited to some number with what each co-insurer can still cover its fixed running costs. For larger $n$, it is rational for the co-insurers to either leave the agreement or to charge a higher cost-loading. The concrete value for such a threshold on $n$ is possibly not unique but varies from risk to risk. For instance, a heavy corporate risk or a comprehensive reinsurance treaty requires automatically several co-(re)insurers, since a single (re)insure cannot bear the risk alone. Thus, coverage of such risks come implicitly with $n > 1$ and surely allow for higher $n$ than risks that could also be covered by a single-insurer policy and possibly have a lower premium volume. Throughout our analysis we keep the simple premium principle of Equation (9) for all $n$, claim that it is a realistic principle especially for low $n$, e.g, single-digit $n$, but provide for the sake of completeness a numeric example that illustrates how the results may change if the cost-loading is rather an increasing function of $n$.

**The Effect of the Recovery Rate in Multiple Co-Insurance Agreements**

Given an utility-maximizing policyholder, it is aimed at finding the coverage quantity $e^{\star}_{I,n}$ that maximizes $U_n$, when the co-insurance policy comprises $n$ co-insurers. Therefore, we need to solve the first-order condition

$$\frac{dU_n}{de_I} = -(1-p)\pi_I u(W_{NL}) + p\sum_{k=0}^{n} d_{k,n}\left(1-\pi_I-\tau\frac{k}{n}\right)u'\left(W_{L,k}\right)=0. \tag{10}$$

Since the second derivative with respect to $e_I$ is negative – due to the concavity of $u$ – it is ensured that $e^{\star}_{I,n}$ solving Equation (10) maximizes $U_n$ globally. Our first finding is linked to the work of Mahul and Wright (2007) according to which, given an actuarial fair premium in a single-insurer policy under default risk, there is a threshold $\tilde{\tau}_1 := (1-p)/(1-p\tilde{q})$ so that over-insurance is optimal if and only if $\tau$ is below this threshold. In other words, supposing that a default risk is introduced, the policyholder will tend to extend the insurance coverage, if the insurer's recovery rate is sufficiently large, and he or she will reduce insurance coverage, if the recovery rate is too small. For the specific case that both values coincide, full coverage – like in the default-free setting – will be optimal. Since $\tilde{\tau}_1 < 1$ for $\tilde{q} > 0$, it is ensured that a policyholder with a single-insurer policy facing a default risk with a zero-recovery-rate will never choose over-insurance. We generalize this claim by Mahul and Wright (2007) for the $n$-type co-insurance.

**Proposition 1.** *Assuming $\lambda = 0$, then:*

*(i)* $\tau < \tilde{\tau}_1 \Rightarrow e^{\star}_{I,n} > l$, *for all* $n > 1$, *i.e., over-insurance is optimal,*

*(ii)* $\tau > \frac{n-np}{1-npq} \Rightarrow e^{\star}_{I,n} < l$, *for all* $n > 1$, *i.e., under-insurance is optimal,*

*(iii) There is* $\tilde{\tau}_n \in (\tilde{\tau}_1, \frac{n-np}{1-npq})$ *such that:* $\tau = \tilde{\tau}_n \Rightarrow e^{\star}_{I,n} = l$, *for all* $n > 1$, *i.e., full insurance is optimal.*

According to the first claim, it is ensured that a policyholder choosing over-insurance in the single-insurer policy will also choose over-insurance in the co-insurance policy with $n \geq 2$. At the same time, the second claim in conjunction with the third claim indicate that $\tau_1$ is no more a threshold at which the demand switches from over- to under-insurance. Thus, if the single-insurer policy is replaced by a co-insurance policy, over-insurance can become optimal though the actual recovery rate is smaller than the critical threshold for the single-insurer policy. In this regard, co-insurance seems to have for cases with low recovery rates a stimulating effect on the demand for insurance coverage.

It is nearby to suppose that $\tilde{\tau}_n$ increases in $n$, yet, the exact value of $\tilde{\tau}_n$ in the interval $(\tilde{\tau}_1, \frac{n-np}{1-npq})$, and hence its behavior in $n$ depends on the curvature of the utility function. In the specific case of a mean-variance framework, for instance, the value for $\tilde{\tau}_n$ turns out to be

$$\tilde{\tau}_n = \frac{1-p}{(1-\tilde{q})(\theta + \frac{1}{n}(1-\theta)) + q(1-p)}. \tag{11}$$

In this case, the critical threshold indeed increases in $n$ and becomes larger than 1 as soon as $n > n^\star := (1-\theta)/(1-p-\theta)$. Since 1 is a natural upper bound for $\tau$, $n^\star$ is the number of co-insurers that must be reached at least, so that over-insurance is always optimal – even though the failure of insurers would result in a complete loss of the policyholder's claim. Apparently, the default correlation $\theta$ weakens this effect, as $n^\star$ increases in $\theta$, however, if the loss probability $p$ takes realistic values, two co-insurers would commonly suffices for over-insurance as optimal coverage quantity. The rationale is that in a co-insurance arrangement, there is the chance that a deficit from a failing co-insurer can be (partially) compensated from the over-indemnification of a second, solvent co-insurer. In the single-insurer policy, however, over-insurance is useless to combat a loss from failure if the recovery rate is zero, with what $\tilde{\tau}_1 < 1$ is not a surprising result. Aside the specific class of quadratic utility, we can deduce for a more general class of utility functions a condition, for which over-insurance with at least two co-insurers is always optimal even when there is no recovery of the policyholder's claims.

**Proposition 2.** *Let $\lambda = 0$, $u''' > 0$ and $\theta < 1-2p$. Then, over-insurance is optimal for all $\tau \in [0,1]$, if $n \geq 2$.*

From this proposition, we can deduce for the class of utility functions with positive third derivative, which includes among others utility functions with decreasing absolute risk aversion, that merely default correlation and the loss probability, respectively, must be reasonable low, so that two co-insurers already come along with over-insurance as optimal coverage. A crucial assumption in our model framework as well as in the framework of Mahul and Wright (2007) and Doherty and Schlesinger (1990), respectively, is the fixed and previously known recovery rate $1-\tau$. Since the result of Proposition 2 particularly holds true for $\tau = 1$, we can, however, conclude that, given the proposition's parameter conditions, over-insurance would also be optimal under $n \geq 2$, if $\tau$ is previously unknown but drawn from a random distribution that takes values from 0 to 1.

For fixed anticipated recovery rates, which are rather low, Proposition 2 indicates that a switch from a single-insurer policy to a multiple co-insurance policy would properly foster the optimal demand from under- to over-insurance, thus, a diversification by taking up co-insurance policy should have a stimulating effect on the insurance demand. Yet, it is unclear, whether this also holds true for high recovery rates, for which we know from Mahul and Wright (2007) that over-insurance may already be optimal under the single-insurer policy. Furthermore, it remains unsettled, how the optimal coverage develops, if the policyholder had already signed a co-insurance policy with $n$ co-insurers in the past, but can then add an additional co-insurer with what the degree of diversification increases to $n+1$: is it then optimal to increase the optimal coverage, i.e. $e^{\star}_{I,n} < e^{\star}_{I,n}$, or is rather better to decrease the coverage level – questions that results in ambiguous answers which we are going to research later on.

**Utility in Multiple Co-Insurance Agreements**

Beforehand, we examine the policyholder's utility and its dependency on the degree of diversification by multiple co-insurance; here, the results are unambiguous. For instance, from Equation (7), we know already that the variance of the policyholder's wealth strictly decreases in $n$ if $\theta < 1$. In conjunction with the expected wealth being invariant in $n$, a policyholder with quadratic utility will thus find it optimal to set $n$ as high as possible. More general, we show in the next proposition that this does not only hold true for the specific case of quadratic utility, but for any class of concave utility functions.

**Proposition 3.** *Let $u$ be a concave utility function $u$, for which $u(w - \pi_I e_I - l + e_I(1-x))$ is continuous in $x \in [0,1]$, then*

*(i) $U_n$ is non-decreasing in $n$, i.e., $U_{n+1} \geq U_n$ for $n \geq 1$ and*

$$\lim_{n\to\infty} U_n = \begin{cases} (1-p)u(w-\pi_I e_I) + p\int_0^1 x^{\alpha-1}(1-x)^{\beta-1} u(w-\pi_I e_I - l + e_I(1-x\tau))\,\frac{dx}{B(\alpha,\beta)}, & \text{if } \theta > 0 \\ (1-p)u(w-\pi_I e_I) + pu(w-\pi_I e_I - l + e_I(1-\tilde{q}\tau)), & \text{if } \theta = 0, \end{cases} \tag{12}$$

*where the convergence of the function series $U_n$ is uniform.*

*(ii) For $n \in \mathbb{N}_{\geq 1} \cup \{\infty\}$, $U_n$ is a non-increasing functions of $\theta \in (0,1)$ and $\lim_{\theta\to 1} U_n(\theta) = U_1$.*

*Part (i) can be revised to "strictly increasing", if $\theta < 1$ and $u$ is a strictly concave utility function.*

Given claim (i) of this proposition, extending diversification by means of multiple co-insurance would never worsen the position of a risk-averse policyholder, where the result also holds true for the generalized beta binomial model, that is, even if default correlation is at hand – in our framework measured by correlation factor $\theta$ – a risk-averse policyholder would choose $n$ as large as possible. Nonetheless, claim (ii) of Proposition 3 shows that a growing contagion risk among co-insurers negatively effects the policyholder's utility from the co-insurance policy. A growing contagion risk, thus, limits the benefit from diversification. If default correlation increases from $\theta_1$ to $\theta_2$, more co-insurers would be needed in order to achieve the same level of utility – if that is possible at all: the maximum achievable utility from co-insurance is namely bounded above by $U_\infty$ denoting the theoretical utility under infinite co-insurance diversification. By using the arguments of Proposition 3, it can then be shown that $U_\infty(\theta)$ decreases towards $U_1$ as $\theta \to 1$. Thus, there must be $n^\star$ such that $U_n(\theta_2) < U_\infty(\theta_2) < U_{n^\star}(\theta_1)$, with what the loss in utility from increasing contagion risk may possibly not fully be regained from an increased diversification.

The result on the benefits of co-insurance diversification indicate that, given our framework, the policyholder cannot choose too many co-insurers, where default correlation does not change this perception. Yet, it must be mentioned, that this unrestricted benefit from diversification is strongly sensitive to the aforementioned premium principle of Equation (9) that assumes a constant amount of absolute costs for changing $n$. If absolute costs were an increasing function in $n$, the marginal utility gain from diversification might be partially or fully offset by the marginal loss of utility from increasing costs. If increasing costs impair the utility more than the utility can benefit from diversification, the policyholder will find some $n^\star$ that maximizes his or her utility. Whether this is a global maximum certainly depends on the curvature of costs in $n$.

**Example 1.** In order to illustrate the effect of the premium principle on the policyholder's utility, we assume for a numeric example an exponential utility with $u(x) = 1 - \exp(-\beta x)$ and $\beta = 5.5$. Furthermore, we fix for the course of this example $w = 1.5$, $l = 1.0$, $p = 0.05$, $\tilde{q} = 0.1$, $\tau = 1$ and $\lambda = 0.2$. Regarding the calculation of $\pi_I$ we rely on the one hand on the principle of Equation (9), which is the basis of our analytical results in Proposition 3. On the other hand, in order to point out how theses results depend on the proposed principle, the example comprises a modified principle, in which diversification becomes costly once the number of co-insurers exceeds a threshold. We denote this threshold by $n^\star$ and define the modified premium principle as

$$\pi_I^{mod}(n) = \begin{cases} p(1-\tilde{q}\tau)(1+\lambda), & \text{if } n \leq n^\star \\ p(1-\tilde{q}\tau)(1+\lambda+\kappa(n-n^\star)^\gamma), & \text{else,} \end{cases} \tag{13}$$

where cost parameters $\omega$ and $\gamma$ are greater than 0. For $n \leq n^\star$, we thus suppose $\pi_I = \pi_I^{mod}$. If $n$ is above the threshold, every additional co-insurer means higher absolute costs for the policyholder. $\gamma$ controls for the curvature of the cost increase. If $\gamma > 1$, the incremental costs increase, if $\gamma = 1$, costs increase linear, if $\gamma < 1$ the incremental costs would decrease, which is rather

Table 2: Utility $U_n \times 10^2$

| | No diversification costs | | | Diversification costs for $n \geq 5$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\theta = 0$ | $\theta = 0.25$ | $\theta = 0.75$ | $\theta = 0$ | | $\theta = 0.25$ | | $\theta = 0.75$ | |
| $n$ | | - | | $\gamma = 1$ | $\gamma = 2$ | $\gamma = 1$ | $\gamma = 2$ | $\gamma = 1$ | $\gamma = 2$ |
| 1 | 99.9235 | 99.9235 | 99.9235 | 99.9235 | 99.9235 | 99.9235 | 99.9235 | 99.9235 | 99.9235 |
| 2 | 99.9565 | 99.9475 | 99.9311 | 99.9565 | 99.9565 | 99.9475 | 99.9475 | 99.9311 | 99.9311 |
| 3 | 99.9610 | 99.9539 | 99.9338 | 99.9610 | 99.9610 | 99.9539 | 99.9539 | 99.9338 | 99.9338 |
| 4 | 99.9621 | 99.9564 | 99.9353 | <u>99.9621</u> | <u>99.9621</u> | 99.9564 | 99.9564 | 99.9353 | 99.9353 |
| 5 | 99.9626 | 99.9578 | 99.9362 | 99.9621 | 99.9621 | 99.9572 | <u>99.9572</u> | <u>99.9354</u> | <u>99.9354</u> |
| 6 | 99.9629 | 99.9585 | 99.9368 | 99.9618 | 99.9608 | <u>99.9573</u> | 99.9562 | 99.9353 | 99.9338 |
| 7 | 99.9630 | 99.9590 | 99.9372 | 99.9615 | 99.9584 | 99.9573 | 99.9537 | 99.9350 | 99.9305 |
| 8 | <u>99.9631</u> | <u>99.9594</u> | <u>99.9376</u> | 99.9611 | 99.9547 | 99.9571 | 99.9498 | 99.9346 | 99.9255 |
| $\infty$ | 99.9637 | 99.9614 | 99.9399 | - | - | - | - | - | - |

unlikely. $\kappa$ is an additional scaling factor.

The left-hand side of Table 2 shows the policyholder's utility, if co-insurance is for free. For any value of $\theta$ an increasing number of co-insurers results in a higher utility for the policyholder. At the same time, for fixed $n$, the utility decreases in $\theta$. The final row shows the utility, when the policyholder could (theoretically) diversify infinitely. In the case of high contagion risk ($\theta = 0.75$), the policyholder's utility is bounded above by 99.9399, thus the policyholder could, for instance, not achieve the utility he or she would be able to achieve already with $n = 2$ co-insurers for the cases of low contagion risk ($\theta = 0$ and 0.25). The right-hand part of Table 2 illustrates the change of utility, if the policyholder cannot diversify endlessly without accepting an increasing loading factor. For the alternative premium principle of Equation (13) we choose the parameter values $\kappa = 0.05$ and $n^\star = 4$, i.e., the loading factor starts increasing as soon as more than four co-insurers are involved. Given this setting, it is optimal for the policyholder to limit the diversification to a number $n$ that maximizes utility before any additional co-insurer would provoke increasing costs that are not overcompensated by the gain of utility from further diversification.

**Optimal Demand in Multiple Co-Insurance Agreements**

Proposition 3 shows the limit value for the expected utility as the number of insurers goes to infinity. Admittedly, a boundless number of co-insurers is academic, yet, the figures of Table 2 indicate that the marginal diversification effects from co-insurance are rapidly decreasing, thus, the utility under infinite diversification can possibly approximate the utility for co-insurance with a reasonable number of co-insurers quite well. Since the convergence of $U_n$ towards $U_\infty$ is uniform, one can suppose that the convergence of the corresponding modes, regarding the optimal quantity of insurance coverage, should be likewise fast. If we denote by $e^\star_{I,\infty}$ the mode of $U_\infty$, this value should be at least a first landmark, how the optimal insurance demand will be influenced, when the policyholder can choose multiple co-insurance. The following proposition points out an explicit relationship between this optimal quantity of insurance coverage for a co-insurance policy with infinite diversification and the optimal quantity in the default-free setting of Mossin (1968).

**Proposition 4.** *Let $e^\star_{I,\infty}(0)$ be the mode of $U_\infty(0)$ for the no-default-correlation case $\theta = 0$ and define $e^\star_0$ as the optimal quantity of insurance coverage in the default-free setting of Mossin. Then, $U_\infty(0)$ is maximized by $e^\star_{I,\infty}(0) = \frac{e^\star_0}{1-\tilde{q}\tau}$.*

This proposition becomes clear, by noting that under $\theta = 0$ diversification allows to approximate the default-free setting of Mossin, since, theoretically, the default risk can be entirely eliminated by choosing infinite diversification. In this case, as it is indicated by Equation (12), the policyholder's uncertainty is reduced to the binary loss risk. In the case of an occurred loss, every purchased unit of insurance coverage, however, would not indemnify one unit of loss entirely but just $1 - \tilde{q}\tau$ of this unit of loss. In order to reach the same level of optimal indemnification as in Mossin's default-free framework, namely $e^\star_0$, it is therefore required to choose for insurance coverage the scaled version $(1-\tilde{q}\tau)^{-1}e^\star_0$. Since the scalar $(1-\tilde{q}\tau)^{-1}$ appears inverted in $\pi_I$, it cancels out and the policyholder bears the same total costs as in the default-free framework.

Though infinite diversification in co-insurance is the aforementioned purely academic example, the result of Proposition (4) serves as indicator for the optimal quantity of coverage in a co-insurance policy with sufficiently many co-insurers and indicates that this quantity is supposedly higher than the optimal coverage in the default-free setting. On the one hand, if $e^\star_{I,\infty}(0)$ is used as proxy for the optimal demand in a co-insurance policy, the known comparative statics in the default-free setting with respect to wealth, cost loading or risk aversion can be directly applied to the optimal coverage of the co-insurance policy. On the other hand, the denominator of $e^\star_{I,\infty}(0)$ shows that increased default risk, either due to an increase in $\tilde{q}$ or due to an increase in $\tau$, results in an extended insurance coverage. For the specific case $\lambda = 0$, we know from Mahul and Wright (2007) that for $n = 1$ under-insurance is optimal, if $\tau$ is sufficiently small (see above). Without cost-loading, we have $e^\star_{I,\infty}(0) = l/(1-\tilde{q}\tau)$, meaning that in a co-insurance policy with enough co-insurers over-insurance becomes optimal. Besides, the higher $\tau$ is, the more over-insurance is taken up. Thus, particularly for low recovery rates, there seems to be a meaningful incremental effect on the optimal coverage level, when a single-insurer policy is replaced by a co-insurance policy.

**Monotonicity Criteria for Diversification and Correlation Risk**

While $e^\star_{I,\infty}(0)$ can be considered as an approximate benchmark allowing for a (rough) comparison between the co-insurance and the single-insurer policy, we are interested in the behavior of $e^\star_{I,n}(\theta)$ with respect to the two dimensions "degree of diversification", i.e., a change in $n$, and "default correlation", i.e., a change in $\theta$. More precisely, the question at hand is, whether the policyholder finds it optimal to extend or to reduce the coverage, once the degree of diversification can be increased or – something, which is probably less controllable for the policyholder – a change in the default correlation among the co-insurers. The following proposition provides sufficient criteria for this question, where these criteria are based on the expression

$$h(x, e_I) = u'(w - \pi_I e_I - l + e_I - x\tau e_I)(1 - \pi_I - x\tau), \qquad x \in [0,1]. \tag{14}$$

$h$ measures the marginal utility from insurance coverage in the loss state, where $x$ controls for the percentage of insurance coverage that is not indemnified due to a failure. $x$ hereby serves as place holder for $k/n$ indicating how man co-insurers have failed and what percentage of the co-insurance policy is accordingly not indemnified.

**Proposition 5.** *Let $e^\star_{I,n}(\theta)$ denote the optimal quantity of insurance coverage for $n$ co-insurers and default correlation $\theta$.*

*(I) If $h\left(x, e^\star_{I,n}(\theta_1)\right)$ is* <u>*convex*</u> *for $x \in [0,1], n \in \mathbb{N}_{\geq 1} \cup \{\infty\}$ and $\theta_1 \in [0,1]$, then*

*(I.i)* $e^\star_{I,n}(\theta_1) \geq e^\star_{I,n+1}(\theta_1)$,

*(I.ii)* $e^\star_{I,n}(\theta_1) \leq e^\star_{I,n}(\theta_2)$ *for* $\theta_2 \in [\theta_1, 1]$.

*(II) If $h\left(x, e^\star_{I,n}(\theta_1)\right)$ is* <u>*concave*</u> *for $x \in [0,1], n \in \mathbb{N}_{\geq 1} \cup \{\infty\}$ and $\theta_1 \in [0,1]$, then*

*(II.i)* $e^\star_{I,n}(\theta_1) \leq e^\star_{I,n+1}(\theta_1)$,

*(II.ii)* $e^\star_{I,n}(\theta_1) \geq e^\star_{I,n}(\theta_2)$ *for* $\theta_2 \in [\theta_1, 1]$.

The implications *(I.i)* and *(II.i)* are pertained to the diversification effect on the optimal quantity of insurance coverage. If $h$ is a convex function under $e^\star_{I,n}(\theta_1)$, the policyholder will not demand more insurance coverage, when an additional co-insurer can be introduced into the co-insurance agreement. Inversely, if $h$ is concave, $e^\star_{I,n}(\theta_1)$ constitutes a lower bound for the insurance coverage as the number of co-insurers is increased. In both implications, the inequalities are strict, if $h$ is strictly convex and concave, respectively. In this case, $e^\star_{I,n}(\theta_1)$ does not only depict an upper or lower bound, but the quantity of insurance coverage is in fact either reduced or extended, when a co-insurer is added. Similarly, implications *(I.ii)* and *(II.ii)* are related to the dimension "default correlation": Assumed that the number of co-insurers is fixed and that $h$ is convex, a rising default correlation will make the policyholder to demand the same or more quantity of coverage, thus, an increased risk from default correlation rather stimulates the insurance demand. If $h$ were concave, the implication is the other way around meaning that the policyholder tends to demand less insurance coverage.

Since $h$ is a function of $u'$, such implications on reduced or extended insurance coverage are apparently a matter of the policyholder's risk preference. So far, the curvature on $h$ seems to be an abstract starting point for an interpretation of the optimal decision-making. However, if we assume that $u$ is three times differentiable, we can translate the convexity and concavity of $h$ into equivalent criteria, which takes into account the policyholder's degree of prudence. We measure prudence by $\eta(x) = -xu'''(x)/u''(x)$ (cf. Eeckhoudt et al., 2007), which is the policyholder's relative prudence and set with regard to the next proposition $\omega(x, e_I) := w - \pi_I e_I - l + e_I - x\tau e_I$, which is the policyholder's wealth in the loss state as a function of $x \in [0,1]$ and the chosen quantity of insurance coverage.

**Proposition 6.** *Let $u$ be a concave utility function, for which $u'''$ exists. Then,*

*(i)* $h(x, e_I)$ *is convex* $\iff \left(1 - \frac{w-l}{\omega(x,e_I)}\right)\eta(\omega(x, e_I)) \geq 2$, *for all* $x \in [0,1]$,

*(ii)* $h(x, e_I)$ *is concave* $\iff \left(1 - \frac{w-l}{\omega(x,e_I)}\right)\eta(\omega(x, e_I)) \leq 2$, *for all* $x \in [0,1]$.

*$h$ is strictly convex (concave), if $u$ is strictly concave and the inequalities of (i) and (ii), respectively, are strict.*

This proposition allows to interpret the policyholder's optimal decision-making under a changing number of co-insurers or a changing default correlation in terms of prudence. Put simply, we can deduce from the proposed equivalences that policyholders possessing high prudence would rather decrease the insurance coverage, when more co-insurers enter the contract, but increase the insurance coverage, when default correlation rises. On the contrary, policyholders with low prudence would tend to increase the insurance coverage under an increasing number of co-insurers, but decrease it, when default correlation rises. Yet, there may be settings, in which this broad conclusion does not work: the right-hand inequalities in Proposition 6 possibly do not hold for all $x \in [0,1]$, so that $h$ is neither convex nor concave. Then, a deduction via Proposition 5 is not possible anymore and there are cases, in which we have a policyholder with high or low prudence, but other implications as just proposed. Yet, there are specific classes of utility functions, for which unambiguous conclusions can be deduced.

**Quadratic Utility** For quadratic utility, for instance, the third derivative exists but is obviously zero, i.e., the policyholder's relative prudence is zero as well. With this, $h$ is universally convex. Thus, a policyholder with quadratic utility will always extend the quantity of insurance coverage, when new co-insurers enter the contract. At the same time, an increased default-correlation will always make the policyholder to decrease the coverage level.

**Constant Relative Prudence** For utility functions with constant relative prudence $\bar{\eta}$, $h$ is concave, if $\bar{\eta} \leq 2\left(1-(w-l)/\omega(0,e_I)\right)^{-1}$ is fulfilled. The right-hand side is bounded below by 2, thus, any policyholder with constant relative prudence being equal or smaller than 2 will probably demand higher insurance coverage but certainly not less; if default correlation increases, it is the other way around. Regarding the number of co-insurers, we can moreover deduce inductively, that once, the inequality holds for $e^{\star}_{I,N}$, it will also hold for any $n > N$, as the value in the bracket decreases in $e_I$. Therefore the sequence $(e^{\star}_{I,n})_{n\geq N}$ is non-decreasing.

On the contrary, $h$ is convex, when $\left(1-(w-l)/\omega(1,e_I)\right)\bar{\eta} \geq 2$. This inequality cannot hold, however, for $\omega(1,e_I) \leq w-l$, which is the case for $\tau \leq (1-p(1+\lambda))/(1-pq(1+\lambda))$, i.e., for low recovery rates our criteria cannot be applied. If the recovery rate is sufficiently large and $\omega(1,e_I) > w-l$ holds, in order to obtain the convexity of $h$, smaller prudence is required the smaller $\tau$ is, yet, the prudence must not become too small, otherwise, $h$ has no unique curvature or passes into concavity. Examples of utility functions with constant relative prudence are given by the class of power utility functions with $u(x) = (x^{1-\nu}-1)/(1-\nu)$ for $\nu > 0, \nu \neq 1$ and $u(x) = \ln(x)$ for $\nu = 1$, respectively. It has constant relative prudence with $\bar{\eta} = \nu + 1$.

**Constant Absolute Prudence** If the policyholder has constant absolute prudence, i.e., there is a constant $\tilde{\eta} > 0$, such that $\eta(x) = -x\tilde{\eta}$ (cf. Kimball, 1990), the left-hand side of the inequality constraints in Proposition 6 become $(1-\pi_I-\tau x)e_I\tilde{\eta}$. Again, if $\tau \geq (1-p(1+\lambda))/(1-pq(1+\lambda))$, $h$ can never be convex but is possibly concave, given that $\tilde{\eta}$ is small enough. Supposed that $\tau$ is below the proposed threshold, $h$ is convex, whenever $\tilde{\eta} \geq 2\left(1-\pi_I-\tau\right)^{-1} e_I^{-1}$ and concave, whenever $\tilde{\eta} \leq 2\left(1-\pi_I\right)^{-1} e_I^{-1}$. Examples of utility functions with constant absolute prudence are given by the class of exponential utility functions with $u(x) = 1-\exp(-\beta x)$, where $\beta > 0$ and $\eta(x) = -x\beta$.

Both examples, constant absolute as well as constant relative prudence, show that, in order to validate the adaptability of our criteria, it must be apparently distinguished between small and large $\tau$. For large $\tau$, our criteria can thoroughly be applied to policyholders with low prudence but not to policyholders with high prudence. For $\tau$ being small the example of constant absolute and relative prudence indicate, that from zero-prudence up to an upper bound, $h$ is concave and the decision-making of the policyholder can be correspondingly deduced. This bound in turn delimits downwards a range of prudence, for which our criteria cannot be used for a direct conclusion on optimal decision-making. For values of prudence above this range, $h$ is convex and our criteria is again applicable. In spite of this indistinct range of prudence, for which our criteria are indeed not directly adaptable, it can be reckoned, however, that due to continuity reasons a threshold presumably split this range in a lower part, where optimal decision-making is made as $h$ is concave, and an upper part, where optimal decision-making is made as $h$ is convex.

Therefore, it is basically feasible to argue that a policyholder with low prudence will thoroughly tend to increase insurance coverage, when additional co-insurers enter the contract, but will reduce insurance coverage, when the default correlation rises. On the contrary, a policyholder with high prudence, will rather tend to decrease insurance coverage, if the number of co-insurers is increased, but extend it, if default correlation rises – yet, only if $\tau$ is small enough; if $\tau$ is large, the decision-making of the high-prudent policyholder can coincide with the decision-making of the low-prudent policyholder.

**Conclusive Discussion**

It remains to elaborate on the rationale of this observed result. Doherty and Schlesinger (1990) use a stylized example in order to illustrate that buying an insurance policy twice is a way of eliminating a default risk entirely – given that the recovery rate for each policies amounts to 50 per cent. This example, though purely illustrative, shows quite well that over-insurance works as hedge against the default risk, since the recovered percentage of over-insurance compensates for the default from the basic coverage. An exception is given, if $\tau$ is high. Then, the recovery rate is low, with what recovered compensation from the over-insurance is low or, if $\tau = 1$, even nonexistent. In our point of view, this reasoning is the best way to retrace the threshold of Mahul and Wright (2007) on the recovery rate that splits the optimal demand in over- and under-insurance, respectively. The link to our results is drawn by noting that prudence is associated with precautionary saving (cf. Kimball, 1990). By relying on the interpretation of over-insurance as hedging instrument, it is nearby that under a given default risk a high-prudent policyholder will take up more insurance coverage than a policyholder possessing low prudence, if the recovery rate is high enough. This claim is verified for the single-insurer policy by the following corollary, which is a composition of Propositions 4, 5 and 6.

**Corollary 1.** *Let $u$ be a concave utility function, for which $u'''$ exists. If $e^\star_{I,\infty}(0)$ is as defined in Proposition 4 and if $e^\star_{I,1}$ denotes the optimal quantity of insurance coverage for the single-insurer policy, we can conclude that* *(i)* $\left(1 - \frac{w-l}{\omega(x,e_I)}\right)\eta(\omega(x,e_I)) \geq 2$*, for all* $x \in [0,1] \Longrightarrow e^\star_{I,1} \geq e^\star_{I,\infty}(0)$,

*(ii)* $\left(1 - \frac{w-l}{\omega(x,e_I)}\right)\eta(\omega(x,e_I)) \leq 2$*, for all* $x \in [0,1] \Longrightarrow e^\star_{I,1} \leq e^\star_{I,\infty}(0)$.

If prudence is low, $e^\star_{I,\infty}(0)$ will serve as upper bound for the optimal insurance quantity in a single-insurer policy under default risk. For high prudence and sufficiently low $\tau$, the optimal quantity is at least $e^\star_{I,\infty}(0)$ meaning that more coverage than in the default-free setting is demanded. There is no contradiction with the result of Mahul and Wright (2007) according to which under-insurance is optimal, if there is no cost-loading and $\tau$ is too high, because then, the inequality in claim *(i)* would not hold and $e^\star_{I,\infty}(0)$ is not a lower bound for the optimal insurance coverage. Overall, aside from settings with high $\tau$, policyholders with high prudence tend to demand more coverage in a single-insurer policy than policyholders with low prudence – a result, which we trace back to the precautionary motive of hedging the default risk.

Assumed that co-insurance is introduced in order to diversify this default risk, the precautionary motive obviously vanishes and the policyholder can divest the costly over-coverage[4] gradually as more co-insurers enter the policy. The situation is different, if the recovery rate is high. Then, the policyholder – though its prudence is high – has not demanded over-coverage in the single-insurer policy, with what there is no reason for divestment, when the default risk is diversified by switching to a co-insurance policy. Nevertheless, a zero-recovery-rate does not exclude that $e^\star_{I,n}$ decreases for sufficiently large $n$. Remember that according to Proposition 2 the recovery-rate-threshold of Mahul and Wright (2007) quickly becomes 0, if co-insurance is introduced. It is therefore possible that under $\tau = 1$ a prudent policyholder would demand comprehensive over-coverage, when $n = 2$, which subsequently abolished again for increasing $n$. Having said this, it is not surprising that a policyholder with high prudence tends to increase the insurance coverage, when $\tau$ is small and and the default correlation increases. From Equation (7), namely, we know that risk – measured in terms of the indemnification's variance – grows for increasing $\theta$, thus, there is an emerging uncertainty that is handled by the prudent policyholder with extended coverage. For a less prudent policyholder, it appears that the decreased utility of the policy (cf. Proposition 3) is in the foreground, with what the policyholder finds it more attractive to resign parts of the coverage rather than extending the coverage as hedge against the default risk.

**Example 2.** Let us assume for a final numeric example illustrating the above interpretation an exponential utility with $u(x) = 1 - \exp(-\beta x)$, where $\beta > 0$. Furthermore, we fix for the course of this example $w = 1.5$, $l = 1.0$, $p = 0.05$, $\tilde{q} = 0.1$ and $\lambda = 0$. Given these parameters, Figure 1

[4]We explicitly refer to $over-coverage$ here meaning more coverage than in the default-free setting. Over-coverage includes $over-insurance$ as special case, which is more than full coverage.

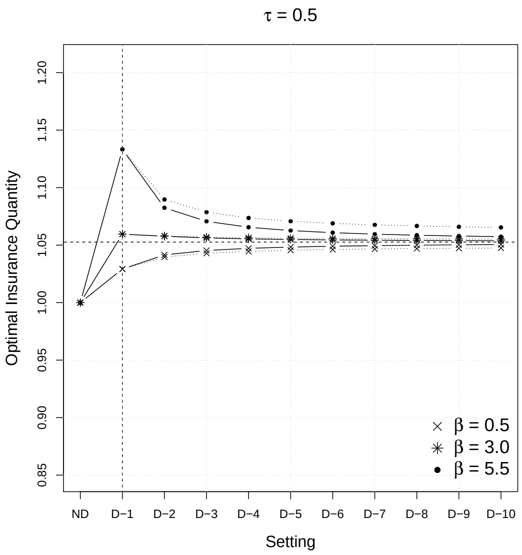


Figure 1: ND corresponds to a default-free single-insurer policy; D-$n$ to a policy at default risk with $n$ co-insurers; $w = 1.5$, $l = 1.0$, $p = 0.05$, $\tilde{q} = 0.1$ and $\lambda = 0$.

shows the optimal quantity of insurance coverage. It is distinguished between settings with and without default risk, where for the former settings the number of co-insurer is varied from $n = 1$ to $n = 10$. On the left-hand side, the case of a positive recovery rate with $\tau = 0.5$ is depicted, whereas on the right-hand side, a case of total default is supposed, that is $\tau = 1.0$. The solid lines belong to the correlation-free setting with $\theta = 0$, the dashed lines to an assumed default-correlation of $\theta = 0.15$. The horizontal lines in both plots display $e^{\star}_{I,\infty}$ which is consistently the limit value for the sequences of the solid lines.

The left-hand plot of Figure 1 verifies that a policyholder with low prudence ($\beta = 0.5$) increase the insurance coverage, when default risk can be released by introducing additional co-insurers. For a high-prudent policyholder ($\beta = 5.5$) we correctly observe a decreasing sequence of the optimal quantity of insurance coverage. Moreover, a rising default correlation results in higher coverage for the low-prudent policyholder, but in less coverage for the high-prudent policyholder. If $\tau = 1$, the proposed monotonicity is still at hand for the low-prudent policyholder, but not for the high-prudent policyholder, for which the relationship between quantity of insurance coverage and number of co-insurers turns out to be non-monotonous. Similarly, higher default correlation does not imply in general, more insurance coverage for the high-prudent policyholder. Ordered monotonous relationships for $\tau = 1$ and high prudence can again be observed, when $n$ has be-

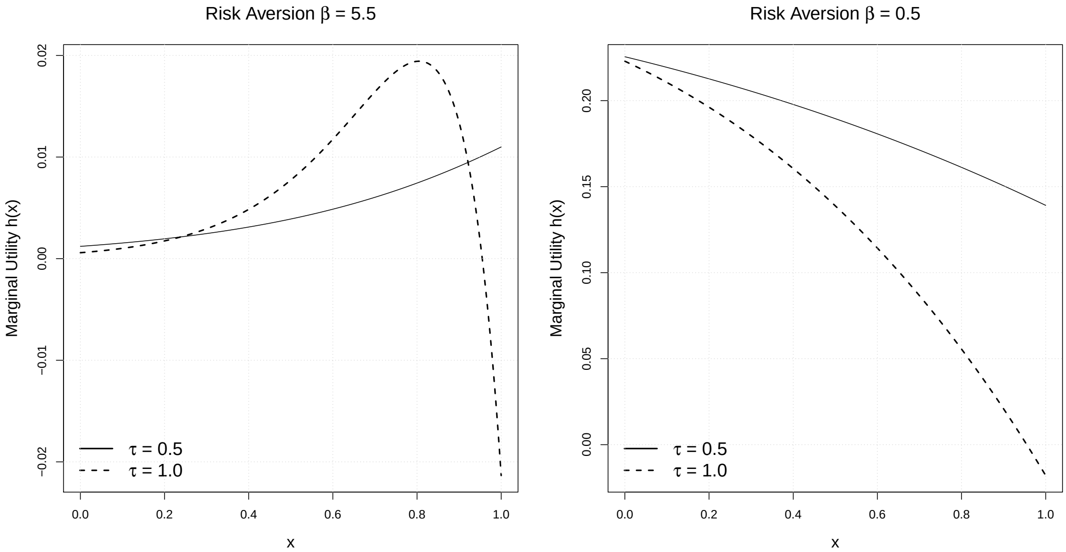


Figure 2: shows $h$ at the optimal quantity $e^{\star}_{I,5}$; $w = 1.5$, $l = 1.0$, $p = 0.05$, $\tilde{q} = 0.1$ and $\lambda = 0$.

comes sufficiently large.

Further insights can be drawn from this example by researching the marginal utility $h$ from Equation (14). It is plotted for both risk aversion parameters $\beta = 5.5$ and $\beta = 0.5$, respectively, in Figure 2. In accordance with Proposition 6, $h$ is convex for the high-prudent policyholder ($\beta = 5.5$) and it is concave for the low-prudent policyholder ($\beta = 0.5$), when $\tau = 0.5$. Apparently, this policy with a 50 percent recovery rate provides a high marginal utility for the high-prudent policyholder, when $x$ is large, that is, when (almost) all co-insurers fail at the same time. Note that a change in $\theta$ and $n$ does not change neither the first term of Equation (10) nor $h$ itself, yet, the probability weights of the sum in Equation (10) are changed. Thus, from the perspective of the high-prudent policyholder, any change of the density that allocates more weights to events with simultaneous co-insurer failures, that is the case, when $\theta$ increases or when $n$ decreases, will make him to buy more of the insurance coverage; given the above discussion the policyholder uses this additional coverage for hedging the tail event of a simultaneous co-insurer failure. On the contrary, the policyholder with low-prudence apparently does not have the need to hedge tail events, but finds the policy rather unattractive when such tail events become more likely, with what the coverage is reduced. Figure 2 also illustrates, why the demand of the low-prudent policyholder does not change structurally, when there is a switch to a policyholder with complete default ($\tau = 1$). In this case, the marginal utility $h$ remains decreasing and concave. For the high-prudent policyholder, however, $h$ drops in the range of tail events; thus, as long as, the probability weights are located in this range of $x$, where $h$ is concave, the claims of Proposition 6 does not hold true, yet, if $n$ becomes large, the probability weights are more allocated to the range, where $h$ is convex, so that we can for instance observe for sufficiently large $n$ a change of the optimal demand such as $\tau$ were small.

# A Appendix

### Proof on Proposition 1

Given $\lambda = 0$ and noting that $-(1-p)\pi_I$ can be rewritten as $-p\left(1 - \pi_I - \sum_{k=0}^n d_{k,n}\frac{k}{n}\tau\right)$, since $\tilde{q}\tau = \sum_{k=0}^n d_{k,n}\frac{k}{n}\tau$, we have

$$\begin{aligned}\left.\frac{dU_n}{de_I}\right|_{e_I=l} &= -(1-p)\pi_I u(W_{NL}) + p\sum_{k=0}^n d_{k,n}\left(1-\pi_I-\tau\frac{k}{n}\right)u'\left(W_{L,k}\right)\\ &= p\sum_{k=1}^n d_{k,n}\left(1-\pi_I-\frac{k}{n}\tau\right)\left\{u'(W_{L,k})-u'(W_{NL})\right\},\end{aligned} \tag{A.1}$$

where it has additionally been used that $W_{NL} = W_{L,0}$ at $e_I = l$. Furthermore, in this point $u'(W_{L,k}) > u'(W_{NL})$ holds true for all $1 \le k \le n$, thus, we can conclude:

$$1-\pi-\tau > 0 \Longrightarrow 1-\pi_I-\tau\frac{k}{n} > 0,\ \text{for } 1 \le k \le n \Longrightarrow \left.\frac{dU_n}{de_I}\right|_{e_I=l} > 0. \tag{A.2}$$

Since $1-\pi-\tau > 0 \Longleftrightarrow \tau <$, (i) is proved.

Analogously, (ii) is proved by noting that

$$1-\pi-\tau\frac{1}{n} < 0 \Longrightarrow 1-\pi_I-\tau\frac{k}{n} < 0,\ \text{for } 1 \le k \le n \Longrightarrow \left.\frac{dU_n}{de_I}\right|_{e_I=l} < 0, \tag{A.3}$$

and $1-\pi-\tau\frac{1}{n} < 0 \Longleftrightarrow \tau <$.

(iii) follows from the intermediate value theorem.

### Proof on Proposition 2

Due to the assumption $u''' > 0$, it is ensured that $u'$ is convex, thus, we have at at $e_I = l$:

$$u'(W_{L,k}) - u'(W_{NL}) > -lu''(W_{NL})\tau\frac{k}{n}. \tag{A.4}$$

Applying this to Equation (A.1) yields

$$\begin{aligned}\left.\frac{dU_n}{de_I}\right|_{e_I=l} &> -lp\tau u''(W_{NL})\sum_{k=1}^{n} d_{k,n}\left(1-\pi_I-\frac{k}{n}\tau\right)\frac{k}{n} \\ &= -lp\tilde{q}\tau u''(W_{NL})\left(1-\pi_I-\frac{\tau}{n}\Pi(n)\right),\end{aligned} \tag{A.5}$$

where $\Pi(n) := ((1-\tilde{q})(1-\theta+n\theta)+n\tilde{q})$. Since $u'' < 0$, the right-hand side of Equation (A.5) becomes positive, if

$$1-\pi_I-\frac{\tau}{n}\Pi(n) > 0 \Longleftrightarrow \tau < \frac{1-p}{\Pi(n)n^{-1}-pq} := \kappa(n). \tag{A.6}$$

$\kappa(n)$ increases in $n$ and we have $\kappa(2) = \frac{1-p}{0.5(1-\tilde{q})(1+\theta)+(1-p)\tilde{q}}$, which becomes greater than 1 for $\theta < 1-2p$.

## Proof on Proposition 3

In order to prove this proposition we at first allude to the following two lemmas.

**Lemma 1.** *Given the probability weights $d_{k,n}$ as defined in Equation* (5) *and a function $h(x)$ that is convex in* $[0,1]$, *we have for $n \geq 2$*

$$S_n := \sum_{k=0}^{n} h\left(\frac{k}{n}\right) d_{k,n} \leq \sum_{k=0}^{n-1} h\left(\frac{k}{n-1}\right) d_{k,n-1}. \tag{A.7}$$

*Proof.* For the correlation-free model, where $\theta = 0$, the probability weights belong to a Binomial distribution, with what the sum $S_n$ in Equation (A.15) is a Bernstein polynomial, for which the proof on the inequality is, e.g., provided by Theorem 6.3.4 in Davis (1963). For $\theta = 1$, we have full correlation, i.e., either no or all co-insurers fail; in this case, the claim is trivial as $\tilde{q}$ is fixed.

For $\theta \in (0,1)$, we use the convexity of $f$ and deduce for $0 < k < n$ (cf. Bennett and Jameson, 2000)

$$h\left(\frac{k}{n}\right) \leq \frac{k}{n}h\left(\frac{k-1}{n-1}\right) + \frac{n-k}{n}h\left(\frac{k}{n-1}\right). \tag{A.8}$$

Thus,

$$\begin{aligned} S_n &= h(0)d_{0,n} + h(1)d_{n,n} + \sum_{k=1}^{n-1} h\left(\frac{k}{n}\right) d_{k,n} \\ &\le h(0)d_{0,n} + h(1)d_{n,n} + \sum_{k=1}^{n-1} \left\{\frac{k}{n} h\left(\frac{k-1}{n-1}\right) + \frac{n-k}{n} h\left(\frac{k}{n-1}\right)\right\} d_{k,n} \\ &= h(0)d_{0,n} + h(1)d_{n,n} + \sum_{k=0}^{n-2} \frac{k+1}{n} h\left(\frac{k}{n-1}\right) d_{k+1,n} + \sum_{k=1}^{n-1} \frac{n-k}{n} h\left(\frac{k}{n-1}\right) d_{k,n} \\ &= \sum_{k=0}^{n-1} h\left(\frac{k}{n-1}\right)\left(\frac{k+1}{n} d_{k+1,n} + \frac{n-k}{n} d_{k,n}\right). \end{aligned} \tag{A.9}$$

Note that the probability weight $d_{k,n}$ can be written as

$$d_{k,n} = \binom{n}{k} \frac{B(\alpha+k, \beta+n-k)}{B(\alpha,\beta)}, \tag{A.10}$$

where $\alpha = \tilde{q}\frac{1-\theta}{\theta}$, $\beta = (1-\tilde{q})\frac{1-\theta}{\theta}$ and $B$ is the beta function. By using the recursion properties

$$\frac{k+1}{n}\binom{n}{k+1} = \binom{n-1}{k}, \tag{A.11}$$

$$\frac{k+1}{n}\binom{n}{k+1} = \binom{n-1}{k}, \tag{A.12}$$

$$B(\alpha+k+1, \beta+n-k-1) = B(\alpha+k, \beta+n-k-1)\frac{\alpha+k}{\alpha+\beta+n-1}, \tag{A.13}$$

$$B(\alpha+k, \beta+n-k) = B(\alpha+k, \beta+n-k-1)\frac{\beta+n-k-1}{\alpha+\beta+n-1} \tag{A.14}$$

we obtain $\frac{k+1}{n} d_{k+1,n} + \frac{n-k}{n} d_{k,n} = d_{k,n-1}$, which finishes the proof. Additionally, note that, with Equation (A.8) in mind, the inequality of the lemma is strict, if $h$ is strictly convex. □

**Lemma 2.** *Let $f$ be a real-valued function and set*

$$B_t(f,y) = \int_0^1 f(x) x^{ty-1}(1-x)^{t(1-y)-1} \frac{dx}{B(ty, t(1-y))}, \qquad t \ge 0, \qquad y \in [0,1]. \tag{A.15}$$

*For the beta operator $B_t$ the following claims hold true:*

*(i) If $f$ is real continuous function on $[0,1]$, then*

$$\lim_{t\to\infty} B_t(f,y) = f(y), \quad \textit{for } y \in (0,1). \tag{A.16}$$

*(ii) If $f$ is real continuous function on $[0,1]$, then*

$$\lim_{t\to 0} B_t(f,y) = (1-y)f(0) + yf(1), \quad \text{for } y \in (0,1). \tag{A.17}$$

*(iii) For $y \in (0,1)$, $0 < t_1 < t_2$ and a convex function $f$ defined on $(0,1)$ such that $B_s(|f|,y) < \infty, s \in \{r,t\}$, we have*

$$B_{t_1}(f,y) \geq B_{t_2}(f,y). \tag{A.18}$$

*Proof.* Claim (i) is a consequence of Theorem 6 in Khan (1991). Regarding (ii) we allude to the beginning of the proof on Theorem 2 in Adell et al. (1996), which points out that this claim results from the Helly-Bray theorem. Claim (iii) corresponds to Theorem 1 in Adell et al. (1996).

Remark: Lemma 1 and the third claim of Lemma 2 hold also true for concave functions, where the inequality signs are just reversed. □

Regarding claim (i) of Proposition 3 we observe in the definition for $U_n$ (Equation (8)) that the first term (no-loss-state) is constant for $n$; for the sum $\sum_{k=0}^{n} d_{k,n} u\left(W_{L,k}\right)$ we can apply Lemma 1 by noting that

$$h(x) := u(w - \pi_I e_I - l + e_I - x\tau e_I) \tag{A.19}$$

is a concave function in $[0,1]$, which proves that $U_n$ is non-decreasing in $n$. The convergence for $\theta$ is deduced by using that $\sum_{k=0}^{n} d_{k,n} u\left(W_{L,k}\right)$ is a Bernstein polynomial (cf. Davis, 1963) with respect to the continuous function $h(x) := u(w - \pi e_I - l + e_I - x\tau e_I)$. It is a well-known result from approximation theory that the Bernstein polynomial $\sum_{k=0}^{n} d_{k,n} h(k/n)$ converges uniformly towards $h(\tilde{q})$. For a proof, see, e.g., Theorem 6.2.2 in Davis (1963). By noting that the first term of $U_n$ (see Equation (8), no-loss-state) is constant for $n$, we can then use this convergence theorem on the Bernstein polynomial in order to deduce the uniform convergence

$$U_\infty = \lim_{n\to\infty} U_n = (1-p)u(w - \pi_I e_I) + pu(w - \pi_I e_I - l + e_I(1 - \tilde{q}\tau)). \tag{A.20}$$

For $\theta \in (0,1)$, i.e., the probability weights $d_{k,n}$ belong to a beta-binomial distribution. Grinshpan (2010) indicates in his Formula (38) the beta-binomial version of the Bernstein polynomial's convergence theorem:

$$\lim_{n\to\infty} \sum_{k=0}^{n} d_{k,n} h\left(\frac{k}{n}\right) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1} h(x) \, \frac{dx}{B(\alpha,\beta)}, \tag{A.21}$$

where the convergence is uniform for $h$ being continuous. This is in turn a consequence of Theorem 1.2.1 in Lorentz (1986). Applying this to the sum of $U_n$ provides the the first part of Proposition 3.

Regarding claim (ii) of Proposition 3 we use that

$$g(x) = \sum_{k=0}^{n} h\left(\frac{k}{n}\right)\binom{n}{k}x^k(1-x)^{n-k} \tag{A.22}$$

is concave in $x \in (0,1)$, if $h$ is the concave function from Equation (A.19). In order to see this, we refer to Lorentz (1986, p. 23) noting that the concavity of $h$ yields for the second difference operator

$$\Delta^2 h\left(\frac{k}{n}\right) := h\left(\frac{k+2}{n}\right) - 2h\left(\frac{k+1}{n}\right) + h\left(\frac{k}{n}\right) \leq 0, \qquad k = 0,\dots,n-2. \tag{A.23}$$

Since (cf. Lorentz, 1986, p. 12)

$$g''(x) = n(n-1)\sum_{k=0}^{n-2} \Delta^2 h\left(\frac{k}{n}\right)\binom{n-2}{k}x^k(1-x)^{n-k-2}, \tag{A.24}$$

it is shown that the concavity of $h$ implies the concavity of $g$. We furthermore observe that

$$U_n = (1-p)u\left(W_{NL}\right) + p\sum_{k=0}^{n} d_{k,n}(\theta,\tilde{q})u\left(W_{L,k}\right) \tag{A.25}$$

$$= (1-p)u\left(W_{NL}\right) + p\int_0^1 g(x)x^{\alpha-1}(1-x)^{\beta-1}\,\frac{dx}{B(\alpha,\beta)}. \tag{A.26}$$

The first term is non-dependent on $\theta$. The second part is non-increasing as $\theta$ increases on $(0,1]$; that is a consequence of claim (iii) in Lemma 2 applied to the convex function $-g$. Therefore, also note that with $\alpha = \tilde{q}\frac{1-\theta}{\theta}$ and $\beta = (1-\tilde{q})\frac{1-\theta}{\theta}$, we can choose the substitution $y = \tilde{q}$ and $t = b(\theta) = \frac{1-\theta}{\theta}$, which yields the parametrization of Lemma 2. For the bijection $b$ it holds true: for any $1 > \theta_1 > \theta_2 > 0$ there are $0 < t_1 < t_2$ with $b(\theta_1) = t_1$ and $b(\theta_2) = t_2$.

For the convergence claim we rely on claim (ii) of Lemma 2 and note that $t \to 0$ is equivalent with $\theta \to 1$, if we use the substitution $t = b(\theta)$ from above.

### Proof on Proposition 4

Let $U_0$ denote the expected utility function in the default-free setting of Mossin, which is maximized by the coverage quantity $e_0^\star$. Setting $\pi_{I,0} = \pi_I(1-\tilde{q}\tau)^{-1} = p(1+\lambda)$ and writing for convenience $U_\infty(0) = U_\infty$ as well as $e_0^\star(1-\tilde{q}\tau)^{-1} = e_{I,\infty}^\star(0) = e_{I,\infty}^\star$, we have

$$\begin{aligned} 0 &= -(1-p)\pi_{I,0}u'(w - e_0^\star\pi_{I,0}) + p(1-\pi_{I,0})u'(w - e_0^\star\pi_{I,0} - l + e_0^\star) \\ &= -(1-p)\pi_I(1-\tilde{q}\tau)^{-1}u'(w - e_{I,\infty}^\star\pi_I) + p(1-\tilde{q}\tau-\pi_I)(1-\tilde{q}\tau)^{-1}u'(w - e_{I,\infty}^\star\pi_I - l + e_{I,\infty}^\star) \\ &= (1-\tilde{q}\tau)^{-1}\left.\frac{dU_\infty}{de_I}\right|_{e_I = e_{I,\infty}^\star}, \end{aligned} \tag{A.27}$$

which proves the claim, since $U_\infty$ is concave in $e_I$.

### Proof on Proposition 5

Regarding claim (I.i), we have for $\theta \in [0,1]$, $e_{I,n}^\star(\theta) = e_{I,n}^\star$, $e_{I,n+1}^\star(\theta) = e_{I,n+1}^\star$,

$$\begin{aligned} 0 &= -(1-p)\pi_I u'(w - e_{I,n}^\star\pi_I) + p\sum_{k=0}^{n} d_{k,n} h\left(k/n, e_{I,n}^\star\right) \\ &\geq -(1-p)\pi_I u'(w - e_{I,n}^\star\pi_I) + p\sum_{k=0}^{n+1} d_{k,n+1} h\left(k/(n+1), e_{I,n}^\star\right) \\ &= \left.\frac{dU_{n+1}}{de_I}\right|_{e_I = e_{I,n}^\star} \end{aligned} \tag{A.28}$$

The inequality is deduced from Lemma 1 using that $h$ is convex by assumption. Since $U_{n+1}$ is concave, we can then conclude that $e_{I,n}^\star \geq e_{I,n+1}^\star$.

Regarding claim (I.ii), we have for $\theta_1 \in (0,1)$, $\alpha_1 :=$, $\beta_1 :=$

$$\begin{aligned} 0 &= -(1-p)\pi_I u'(w - e_{I,n}^\star(\theta_1)\pi_I) + p\sum_{k=0}^{n} d_{k,n} h\left(k/n, e_{I,n}^\star(\theta_1)\right) \\ &= -(1-p)\pi_I u'(w - e_{I,n}^\star(\theta_1)\pi_I) + p\int_0^1 g\left(x, e_{I,n}^\star(\theta_1)\right) x^{\alpha_1 - 1}(1-x)^{\beta_1 - 1}\frac{dx}{B(\alpha_1,\beta_1)}, \end{aligned} \tag{A.29}$$

where $g\left(x, e^{\star}_{I,n}(\theta_1)\right) = \sum_{k=0}^{n} h\left(k/n, e^{\star}_{I,n}(\theta_1)\right)\binom{n}{k}x^k(1-x)^{n-k}$. From the proof on Proposition 3, we know, given the presumed convexity of $h$, that $g$ is convex. Thus, we can apply claim (iii) of Lemma 2 resulting in

$$\begin{aligned}0 &= -(1-p)\pi_I u'(w - e^{\star}_{I,n}(\theta_1)\pi_I) + p\int_0^1 g\left(x, e^{\star}_{I,n}(\theta_1)\right)x^{\alpha_1-1}(1-x)^{\beta_1-1}\,\frac{dx}{B(\alpha_1,\beta_1)} \qquad \text{(A.30)}\\ &\leq -(1-p)\pi_I u'(w - e^{\star}_{I,n}(\theta_1)\pi_I) + p\int_0^1 g\left(x, e^{\star}_{I,n}(\theta_1)\right)x^{\alpha_2-1}(1-x)^{\beta_2-1}\,\frac{dx}{B(\alpha_2,\beta_2)}\\ &= \left.\frac{dU_n(\theta_2)}{de_I}\right|_{e_I=e^{\star}_{I,n}(\theta_1)}, \qquad \text{(A.31)}\end{aligned}$$

for $\alpha_2 =$ and $\beta_2 =$ with $\theta_2 \in [\theta_1, 1)$. Claim (ii) of Lemma 2 yields

$$\begin{aligned}\lim_{\theta_2\to 1}\left.\frac{dU_n(\theta_2)}{de_I}\right|_{e_I=e^{\star}_{I,n}(\theta_1)} &= -(1-p)\pi_I u'(w - e^{\star}_{I,n}(\theta_1)\pi_I) + (1-\tilde{q})h\left(0, e^{\star}_{I,n}(\theta_1)\right) + \tilde{q}h\left(1, e^{\star}_{I,n}(\theta_1)\right) \qquad \text{(A.32)}\\ &= \left.\frac{dU_1}{de_I}\right|_{e_I=e^{\star}_{I,n}(\theta_1)},\end{aligned}$$

which shows that claim (I.ii) can be generalized to $\theta_1 = 1$. Furthermore, according to claim (i) of Lemma 2

$$\begin{aligned}&\lim_{\theta_1\to 0}\left\{-(1-p)\pi_I u'(w - e^{\star}_{I,n}(\theta_1)\pi_I) + p\int_0^1 g\left(x, e^{\star}_{I,n}(\theta_1)\right)x^{\alpha_1-1}(1-x)^{\beta_1-1}\,\frac{dx}{B(\alpha_1,\beta_1)}\right\}\\ &= -(1-p)\pi_I u'(w - e^{\star}_{I,n}(0)\pi_I) + g\left(\tilde{q}, e^{\star}_{I,n}(0)\right)\\ &= \left.\frac{dU_n(0)}{de_I}\right|_{e_I=e^{\star}_{I,n}(0)}. \qquad \text{(A.33)}\end{aligned}$$

Thus, claim (I.ii) also holds for $\theta_1 = 0$.

Claims (II.i) and (II.ii) can be analogously proved with reversed inequality signs.

## Proof on Proposition 6

For $h(x, e_I) = h(x)$ it can be shown that

$$h''(x) = \tau^2 e_I u''(\omega(x, e_I))\left\{2 - \frac{(1-\pi_I-\tau x)e^{\star}_{I,n}}{\omega(x,e_I)}\eta(\omega(x,e_I))\right\}. \qquad \text{(A.34)}$$

Since $u'' \leq 0$,

$$h'' \geq 0 \Leftrightarrow 2 - \frac{(1-\pi_I-\tau x)e^{\star}_{I,n}}{\omega(x,e_I)}\eta(\omega(x,e_I)) \leq 0 \Leftrightarrow \left(1-\frac{w-l}{\omega(x,e_I)}\right)\eta(\omega(x,e_I)) \geq 2, \quad \text{(A.35)}$$

$$h'' \leq 0 \Leftrightarrow 2 - \frac{(1-\pi_I-\tau x)e^{\star}_{I,n}}{\omega(x,e_I)}\eta(\omega(x,e_I)) \geq 0 \Leftrightarrow \left(1-\frac{w-l}{\omega(x,e_I)}\right)\eta(\omega(x,e_I)) \leq 2, \quad \text{(A.36)}$$

which finishes the proof.

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