Insurance and Self-protection for Increased Risk Aversion

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Abstract

We re-examine the classic problem of risk aversion and self-protection in this paper. By applying interval dominance order (Quah and Strulovici,2009) method, we find that an increased risk aversion induces the level of protection if the value of hazard rate is higher than the 'boldness'coefficient(Aumann and Kurz,1977). This new condition is effective when the optimal protection is not interior solution. We also show this new condition is effective with self-insurance-cum-protection model (Lee, 1998), in which the actions to prevent the risk sever both as self-insurance and self-protection.

1 Research background and introduction

In order to mitigate risks, individuals may take actions either reducing the severity of potential loss (self-insurance or loss reduction)or reducing the probability of occurrence for a risk(self-protection or loss prevention).It is intuitive that people would take more efforts to reduce risk when they become more unwilling to take risk. Ehrlich and Becker(1972) are the first to study the demand of self-insurance and self protection. They focus on the interaction between market insurance, self-insurance and self-protection.In another classical paper,Dionne and Eeckhoudt(1985) show that a more risk-averse decision maker would take higher self-insurance activities, while the effect of increased risk aversion on self-protection is ambiguious.Later studies have tried to clarify the ambiguous link between risk aversion and self-protection.Boyer and Dionne (1989) study the relation between increased exogenous risk and self-protection actions, but they find that impact on the self-protection activities by increased risk is ambiguous and increased self-protection maybe the result of increased risk under non-DARA utility functions.Briys and Schlesinger(1990) prove that the relation between risk aversion and self-insurance are still robust in several distinct settings, such as state-dependent utility, the presence of background and random initial wealth,while the self-protection cannot hold in these settings.Jullien and et al.(1999) suggest a utility-dependent threshold of probability,beneath which more self-protection is the result of increased risk aversion.Chiu(2000)analyzed the effect of prudence on this threshold.Eeckhoudt and Gollier(2005) propose some assumptions under which a risk-neutral agent invests less in self-protection than a prudence agent.

Interestingly, the method in comparative statics of risk aversion and prevention efforts is mainly confined to first order condition,which requires some technical assumptions of second order conditions for utility function [1].Single cross-

[1]for example, concave of utility function

ing property(Milgrom and Shannon,1994) and interval dominance order(Quah and Strulovici,2009) enables one to better analyze this comparative statics problem of risk aversion and self protection with less assumptions, such as no assumptions on the second order conditions for utility function.Early contributions in literature of monotone comparative statics are Milgrom and Roberts(1990),Vives(1990)and Topkis(1998).Milgrom and Shannon (1994) characterize the single crossing condition and demonstrate its application to several settings,such as competitive firm,the Bertrand oligopolistist and so on. However,single crossing property maybe invalid in some situations.Quah and Strulovici(2009) identify the interval dominance order,which requires weaker condition than single crossing property and complement the cases when conditions do not guarantee single crossing in its dominance.

By applying single cross property and interval dominance order, we show the relationship between increased risk aversion and self-insurance is still effective without second order assumptions and under a new condition an increased risk aversion increases self-protection efforts.

The structure of this paper is as follows.Section 2 provides settings of the model and derive the comparative statics on changes in risk aversion.Section 3 concludes the paper.

2 The models

Consider an individual with an initial wealth w_0 is facing an event with potential risk. Let $p \in [0, 1]$ defines the probability of risk happens and $(1-p)$ is the state when risk does not happen.Assume this individual has von Neumann-Morgenstern utility function u,and utility function is differentiable and strictly increasing ($u^{'} > 0$). The individual can engage in self-insurance activities to reduce the potential losses of the accident,self-protection activities to guarantee a better chance of accidents do not happen or self-insurance-cum-protection activities to reduce the size and probability of losses at the same time.The decision maker's effort in self-insurance or self-protection doesnot beyond his initial wealth.

We will first consider self-insurance activities and self-protection activities separately,then we study SICP model.

2.1 Self-Insurance

Assume an individual's level of self-insurance is x.Monetary cost for self-insurance activity is $c(x)$ and increasing marginal cost indicates $c^{'}(x) > 0$.Potential loss is reduced by self-insurance activities,thus loss is a function of level of self-insurance $l(x)$.Because marginal effect of self-insurance is decreasing,$l^{'} < 0$ holds.

The expected utility function for individual u can be put:

$$U(x) = pu(w_0 - c(x) - l(x)) + (1-p)u(w_0 - c(x))$$

Assume another individual v is more risk-averse than u and his expected utility can be represented by a concave transformation $(k(u), k^{'} > 0, k^{''} < 0)$ of u's expected utility.

$$\begin{aligned} V(x) &= pv(w_0 - c(x) - l(x)) + (1-p)v(w_0 - c(x)) \\ &= pk(u((w_0 - c(x) - l(x))) + (1-p)k(u(w_0 - c(x))) \\ &= k(U(x)) \end{aligned}$$

In Dionne and Eeckhoudt's 1985 paper,they resort to first order condition to show v takes higher self-insurance activites than u does and finds v's marginal utility is larger than zero at the optimal self-insurance level for u $(V^{'}(x_u > 0)$.A major limitation for their conclusion is they require the first order derivatives of $U(x)$ and $V(x)$ are monotonic $(U^{''}(x) < 0, V^{''}(x) < 0)$.Otherwise,$V^{'}(x_u) > 0$ does not guarantee higher $x_v > x_u$.

Here, we resort to single crossing condition, which is still effective even when monotonicity of first order condition is released. Our proposition is

Proposition 2.1 *If v is more risk averse than u in Arrow-Pratt sense,regardless of u's preference towards risk ($u^{''} = 0, > 0, < 0$),v will exert more self-insurance than u do.*

Proof For the sake of convenience, define $A = w_0 - c(x) - l(x)$ and $B = w_0 - c(x)$. $U(x), V(x)$ are both in the family $\{W(x)\}$.Because $V(x)$ can be gained by an increasing concave transformation $k(x)(k^{'}(x) > 0)$ of $U(x)(V(x) = k(U(x)))$. According to Theorem **??**, if $\{W(x)\}$ obeys single crossing property, then

$$argmax_{x \in R^+} V(x) \geq_{SSO} argmax_{x \in R^+} U(x)$$

holds.Based on Definition 2.1.3

$\{W(x)\}$ obeys single crossing property

$\Leftrightarrow$

$$U(x'') - U(x') \geq (>)0 \Rightarrow V(x'') - V(x') \geq (>)0. \tag{1}$$

A sufficient condition for (1) is

$$\textit{U(x) is increasing with } x \Rightarrow \textit{ V(x) is increasing with } x. \tag{2}$$

Note that

$$\begin{aligned} & \textit{U(x) is increasing with } x \\ \Leftrightarrow \quad & -pu^{'}(A)(c^{'}(x) + l^{'}(x)) - (1-p)u^{'}(B)c^{'}(x) \geq (>)0 \end{aligned} \tag{3}$$

and

$$\begin{aligned} & \textit{V(x) is increasing with } x \\ \Leftrightarrow \quad & -pv^{'}(A)(c^{'}(x) + l^{'}(x)) - (1-p)v^{'}(B)c^{'}(x) \geq (>)0 \end{aligned} \tag{4}$$

From $V(x) = k(U(x))$, we have

$$\begin{aligned} & -pv^{'}(A)(c^{'}(x) + l^{'}(x)) - (1-p)v^{'}(B)c^{'}(x) \geq (>)0 \\ \Leftrightarrow \quad & -pk^{'}(u(A))u^{'}(A)(c^{'}(x) + l^{'}(x)) - (1-p)k^{'}(u(B))u^{'}(B)c^{'}(x) \geq (>)0 \\ \Leftrightarrow \quad & k^{'}(u(A))[-pu^{'}(A)(c^{'}(x) + l^{'}(x)) - (1-p)\frac{k^{'}(u(B))}{k^{'}(u(A))}u^{'}(B)c^{'}(x)] \geq (>)0. \end{aligned} \tag{5}$$

From $k^{'} > 0, k^{''} < 0$ and $u(A) < u(B)$, we obtain $\frac{k^{'}(u(B))}{k^{'}(u(A))} < 1$, which implies

$$-pu^{'}(A)(c^{'}(x)+l^{'}(x))-(1-p)\frac{k^{'}(u(B))}{k^{'}(u(A))}u^{'}(B)c^{'}(x) \geq -pu^{'}(A)(c^{'}(x)+l^{'}(x))-(1-p)u^{'}(B)c^{'}(x)$$

Therefore (3) implies (4).

Q.E.D

An advantage of our result is: this result is the result is independent of the restrictions on second order derivatives of utility functions. Thus,this result extends the effectiveness of conclusions by Dionne and Eeckhoudt(1985)to the situations where the optimal solutions are not inertial solutions.

2.2 Self-Protection

Now we consider self-protection case.Self-protection decreases the possibility of loss. Assume the level of self-protection is x,and the probability of loss can be expressed as a function of self-protection level $p(x)$.$p^{'} < 0$ because the marginal effect of self-protection is decreasing.The monetary cost $c(x)$ and $c^{'} > 0$ for increasing marginal cost. Here, we assume self-protection can only influence the probability of risk occurrence,while it does not affect the severity of loss.The size of loss is independent of self-protection and represented by a constant $l > 0$.

The expected utility function for decision maker u is:

$$U(x) = p(x)u(w_0 - c(x) - l) + (1 - p(x))u(w_0 - c(x)). \tag{6}$$

The expected utility function for a more risk-averse decision maker v is:

$$V(x) = p(x)v(w_0 - c(x) - l) + (1 - p(x))v(w_0 - c(x)). \tag{7}$$

In Dionne and Eeckhoudt's paper,first order condition does not suggest a clear comparison for the optimal self-protection level.Jullien and et al.(1999) suggested a more risk-averse agent will take higher self-protection activities if and only if the probability of loss is lower than a threshold.

With the help of interval dominance order,we find a new condition,which requires one assumption concerning fear of ruin coefficient and marginal effect of self-protection on probability and marginal cost of self-protection.In fact, our result is consistent with previous Jullien and et al's result, that if the initial probability is lower than a certain value (in our study this threshold can be put as $1+\frac{P'}{c'}\frac{u(w_0-c(x))}{u'(w_0-c(x))}=p_0(x)$),the decision maker will take higher self-protection activities.

Our proposition for self-protection is

Proposition 2.2 *Assume v is more risk averse than u. Under the condition that hazard rate of the loss is higher than 'boldness' coefficient of u in no risk state ($\frac{p'(x)}{1-p(x)} \geq -c'(x)\frac{u'(w_0-c(x))}{u(w_0-c(x))}$), v will take higher self-protection efforts than u do.*

Proof Define $A = w_0 - c(x) - l$ and $B = w_0 - c(x)$.
Assume $U(x), V(x)$ are both in the family $\{R(x)\}$.There is an increasing concave transformation $k(x)(k^{'} > 0)$ such that $V(x) = kU(x)$.
From Theorem **??** and Theorem **??**, we note that if there is a nondecreasing positive function $\alpha(x)$ for all x in the interval $[0, w_0]$ such that:

$$V'(x) \geq \alpha(x)U'(x) \tag{8}$$

we can obtain the following results based on interval dominance order:

$$argmax_{x\in R^+}V(x) \geq argmax_{x\in R^+}U(x)$$

Define $g(x) = k(x) - k'(x)x$

$$g'(x) = k'(x) - k''(x)x - k'(x) = -k''(x)x \geq 0 \text{ for all } x \geq 0$$

Thus

$$g(u(A)) = k(u(A)) - k'(u(A))u(A) < g(u(B)) = k(u(A)) - k'(u(A))u(A)$$

First note that

$$U'(x) = p'(x)u(A) - p(x)c'(x)u'(A) - p'(x)u(B) - (1 - p(x))c'(x)u'(B). \tag{9}$$

Substitute $v(x) = k(u(x))$ in $V^{'}(x)$

$$\begin{aligned}
V'(x) &= p'(x)v(A) - p(x)c'(x)v'(A) - p'(x)v(B) - (1 - p(x))c'(x)v'(B) \\
&= p'(x)k(u(A)) - p(x)c'(x)k'(u(A))u'(A) - p'(x)k(u(B)) \\
&\quad -(1 - p(x))c'(x)k'(u(B))u'(B) \\
&= p'(x)\{[k(u(A)) - k'(u(A))u(A)] - [k(u(B)) - k'(u(B))u(B)]\} \\
&\quad +k'(u(A))[p'(x)u(A) - p(x)c'(x)u'(A)] \\
&\quad -k'(u(B))[p'(x)u(B) + (1 - p(x))c'(x)u'(B)] \\
&\geq k'(u(A))[p'(x)u(A) - p(x)c'(x)u'(A)] \\
&\quad -k'(u(B))[p'(x)u(B) + (1 - p(x))c'(x)u'(B)] \\
&\geq k'(u(A))[p'(x)u(A) - p(x)c'(x)u'(A)] \\
&\quad -k'(u(A))[p'(x)u(B) + (1 - p(x))c'(x)u'(B)] \\
&= k'(u(A))U'(x), \qquad (10)
\end{aligned}$$

where the first inequality holds because $p'(x) \leq 0$, $g'(x) \geq 0$ and $u(A) < u(B)$, and the second inequality holds because $k''(x) \leq 0$, $u(A) < u(B)$ and $\frac{p'(x)}{1-p(x)} \geq -c'(x)\frac{u'(B)}{u(B)}$. And, with $\frac{dk'(u(A))}{dx} = -k''(u(A))u'(A)c'(x) \geq 0$, (8) holds.

Q.E.D

Though this result requires a very technical assumption, it suggests more risk averse decision maker will resort to higher self-protection efforts regardless of risk preference of this decision maker. Moreover, this result is consistent with previous conclusion that more self-protection under increased risk aversion if loss probability is under a utility dependent threshold and we extend the effectiveness of this result to case when optimal self-protection level is not interior solution.

This condition can be put:

$$\frac{p'(x)}{1-p(x)} \geq -c'(x)\frac{u^{'}(w_0 - c(x))}{u(w_0 - c(x))} \tag{11}$$

$$HR(x) \geq boldness(x)$$

$$1 \geq HR(p(x))FR(u(x)) > 0$$

The term of left side of first inequality is *hazard rate*,measures change in probability for risk when no risk happens at the current activity level. The term of right side is the boldness rateand its inverse as *fear of ruin*(Aumann and Kurz,1977)[2],which indicates an agents willingness to risk all his fortune against a small potential gain. This condition can be put in another way that the less risk averse decision maker's marginal decrease in probability to survival probability is higher than the marginal cost of utility to total utility.

2.3 Self-insurance-cum-protection model

The Self-insurance-cum-protection model (SICP) is formally studied by Lee (1998). This model examines the case when one's effort simultaneously influences both probability of risk and the size of loss.In practice,SICP model is more consistent with some real world examples.For example, those who wear helmet while cycling are tend to be more cautious about their behavior than those do not wear any protection. Thus they are less likely to suffer from the accidents and exposure to lower probability of accidents.The helmet can reduce the seriousness of potential injures. Given this,these cyclists exposure to both lower probability of accidents and less potential injure.Another example can be found with high quality brakes on viechles reduce both the probability of an automobile accident(such as. ABS system can guarantee the viechle tractable in harsh situations) and the magnitude of a loss if an accident occurs.

[2]also see Foncel and Treich,2005

The model is as following.The expected utility for two agents are:

$$U(x) = p(x)u(w_0 - c(x) - l(x)) + (1 - p(x))u(w_0 - c(x))$$
$$V(x) = p(x)v(w_0 - c(x) - l(x)) + (1 - p(x))v(w_0 - c(x))$$

Assume agent v is more risk averse than u,and the utility function for v is a concave transformation of utility function for u,$V(x) = k(U(x))$.In our model,there is no restrictive assumptions on the second order derivative of u,which can be risk-averse,risk-neutral or even risk-loving.x is the level of self-insurance-cum-protection (SICP) activities by the decision maker.The probability and size of losses are influenced by SICP activities.Thus,$p(x) \in [0, 1], p^{'}(x) < 0$ and $l(x) > 0, l^{'}(x) < 0$. Monetary cost for SICP activites is represented by $c(x)$ and its marginal cost is increasing ($c^{'}(x) > 0$).

Lee's(1998) paper has studied the effect of an increase in risk aversion on SICP activities and shows that the effect depends in part on the shape of the loss function, relating the size of a potential loss to SICP expenditures.Particularly,if the marginal reduction for a loss is larger than the marginal increase in the cost of SICP expenditures, more risk-averse individuals invest more in SICP. In Lee's proposition,he considers when mainly two different situations,based on the sign of $c^{'}(x_u) + l^{'}(x_u)$. When $c^{'}(x_u) + l^{'}(x_u) \leq 0$, the results is much like that for self-insurance case, while $c^{'}(x_u) + l^{'}(x_u) > 0$, some additional assumptions are needed to guarantee relation between SICP and increased risk-aversion, since the increase in probability of loss p will decrease DM's benefit from SICP activities.

In our proposition,we suggest a new condition,which combine the two situations in Lee's paper.

Proposition 2.3 *Assume agent v is more risk averse than u.Under the condition that hazard rate of the loss is higher than 'boldness' coefficient of u in no risk state ($\frac{p'(x)}{1-p(x)} \geq -c'(x)\frac{u'(w_0-c(x))}{u(w_0-c(x))}$), v will take higher SICP efforts than u do.*

Proof The first order derivatives for two agents' utility function are

For convenience, let $A = w_0 - c(x) - l(x), B = w_0 - c(x)$.

$$\begin{aligned}
U^{'}(x) &= p'(x)u(A) - p(x)(c'(x) + l^{'}(x))u'(A) - p'(x)u(B) - (1-p(x))c'(x)u'(B) \\
V'(x) &= p'(x)v(A) - p(x)(c'(x) + l^{'}(x))v'(A) - p'(x)v(B) - (1-p(x))c'(x)v'(B) \\
&= p'(x)k(u(A)) - p(x)(c'(x) + l^{'}(x))k'(u(A))u'(A) \\
&\quad -p'(x)k(u(B)) - (1-p(x))c'(x)k'(u(B))u'(B) \\
&= p'(x)\{[k(u(A)) - k'(u(A))u(A)] - [k(u(B)) - k'(u(B))u(B)]\} \\
&\quad +k'(u(A))[p'(x)u(A) - p(x)(c'(x) + l^{'}(x))u'(A)] \\
&\quad -k'(u(B))[p'(x)u(B) + (1-p(x))c'(x)u'(B)] \\
&\geq k'(u(A))[p'(x)u(A) - p(x)(c'(x) + l^{'}(x))u'(A)] \\
&\quad -k'(u(B))[p'(x)u(B) + (1-p(x))c'(x)u'(B)] \\
&\geq k'(u(A))[p'(x)u(A) - p(x)(c'(x) + l^{'}(x))u'(A)] \\
&\quad -k'(u(A))[p'(x)u(B) + (1-p(x))c'(x)u'(B)] \\
&= k'(u(A))U'(x), \qquad (12)
\end{aligned}$$

The first inequality holds because $p^{'}(x) < 0$ and $[k(u(A)) - k'(u(A))u(A)] - [k(u(B)) - k'(u(B))u(B)] < 0$.The secondly inequality holds if $(k'(u(A)) - k'(u(B)))[p'(x)u(B) + (1-p(x))c'(x)u'(B)] > 0$.Since $k^{''} < 0$,$k'(u(A)) - k'(u(B)) > 0$.However,conditions do not guarantee $p'(x)u(B) + (1-p(x))c'(x)u'(B)$ is positive.So we assume $p'(x)u(B) + (1-p(x))c'(x)u'(B) > 0$.

Compare (10) and (12),the second inequality in SICP model holds under the same condition with the case in previous subsection (Proposition 2.2).

Given the above (12) holds,

$$V(x) \succeq_I U(x)$$

$$argmax_{x \in R^+} V(x) \geq argmax_{x \in R^+} U(x)$$

More risk-averse decision maker will take higher SICP effort.

Q.E.D

In this way, we conclusion extends to the situation, when probability and size of loss are both influenced by effort.The intuition for our result is: an increase in SICP expenditures makes the distribution of utility less risky or induces second-order stochastic dominance in the distribution of utility.

3 Conclusion

We re-examine the impact of risk aversion on self-insurance,self-protection activities and SICP activities.With single crossing condition and interval dominace order,we conclude a new condition for the positive relation between risk aversion and self-protection activities.Additionally, we extend our results to self-insurance-cum-protection (SICP) model.

This study still opens to further examination. For example,it is of great interest to investigate continuous states of nature and the general type of distribution function.Furthermore,bi-variate or even multivariate utility function is not included,which can more effectively suggest injurers and victims' behavior and care level.

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