**Evaluation of Portfolio-Level Liquidity Adjusted Value at Risk Model Formulated By Accounting for Non-Normality in Liquidity Risk**

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*In this paper, a portfolio-level Liquidity Adjusted Value at Risk model is formulated by using the adapted approach based on the Cornish-Fisher expansion technique to account for non-normality in liquidity risk. Prior studies have analyzed the importance of liquidity risk using a comprehensive liquidity measure in a Value at Risk framework. Nevertheless, most models ignore the fact that liquidity costs which measure market liquidity are non-normally distributed and this leads to a severe underestimation of the total risk. The empirical evidence obtained in this study shows that accounting for non-normality at portfolio level and using the modified approach produces much more accurate results than alternative risk estimation methodologies. The model is tested using emerging markets’ data as research on liquidity that primarily focuses on emerging markets yield particularly powerful tests and useful independent evidence since liquidity premium is an important feature of these data.*

**Keywords:** Value at Risk, Liquidity Costs, Non-normality, Emerging Markets

**JEL Codes:** G11, G32

# Introduction

The last decade has seen considerable amount of research work directed towards managing liquidity risk. The financial crisis and the subsequent global recession of 2008-2012 have demonstrated how “a complete evaporation of liquidity”[[3]](#footnote-3) can cause the collapse of many financial institutions. Large and random security price movements during financial crises cause liquidity gaps and most hedging strategies tend to fail when these crises occur. Various hedging strategies have been proposed to mitigate the risk of sudden jump in security prices (He et al. 2006, Kennedy et al. 2009, Carr et al. 1998, Carr and Wu 2013). However, the hedging errors from both the static and the dynamic strategies become larger when the maturity of the target call/security increases, indicating that the availability and liquidity of the relevant option contracts is an important risk factor affecting option prices. Existing literature shows that investors should worry about a security’s performance and tradability both in market downturns and when liquidity “dries up” (Amihud 2002, Chordia et al. 2001, Acharya and Pedersen 2005, Bekaert et al. 2007). There are many alternative measures of liquidity in the literature such as quoted bid-ask spreads, effective bid-ask spreads, turnover, the ratio of absolute returns-to-volume, and adverse-selection and market-making cost components of price impact (Korajczyk and Sadka 2008).

Prior studies have analyzed the importance of liquidity risk using a comprehensive liquidity measure in a Value‐at‐Risk (VaR) framework (Jarrow and Subramaniam 1997, Bangia et al. 2002, Angelidis and Benos 2006, Stange and Kaserer 2011). However, most LVaR models ignore the fact that liquidity costs, which measure market liquidity, are non-normally distributed displaying fat tails and skewness. Many studies show that the assumption of normally distributed returns is rejected for most financial time series, including those for individual stocks, stock indexes, exchange rates and precious metals. The argument of non-normality holds equally for liquidity costs. Stange and Kaserer(2008) analyze the distributional properties of liquidity costs and show that they are heavily skewed and fat-tailed. Ernst et al. (2012) suggest a parametric approach based on the Cornish–Fisher approximation to account for non-normality in liquidity risk.

The goal of this paper is to extend the concept of including liquidity measure in centralized risk in a Value‐at‐Risk framework in order to formulate a portfolio-level Liquidity Adjusted Value at Risk model. The Cornish-Fisher expansion technique, as proposed by prior studies is used for correcting the percentiles of a standard normal distribution for non-normality and is simple to implement in practice. Indian stocks belonging to diverse sectors are selected for the analysis based on data availability in the period from January 2010 to December 2014.

In recent years, many financial institutions have seen growth in their emerging markets trading activity due to higher margins. A risk-adjusted view of performance in those markets should account for liquidity risk as it is usually found to be higher in emerging markets due to lower volumes. Thus, research on liquidity that primarily focuses on emerging markets yield particularly powerful tests and useful independent evidence as the liquidity premium is an important feature of these data (Bekaert et al. 2007).

The paper is organized as follows; Section 2 provides a comprehensive literature review on various hedging methodologies and the relevance of including the liquidity risk component in pricing models Section 3 discusses the research methodology, Section 4 describes the data, Section 5 discusses the empirical performance of the modified LVaR model at the portfolio level, Section 6 presents robustness checks and Section 7 concludes.

# Literature Review

1.

## Efficient Hedging Methods

A number of hedging strategies such as delta or dynamic hedging, static hedging and semi-static hedging have been developed over the years though firms generally do not give enough details about the types of hedging methods used in their annual reports. Delta hedging or dynamic hedging is the process of keeping the delta[[4]](#footnote-4) of a portfolio as close to zero as possible. The existence of a delta neutral[[5]](#footnote-5) portfolio was first shown as a part of the original proof of the option pricing formula developed by Black-Scholes (1973) in which the option’s payoff is replicated by a continuously-adjusted hedge portfolio composed of the underlying asset and short-term bonds. The key distributional assumption of the formula is that the price of the asset on which the option is written follows a lognormal diffusion, the instantaneous variance of which depends at most upon the asset price and time. In an ideal setting or under the complete market scenario, the price of the underlying asset moves continuously (such as in a diffusion with known instantaneous variance) or with fixed and known size steps and the risks inherent in the derivatives position can be eliminated via frequent trading in small number of securities. In reality, however, markets are incomplete and large random price movements (jumps) happen much more often than typically assumed in the ideal settings and a dynamic hedging strategy based on small or fixed size movements often breaks down.

In response to these deficiencies of dynamic hedging, Breeden and Litzenberger (1978) pioneered an alternative approach called static hedging. This strategy (static hedging) shows that a path-independent payoff can be hedged using a portfolio of standard options maturing with the claim. It is effective even in the presence of jumps of random size and can be used to avoid the high costs of frequent trading. It is completely robust to model misspecification but cannot deal with path-dependent options. Carr et al. (1998) focus on path-dependent options that change characteristics at one or more critical price levels, for example, barrier and look-back options and their extensions. Put-call symmetry (PCS) which is both an extension and a restriction of the widely known put-call parity result is used to develop a method for valuation and static hedging of exotic options. Simple portfolios are engineered to mimic the values of standard options along barriers using PCS. Static positions in standard options are invariant to volatility, interest rates and dividends, bypassing the need to estimate them (unlike dynamic hedging). The only real drawback of this strategy is that the class of claims it can hedge is fairly narrow.

A dynamic hedging strategy that can be used under a jump diffusion model was explored in He et al. (2006). This method seeks to mitigate the jump risk by holding instruments in the hedge portfolio that protect against a sudden, extreme movement in the stock price. The strategy handles contracts with path dependent features better but in the presence of transaction costs, the cumulative expense of the necessary updates generally become large as the rebalancing frequency increases. Kennedy et al. (2009) devise a dynamic hedging strategy that protects against the diffusion and jump risk while not costing too much to maintain. The objective function in the dynamic strategy of He et al. (2006) is augmented to include a component that takes into account transaction costs and a multi-objective optimization problem is defined. The results of the hedging simulations of this procedure indicate that the dynamic hedging strategy provides sufficient protection against the diffusion and jump risk while not incurring large transaction costs. Carr and Wu (2013) propose a semi-static[[6]](#footnote-6) approach for hedging derivative securities; it lies between dynamic hedging and static hedging in terms of both range and robustness. It is found that under purely continuous price dynamics, discretized static hedges with as few as three to five options perform comparably to the dynamic delta hedge with the underlying futures and daily updating, but the static hedges strongly outperform the daily delta hedge when the underlying price process contains random jumps. A historical analysis using over 13 years of data on S&P 500 index options further validates the superior performance of the static hedging strategy in practical situations.

However, the hedging errors from both the static and the dynamic strategies become larger when the maturity of the target call increases, indicating the potential existence of additional risk factors affecting option prices. Availability and liquidity of the relevant option contracts is one such important risk factor affecting option prices. Bongaerts et al. (2011) generate interaction effects between hedging demand and liquidity premia. It is shown that, on one hand, increased hedging demand implies that some investors will short hedge assets for hedging purposes, but that on the other hand, this hedging pressure pushes up expected returns, which increases the speculative demand of investors.

## Liquidity Risk

*The risk that a given security or asset cannot be traded quickly enough in the market to prevent or minimize a loss is termed liquidity risk*. The last decade has seen considerable amount of research work directed towards managing liquidity risk while pricing an option. According to Acharya and Pedersen (2005), liquidity is risky and has commonalty: it varies over time both for individual stocks and for market as a whole. Their Liquidity –Adjusted Capital Asset Pricing Model provides a unified theoretical framework that explains the empirical findings that return sensitivity to market liquidity is priced (Pastor and Stambaugh, 2003), that average return is priced (Amihud and Mendelson, 1986) and that liquidity commoves with returns and predicts future returns (Amihud, 2002; Chordia et al., 2001; Bekaert et al., 2007). Said differently, the model implies that investors should worry about a security’s performance and tradability both in market downturns and when liquidity “dries up”. Brunnermeier and Pedersen (2009) provide a model that links an asset’s market liquidity and trader’s funding liquidity. The model explains empirically documented features that market liquidity (i) can suddenly dry up, (ii) has commonality across securities, (iii) is related to volatility, (iv) is subject to “flight to quality” and (v) co-moves with the market. Importantly, the model links a security’s market illiquidity and risk premium to its margin requirement (i.e. funding use) and the general shadow cost of funding.

There are many alternative measures of liquidity in the literature. Measures that have appeared in the literature include quoted bid-ask spreads, effective bid-ask spreads, turnover, the ratio of absolute returns-to-volume, and adverse-selection and market-making cost components of price impact. Korajczyk and Sadka (2008) estimate latent factor models different measures of liquidity and a measure of global, across-measure systematic liquidity by estimating a latent factor model pooled across all measures. The results show that there is commonality, across assets, for each individual measure of liquidity and that these common factors are correlated across measures of liquidity. Return shocks are contemporaneously correlated with liquidity shocks and lead changes in liquidity. Additionally, shocks to liquidity tend to die out slowly over time.

Liquidity risk is neglected by widely used risk management measures such as Value at Risk (VaR). Derivatives users generally calculate a VaR measure for their derivatives portfolio and by not taking into account the liquidity risk component they underestimate the portfolio risk exposures. *VaR is an estimate of the maximum potential loss that may be incurred on a position for a given time horizon and a specified level of confidence.* Since the publication of the market-risk-management system RiskMetrics[[7]](#footnote-7) of JP Morgan in 1994, VaR has gained increasing acceptance and is now considered as industry’s standard tool to measure market risk. In calculating VaR, it is assumed that the positions concerned can be liquidated or hedged within a fixed and fairly short timeframe (in general one day to ten days), that the liquidation of positions will have no impact on the market and that the bid-ask spread will remain stable irrespective of the size of the position, in essence a perfect market is assumed. The price referred to is often the mid-price or the last known market price. However, the quoted market price cannot be used as a basis for valuating a portfolio that is to be sold on a less than perfectly liquid market: in practice, account must be taken of its orderly liquidation value or even its distress liquidation value.

Jarrow and Subramaniam (1997) were among the first to estimate liquidity-adjusted VaR (LVaR), taking account of the expected execution lag in closing a position and the market impact of prices being adversely effected by a quantity discount that varies with the size of the trade. The model requires three quantities which increase the loss level – namely a liquidity discount, the volatility of the liquidity discount and the volatility of the time horizon to liquidation. The authors themselves acknowledge that traders or firms must collect time series data on the shares traded, prices received and time to execution in order to estimate these quantities. Whilst this model is robust and fairly easy to implement, estimating these quantities is by no means trivial. Indeed, some may only be determined empirically with the accompanying introduction of significant bias. Bangia et al. (2002) propose similar measures of LVaR, they classify the liquidity risk into two diﬀerent categories: (i) the exogenous illiquidity that depends on the general conditions of the market and (ii) the endogenous which relates the position of a trader with the bid-ask spread. By focusing on the exogenous risk, they construct an LVaR measure for both the underlying instrument and the bid-ask spread. Specifically, they adjust the VaR number for “fat” tails and for the variation of the bid-ask spread.

Hisata and Yamai (2000) propose a practical framework for the quantification of LVaR which incorporates the market liquidity of financial products. The framework incorporates the mechanism of the market impact caused by the investor’s own dealings through adjusting VaR according to the level of market liquidity and the scale of the investor’s position. In addition, they propose a closed-form solution for calculating LVaR as well as a method of estimating portfolio LVaR. Angelidis and Benos (2006) relax the traditional, yet unrealistic, assumption of a perfect, frictionless financial market where investors can either buy or sell any amount of stock without causing significant price changes. They extend the work of Madhavan et al. (1997) (who argue that traded volume can explain price movements) and develop a liquidity VaR measure based on spread components. Under this framework, the liquidity risk is decomposed into its endogenous and exogenous components, thereby permitting an assessment of the liquidation risk of a specific position.

Stange and Kaserer (2011) analyze the importance of liquidity risk using a comprehensive liquidity measure, weighted spread, in a Value‐at‐Risk (VaR) framework. The weighted spread measure extracts liquidity costs by order size from the limit order book. Using a unique, representative data set of 160 German stocks over 5.5 years, they show that liquidity risk is an important risk component. Liquidity risk is increases the total price risk by over 25%, even at 10‐day horizons and for liquid blue chip stocks and especially in larger, yet realistic order sizes beyond €1 million. When correcting for liquidity risk, it is commonly assumed that liquidity risk can be simply added to price risk. The empirical results show that this is not correct, as the correlation between liquidity and price is non‐perfect and total risk is thus overestimated. According to Ernst et al. (2012) liquidity costs, which measure market liquidity, are non-normally distributed, displaying fat tails and skewness. Most liquidity risk models either ignore this fact or use the historical distribution to empirically estimate worst losses. Therefore, they propose a parametric approach based on the Cornish–Fisher approximation to account for non-normality in liquidity risk. Using the bid-ask spread data of a large number of stocks, they demonstrate the superiority of the suggested liquidity risk estimation technique.

# Research Methodology

Value-at-Risk (VaR) is a number that represents the potential change in a portfolio’s/asset’s future value. This change is defined based on (1) the horizon over which the portfolio’s change in value is measured and (2) the “degree of confidence” chosen by the risk manager. Since the publication of the market-risk-management system RiskMetrics of JP Morgan in 1994[[8]](#footnote-8), VaR has gained increasing acceptance and is now considered as industry’s standard tool to measure market risk.

To compute the VaR of an asset over a 1-day horizon with α% chance (confidence interval) that the actual loss in the asset’s value does not exceed VaR estimate consists of the following steps:

Asset returns rt are computed as the log difference of mid-prices (the average of bid ask values of the asset at time t)

The α% worst case value assuming normal returns is

Where

VaR is a number that represents potential change in asset’s future value. Assuming the return on this asset is distributed conditionally normal, the relative VaR estimate is

……………… (Equation 1)

The above expressions for α% worst case value () and potential loss (relative VaR estimate) only consider the volatility of the mid-price, whereas on an average the bid-price is expected to be ½ times average spread below that. Moreover, in unusual tail-event circumstances due to overall market conditions liquidity risk is defined in terms of a confidence interval or a tail probability. Bangia et al. (2002) define the exogenous cost of liquidity (COL) based on average spread plus a multiple of the spread volatility to cover α% of the spread situations

The achievable transaction price accounting for liquidity cost is

Where

Applying the simplification that is almost equal to 1, the price is

The relative Liquidity-adjusted VaR measure (assuming a normal distribution with mean as zero) according to Bangia et al. (2002) is

 ...................................................... (Equation 2)

A normal distribution is fully described by its first two moments: mean and variance. Higher centralized moments like skewness and excess kurtosis are zero. However, if the distribution is non-Gaussian, higher moments will also determine loss probabilities. For this reason, it is not accurate to use standard percentiles of a normal distribution for the calculation of the LVaR of nonnormally distributed returns. Cornish and Fischer (1937) were the first to modify the standardized percentiles of a normal distribution in a manner that accounted for higher moments. They obtained explicit polynomial expansions for standardized percentiles of a general distribution in terms of its standardized moments and the corresponding percentiles of the standard normal distribution. Their procedure is commonly known as the Cornish-Fischer expansion. Using the first four moments (mean, variance, skewness and kurtosis), the Cornish-Fischer expansion approximating the α-percentile of a standardized random variable is calculated as:

....... (Equation 3)

Where is the α-percentile of an N (0,1) distribution, where denotes skewness and denotes the excess kurtosis of the random variable. The skewness of y is computed from historical data over n days as:

................................................................................................ (Equation 4)

With being the expected value and being the volatility of y. The excess kurtosis for y is:

........................................................................................ (Equation 5)

Ernst et al. (2012) propose an adapted model based on the Cornish Fisher expansion technique used to correct the percentiles of a standard normal distribution. They apply the Cornish-Fischer approximation to the basic spread model of Bangia et al. (2002) to obtain the following modified LVaR estimate:

............................................... (Equation 6)

where is the percentile of the return distribution accounting for its skewness and kurtosis, is the corresponding spread distribution percentile. The methodology described in Equation 6 is then used to compute LVaR estimates at instrument level and simply take the mean of the LVaR estimates for the analysis of more than one instrument. There is no explicit methodology suggested in their paper to compute a portfolio level LVaR model.

One approach for a full portfolio level treatment for liquidity risk is suggested in Bangia et al. (2002). They suggest computing the portfolio-level bid and ask series by taking the weighted sum of the bids and asks of the instruments. However, Bangia et al. (2002) assume that the returns are normally distributed while computing the portfolio LVaR estimates using this approach. Many studies (Stange and Kaserere 2011, Ernst et al. 2012) show that the assumption of normally distributed returns is rejected for most financial time series, including those for individual stocks, exchange rates, precious metals etc.

In this paper, the portfolio level bid and ask series is computed by taking the weighted sum of the bids and asks of the instruments (suggested by Bangia et al. 2002) and this bid-ask data is used for calculating the portfolio-level estimate LVaR (Modified) using Equation 6 (discussed by Ernst at al. 2012). Therefore, this paper discusses the approach for calculating a portfolio-level LVaR (Modified) measure by using the adapted model based on the Cornish-Fisher expansion technique used for correcting the percentiles of a standard normal distribution for non-normality.

# Data Description

The required price and bid-ask spread data of the stocks is obtained from the database Datastream for the time period from January 2010 to December 2014. Table 1 contains the exact description of the sample portfolios used for the analysis. Indian stocks belonging to diverse sectors are selected based on data availability during the analysis period. Descriptive statistics of relative bid-ask spreads for the stocks in the Nifty portfolio are presented in Table 2. The analysis for the remaining portfolios is included in the Appendix.

The relative bid-ask is found via formula,

................................................... (Equation 7)

Table 1 Compositions of equally-weighted portfolios for analysis

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Nifty** | **Infra** | **Service** | **Midcap** | **Smallcap** |
| Bajaj Auto | JSW Energy | Infosys | Apollo Hospitals | Bombay Dyeing |
| Cipla | Crompton Greaves | Adani Ports | DLF | Escorts |
| ITC | Tata Communications | Axis Bank | Jindal Steel | Chambal Fertilizers |
| Gail | IRB Infra. | Bharti Airtel | SUN TV | Gujarat Fluorochemicals |

Table 2 Descriptive Statistics of relative bid-ask spreads calculation using Equation 7 (in percent)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | **2010** | **2011** | **2012** | **2013** | **2014** |
|
| **BAJAJ AUTO**  |   |   |   |   |   |
| Mean | 0.113697 | 0.120219 | 0.099602 | 0.11798251 | 0.10985539 |
| Standard deviation | 0.093527 | 0.112087 | 0.084476 | 0.11388243 | 0.08982457 |
|   |   |   |   |   |   |
| **CIPLA**  |   |   |   |   |   |
| Mean | 0.098872 | 0.09883 | 0.08406 | 0.08636526 | 0.0887063 |
| Standard deviation | 0.088175 | 0.082422 | 0.067186 | 0.06988485 | 0.07172833 |
|   |   |   |   |   |   |
| **ITC**  |   |   |   |   |   |
| Mean | 0.078364 | 0.06339 | 0.066908 | 0.06300317 | 0.06114429 |
| Standard deviation | 0.059586 | 0.049315 | 0.052912 | 0.05235501 | 0.05165432 |
|   |   |   |   |   |   |
| **GAIL**  |   |   |   |   |   |
| Mean | 0.109129 | 0.133158 | 0.129201 | 0.14582128 | 0.13523665 |
| Standard deviation | 0.097361 | 0.114772 | 0.105513 | 0.13524092 | 0.11173876 |
|   |   |   |   |   |   |
| Observations per firm | 247 | 247 | 247 | 248 | 243 |

Table 2 shows that ITC is the most liquid stock with the smallest spread and GAIL is the least liquid stock with the largest spread for the time period from 2010 to 2014. The spread volatility values show that not only is the spread lowest for ITC but it also varied considerably less over time compared to the other stocks.

# Empirical Performance

In this section, the risk estimates for the individual stocks are computed first using measures suggested by existing research to check whether the results obtained using emerging markets’ data are consistent with the prior theory. Then the empirical estimates for the portfolio are computed using the modified LVaR model.

Conforming to the standard Basel framework, risk is estimated using a one-day horizon and a 99% confidence level. The values of relative spread means and return means required for the LVaR model (refer Equation 6) are estimated using a twenty day rolling procedure.

One day asset returns at time t are calculated as the log difference of mid-prices:

 ................................................. (Equation 8)

Volatilities of relative spread (Equation 7) and return (Equation 8) are also calculated rolling over twenty days. Volatility clustering is accounted for using a common exponential weighted moving average method with a weight δ of 0.94 as:

............................................................... (Equation 9)

Skewness (Equation 4) and excess kurtosis (Equation 5) are calculated as 500-day rolling estimates. The long estimation horizon is chosen as the estimates are heavily influenced by outliers. However, to keep the sample as large as possible and to include the first two years in the results period, shorter rolling windows in the increasing order of 20, 50, 100 & 250-day are included at the beginning of the sample. Skewness and excess kurtosis estimates for Spread and return are presented in Table 3.

Table 3 Relative Spread & Return moment estimates

|  |  |
| --- | --- |
|   | **(a) Spread moment estimates** |
|   | **BAJAJ AUTO**  | **CIPLA**  | **ITC** | **GAIL**  |
| **Skewness** |   |  |   |   |
| Mean | 1.693839209 | 1.822292261 | 2.013397495 | 1.772488258 |
| Median | 1.740354438 | 1.679014116 | 2.106195292 | 1.800388092 |
| Standard deviation | 0.278628996 | 0.384730719 | 0.384833247 | 0.555467708 |
|   |  |  |  |  |
| **Kurtosis** |  |  |  |  |
| Mean | 3.647316173 | 4.922664444 | 6.690311755 | 5.595536253 |
| Median | 3.890146269 | 3.857497105 | 6.814681255 | 4.302191846 |
| Standard deviation | 1.441803241 | 2.462479477 | 2.661017741 | 4.253710592 |
|   |  |  |  |   |
|   | **(b) Return moment estimates** |
|   | **BAJAJ AUTO**  | **CIPLA**  | **ITC** | **GAIL**  |
| **Skewness** |   |  |   |   |
| Mean | 0.126332404 | 0.070600171 | -0.05665188 | 0.045883306 |
| Median | 0.037823072 | 0.081129108 | -0.16960306 | -0.042300135 |
| Standard deviation | 0.25063248 | 0.281448566 | 0.423730486 | 0.262188642 |
|   |  |  |  |  |
| **Kurtosis** |  |  |  |  |
| Mean | 0.851454442 | 1.674145 | 1.882035737 | 0.485880215 |
| Median | 0.902888808 | 1.424672729 | 1.858183712 | 0.337319939 |
| Standard deviation | 0.3489489 | 0.915809932 | 0.977181887 | 0.549955371 |

Empirical 99% percentile estimate of (S) shown in Table 4 are calculated according to the Bangia et al. (2002) framework as:

............................................................................................. (Equation 10)

where is the percentile spread of the past twenty-day historical distribution and and are mean and volatility of the relative spread.

Table 4 Empirical percentile estimates for the Bangia model

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|   | **BAJAJ AUTO**  | **CIPLA**  | **ITC** | **GAIL**  |
| Mean | 1.646200565 | 1.562148885 | 1.555050208 | 1.621059388 |
| Median | 1.600283939 | 1.504682854 | 1.477278938 | 1.595656437 |
| Standard deviation | 0.460316135 | 0.474548719 | 0.510702042 | 0.486299454 |

Using the first four moments (mean, variance, skewness and kurtosis), the percentiles based on the Cornish-Fisher approximation are calculated for relative spreads and returns using Equation 3 (Table 5).

Table 5 Cornish-Fischer percentile estimates – Spread & Return

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|   | **BAJAJ AUTO**  | **CIPLA**  | **ITC** | **GAIL**  |
| **Spread** |  |  |  |  |
| Mean | 3.314796833 | 3.510807394 | 3.788462333 | 3.638404654 |
| Median | 3.379408608 | 3.399985274 | 3.801174484 | 3.437969448 |
| Standard deviation | 0.234776953 | 0.323706586 | 0.44515262 | 0.738801237 |
| **Return** |  |  |  |  |
| Mean | 2.588458345 | 2.737654317 | 2.655693383 | 2.446938434 |
| Median | 2.557345522 | 2.76865779 | 2.500223997 | 2.380306519 |
| Standard deviation | 0.153474096 | 0.192993233 | 0.37992527 | 0.202782626 |

Table 6 shows empirical risk estimates for VaR or Price risk (Equation 1), LVaR measure (Equation 2) according to Bangia et al. (2002) and the LVaR measure (Equation 6) suggested by Ernst et al. (2012). Since, ITC is the most liquid stock with the smallest spread and GAIL is the least liquid stock with the largest spread therefore as expected GAIL has the highest risk estimate and ITC has the lowest. Further the LVaR measure suggested by Ernst et al. (2012) provides highest risk estimates across all the securities.

Table 6 Risk estimates for Individual Stocks

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  % | **BAJAJ AUTO**  | **CIPLA**  | **ITC** | **GAIL**  |
| **Price risk** |   |   |  |   |
| Mean | 3.632656699 | 3.37084671 | 3.258682793 | 3.70610231 |
| Median | 3.467297221 | 3.186048101 | 3.106862453 | 3.549050878 |
| Standard dev. | 1.081814956 | 1.04598625 | 1.032842813 | 1.050924702 |
| **LVaR (Bangia et al.)** |   |   |   |   |
| Mean | 3.803240547 | 3.504543781 | 3.354800877 | 3.904938768 |
| Median | 3.633192289 | 3.327872271 | 3.192487517 | 3.756645786 |
| Standard dev. | 1.087671406 | 1.05800485 | 1.034768181 | 1.060040644 |
| **LVaR (Ernst et al.)** |   |   |  |   |
| Mean | 4.221186275 | 4.139432607 | 3.80161315 | 4.254589843 |
| Median | 4.03003585 | 3.943251974 | 3.612603736 | 4.107947502 |
| Standard dev. | 1.255921982 | 1.300857475 | 1.305408989 | 1.21632208 |

In order to compute the portfolio-level risk estimates, an equally-weighted portfolio is constructed using the stocks Bajaj Auto, Cipla, ITC and Gail. The portfolio level bid-ask series is computed by taking the equally weighted sum of the bids and asks of the instruments (refer section 3 Research Methodology). The bid-ask data thus obtained is used for calculating the portfolio risk estimates following the same approach described earlier. Table 7 shows portfolio risk estimates.

Table 7 Portfolio Risk Estimates (weights: Bajaj Auto = .25, Cipla = .25, ITC = .25 & GAIL = .25)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **Relative Spread** | **Return** | **Skewness(S)** | **Kurtosis(S)** | **Skewness (R)** | **Kurtosis (R)** |
| **Mean** | 0.107504734 | 0.000659609 | 1.467476417 | 2.71448549 | 0.054965396 | 0.569078752 |
| **Median** | 0.092079206 | 0.000591684 | 1.486144595 | 2.688464986 | 0.002645487 | 0.556628963 |
| **Std Dev.** | 0.066141839 | 0.012148188 | 0.291730956 | 1.082309236 | 0.192383672 | 0.244441789 |
|  |  |  |  |  |  |  |
|  | **z-alpha(Bangia)** | **z-cornish(S)** | **z-cornish (R)** | **Price Risk (%)** | **LVaR (Bangia) %** | **LVaR (Modified) %** |
| **Mean** | 1.222216147 | 3.196860544 | 2.48462352 | 2.681242886 | 2.809134596 | 3.026726895 |
| **Median** | 1.195095316 | 3.216251046 | 2.455297707 | 2.564681479 | 2.69314726 | 2.907314097 |
| **Std Dev.** | 0.351456225 | 0.201711476 | 0.16309667 | 0.765116261 | 0.772016994 | 0.860140693 |

The LVaR (Modified) measure provides the highest risk estimates, showing that neglecting liquidity risk leads to a severe underestimation of the total risk. The portfolio-level analysis is repeated using distinct portfolios described in Table 16 and the results are presented in Tables 10, 11, 12 & 13 (refer Appendix). The results remain the same and the LVaR (Modified) measure provides the highest risk estimates in all cases.

# Backtesting Results

Using the close price as the liquidation price of the stocks instead of the mid-value of the bid and ask prices, the return values are calculated as follows:

…………….………….................................................... (Equation 11)

The value of exceedance E is taken as one if the value of the realized loss (computed using Equation 11) is larger than the predicted loss.

……………….......................................................... (Equation 12)

Table 1 contains the exact composition of the equally-weighted portfolios from diverse segments for backtesting analysis. The values of Price Risk or VaR, LVaR (Bangia) and LVaR (Modified) for the portfolios are shown in Tables 7, 10, 11, 12 and 13. The required close price of the stocks is obtained from the database Datastream for the time period from January 2010 to December 2014.

Table 8 Kupiec’s ‘proportion of failures’ (PF) Coverage test

Non-rejection intervals [x1, x2] for various values of q and α+1. The VaR measure is rejected at the .05 significance level if the number of exceedances X is less than x1 or greater than x2[[9]](#footnote-9)

|  |  |
| --- | --- |
|   | **quantile of loss q** |
|   |  | **0.90** | **0.95** | **0.975** | **0.99** |
| **α +1** | **125** | [6, 20] | [2, 12] | [0, 8] | [0, 4] |
| **250** | [16, 35] | [6, 20] | [2, 12] | [0, 7] |
| **500** | [37, 64] | [16, 36] | [6, 20] | [1, 10] |
| **750** | [59, 92] | [26, 50] | [11, 28] | [2, 14] |
| **1000** | [81, 120] | [37, 65] | [15, 36] | [4, 17] |
| **1250** | [104, 147] | [47, 79] | [21, 43] | [6, 20] |

Table 9 shows the magnitude of exceedances E at portfolio level for VaR, LVaR (Bangia) and LVaR (Modified) in period from January 2010 to December 2014 (Number of days = 1212). According to Kupiec’s ‘proportion of failures’ (PF) coverage tests (Table 8), only LVaR (Modified) measure is not rejected at .01 significance level.

Table 9 Magnitude of Exceedances in period from January 2010 to December 2014 (1212 days)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Measure\Portfolio | **Nifty** | **Infra** | **Service** | **Midcap** | **Smallcap** |
| VaR | 18 | 24 | 23 | 26 | 27 |
| LVaR (Bangia) | 12 | 20 | 21 | 23 | 21 |
| LVaR (Modified) | 9 | 20 | 13 | 13 | 10 |

# Conclusion

Large and random security price movements during financial crises cause liquidity gaps and most hedging strategies tend to fail when these crises occur. The risk that a given security or asset cannot be traded quickly enough in the market to prevent or minimize a loss is termed liquidity risk and ignoring it has caused the collapse of many financial institutions. Previous studies have analyzed the importance of liquidity risk by including a liquidity measure in centralized risk management tools such as VAR (Value at Risk).

In this study, a Portfolio-Level Liquidity Adjusted Value at Risk model is developed using a parametric approach based on the Cornish–Fisher approximation to account for non-normality in liquidity risk. The model is tested using the data on Indian stocks as research on liquidity that primarily focuses on emerging markets yield powerful tests. Indian stocks from diverse sectors are selected for the analysis based on data availability over the time period from January 2010 to December 2014.

The empirical evidence shows that LVaR (Modified) measure provides the highest risk estimates at portfolio-level. The Backtesting results demonstrate the superiority of the LVaR (Modified) estimates when compared to alternative estimation techniques. Overall, the results prove that neglecting liquidity risk or assuming that the returns are normally distributed leads to a severe underestimation of the total risk. The Cornish-Fisher procedure used gains accuracy with the length of the estimation horizon hence future research could address this limitation.

# Appendix

Table 10 Portfolio Risk Estimates - Infra (weights: JSW Energy = .25, Crompton Greaves = .25, Tata Communications = .25 & IRB Infra. = .25)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **Relative Spread** | **Return** | **Skewness(S)** | **Kurtosis(S)** | **Skewness (R)** | **Kurtosis (R)** |
| **Mean** | 0.157935569 | 1.41699E-05 | 5.355810542 | 71.75344423 | -0.248326109 | 0.469428191 |
| **Median** | 0.142673258 | 0.000426268 | 7.519791353 | 106.3839229 | -0.288795359 | 0.351296393 |
| **Std Dev.** | 0.100376706 | 0.018994293 | 3.525948658 | 58.09914333 | 0.188124925 | 0.561464923 |
|  |  |  |  |  |  |  |
|  | **z-alpha(Bangia)** | **z-cornish(S)** | **z-cornish (R)** | **Price Risk (%)** | **LVaR (Bangia) %** | **LVaR (Modified) %** |
| **Mean** | 0.962803869 | 7.565285392 | 2.217071326 | 4.103538767 | 4.274740198 | 4.626610064 |
| **Median** | 0.826826105 | 7.052219911 | 2.147985599 | 3.896207647 | 4.03519867 | 4.446329983 |
| **Std Dev.** | 0.51004316 | 3.942126764 | 0.190830865 | 1.322265372 | 1.356173616 | 1.580766816 |

Table 11 Portfolio Risk Estimates – Service (weights: Infosys = .25, Adani Ports = .25, Axis Bank = .25 & Bharti Airtel = .25)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **Relative Spread** | **Return** | **Skewness(S)** | **Kurtosis(S)** | **Skewness (R)** | **Kurtosis (R)** |
| **Mean** | 0.0741 | 0.000435111 | 1.714800637 | 6.503649642 | -0.653438834 | 5.888855817 |
| **Median** | 0.06701 | 0.000322182 | 1.710293617 | 7.150128066 | -0.400893165 | 1.650917704 |
| **Std Dev.** | 0.03893 | 0.013166059 | 0.689939776 | 5.484808131 | 0.547232875 | 7.068327307 |
|  |   |  |   |  |   |   |
|  | **z-alpha(Bangia)** | **z-cornish(S)** | **z-cornish (R)** | **Price Risk (%)** |  **LVaR (Bangia) %** | **LVaR (Modified) %** |
| **Mean** | 0.99109 | 3.820876166 | 2.948796556 | 2.807252427 | 2.885473771 | 3.605684819 |
| **Median** | 0.91061 | 3.972776354 | 2.371200501 | 2.602336958 | 2.673507938 | 3.374473219 |
| **Std Dev.** | 0.36344 | 0.731215626 | 0.942148285 | 1.084436172 | 1.0905948 | 1.566669063 |

Table 12 Portfolio Risk Estimates – Midcap (weights: Apollo Hospitals = .25, DLF = .25, Jindal Steel = .25 & SUN TV = .25)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **Relative Spread** | **Return** | **Skewness(S)** | **Kurtosis(S)** | **Skewness (R)** | **Kurtosis (R)** |
| **Mean** | 0.146051035 | -7.761E-06 | 1.489623316 | 3.54335582 | -0.140138502 | 1.39654264 |
| **Median** | 0.129639101 | 0.00036725 | 1.410208357 | 3.253552569 | -0.134623322 | 1.52654544 |
| **Std Dev.** | 0.082192034 | 0.015546717 | 0.290370656 | 1.609695564 | 0.247151212 | 0.640799716 |
|  |   |  |   |  |   |   |
|  | **z-alpha(Bangia)** | **z-cornish(S)** | **z-cornish (R)** | **Price Risk (%)** |  **LVaR (Bangia) %** | **LVaR (Modified) %** |
| **Mean** | 0.997695483 | 3.382423285 | 2.519277212 | 3.350106665 | 3.504348381 | 3.958999614 |
| **Median** | 0.937836913 | 3.362695885 | 2.524618869 | 3.08827959 | 3.247171337 | 3.615072742 |
| **Std Dev.** | 0.357524515 | 0.251430845 | 0.207740312 | 1.137948206 | 1.149147036 | 1.400492434 |

Table 13 Portfolio Risk Estimates – Smallcap (weights: Bombay Dyeing = .25, Escorts = .25, Chambal Fertilizers = .25 & Gujarat Fluorochemicals = .25)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **Relative Spread** | **Return** | **Skewness(S)** | **Kurtosis(S)** | **Skewness (R)** | **Kurtosis (R)** |
| **Mean** | 0.268267967 | 0.000657802 | 1.48803039 | 3.781139361 | -0.185756732 | 1.713362573 |
| **Median** | 0.221000049 | 0.000629973 | 1.611484413 | 4.326471495 | -0.213558163 | 1.432035791 |
| **Std Dev.** | 0.183821156 | 0.020233165 | 0.421688997 | 2.172755092 | 0.264387721 | 1.006434075 |
|  |   |  |   |  |   |   |
|  | **z-alpha(Bangia)** | **z-cornish(S)** | **z-cornish (R)** | **Price Risk (%)** |  **LVaR (Bangia) %** | **LVaR (Modified) %** |
| **Mean** | 1.11368239 | 3.403449557 | 2.550865548 | 4.322830752 | 4.629891459 | 5.289657154 |
| **Median** | 1.030217681 | 3.511846396 | 2.582819486 | 4.049927902 | 4.370264702 | 4.99341017 |
| **Std Dev.** | 0.393071007 | 0.426229472 | 0.213752974 | 1.492421893 | 1.507556678 | 1.753073502 |

# References

Acharya, Viral and Lasse Pedersen, 2005, Asset pricing with liquidity risk, Journal of Financial Economics 77 (2), 375-410.

Amihud, Y. and H. Mendelson, 1986, Asset pricing and the bid-ask spread, Journal of Financial Economics 17 (2), 223–249.

Amihud, Y., 2002, Illiquidity and stock returns: cross-section and time-series effects, Journal of Financial Markets 5 (1), 31–56.

Angelidis, T. and A. Benos, 2006, Liquidity adjusted value-at-risk based on the components of the bid–ask spread, Applied Financial Economics 16 (11), 835–851.

Bangia, A., Francis X. Diebold, Til Schuermann and John D. Stroughair, 2002, Modeling Liquidity Risk with Implications for Traditional Market Risk Measurement and Management, The New York University Salomon Center Series on Financial Markets and Institutions 8, 3-13.

Bekaert, G., C.R Harvey and C. Lundblad, 2007, Liquidity and Expected Returns: Lessons from Emerging Markets, Review of Financial Studies 20 (6), 1783-1831.

Black, Fischer and Myron Scholes, 1973, The Pricing of Options and Corporate Liabilities, The Journal of Political Economy 81 (3), 637-654.

Bongaerts, Dion, Frank De Jong, and Joost Driessen, 2011, Derivative Pricing with Liquidity Risk: Theory and Evidence from the Credit Default Swap Market, Journal of Finance 66 (1), 1-340.

Breeden, Douglas T., and Robert H. Litzenberger, 1978, Prices of State-Contingent Claims Implicit in Option Prices, Journal of Business 51 (4), 621-651.

Brunnermeier, Markus K. and Lasse Heje Pedersen, 2009, Market Liquidity and Funding Liquidity, Review of Financial Studies, Vol. 22 (6), 2201-2238.

Carr, Peter, and Liuren Wu, 2013, Static hedging of standard options, Journal Of Financial Econometrics 12 (1), 3-46.

Carr, Peter, Katrina Ellis, and Vishal Gupta, 1998, Static hedging of exotic options, Journal of Finance 53 (3), 1165-1190.

Chordia, T., R. Roll and A. Subrahmanyam, 2001, Market liquidity and trading activity, Journal of Finance 56 (2), 501–530.

Ernst, C., Sebastian Stange and Christoph Kaserer, 2012, Accounting for nonnormality in liquidity risk, The Journal of Risk 14 (3), 3–21.

He, C., J. Kennedy, T. Coleman, P. Forsyth, Y. Li, and K. Vetzal, 2006, Calibration and Hedging Under Jump Diffusion, Review of Derivatives Research 9 (1), 1–35.

Jarrow, R. and A. Subramanian, A., 1997, Mopping up liquidity, Risk, 170–173.

Kennedy, J.S., P.A. Forsyth, and K.R. Vetzal, 2009, Dynamic Hedging under Jump Diffusion with Transaction Cost, Operations Research 57 (3), 541-559.

Korajczyk, Robert and Ronnie Sadka, 2008, Pricing the Commonality Across Alternative Measures of Liquidity, Journal of Financial Economics 87 (1), 45-72.

Madhavan, A., M. Richardson, and M. Roomans, 1997, Why Do Security Prices Change? A Transaction-Level Analysis of NYSE Stocks, The Review of Financial Studies 10 (4), 1035–1064.

Pastor, Lubos and Robert F. Stambaugh, 2003, Liquidity Risk and Expect Stock Returns, Journal of Political Economy 111, 642-685.

Stange, S. and C. Kaserer, 2008, The impact of order size on stock liquidity – A representative study, CEFS Working Paper No. 2008-9.

Stange, S. and C. Kaserer, 2011, The impact of liquidity risk: a fresh look, International Review of Finance 11 (3), 269–301.

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3. BNP Paribas terminated withdrawals from three hedge funds citing “a complete evaporation of liquidity” on August 9th, 2007. [↑](#footnote-ref-3)
4. Delta measures the sensitivity of the value of a portfolio to changes in the price of the underlying asset assuming all other variables remain unchanged. [↑](#footnote-ref-4)
5. A portfolio is delta neutral (or, instantaneously delta-hedged) if its instantaneous change in value, for an infinitesimal change in the value of the underlying security is zero. [↑](#footnote-ref-5)
6. A hedging strategy is semi-static if trades only need to occur at the discrete monitoring dates. [↑](#footnote-ref-6)
7. JP Morgan, 1996, RiskMetrics – Technical Document, Fourth Edition, New York. [↑](#footnote-ref-7)
8. JP Morgan, 1996, RiskMetrics – Technical Document, Fourth Edition, New York. [↑](#footnote-ref-8)
9. Source: <http://www.value-at-risk.net/backtesting-coverage-tests/> (last accessed 5th November 2015) [↑](#footnote-ref-9)