Insurance Demand under a Hybrid Model of Regret and Rejoicing[*]

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Abstract:
This study investigates optimal insurance coverage under regret theory. Especially, this study proposes the reference as a convex combination of the highest and lowest alternatives for introducing not highest or lowest but moderate wealth. Then, this study mainly examines how the mixture of regret and rejoicing feelings affect optimal insurance coverage.

From the analysis, we find that (1) the individuals who put more weight on regret (rejoicing) purchase partial (over) insurance, (2) the individuals who equally feel regret and rejoicing purchase over insurance when the accident probability is less than 1/2, (3) Optimal insurance coverage might not be a weakly decreasing function of weight on regret.

Keywords: Regret, Rejoicing, Insurance demand

JEL classification codes: D03, G22

[*] This research was supported by JSPS KAKENHI Grant Numbers JP15K03727 (Okura), JP26380411 (Osaki), JP26705004 (Osaki) and The Focused Researches of Osaka Sangyo University (Osaki).

1. Introduction

Individuals might feel regret or rejoicing through purchasing or not purchasing insurance after they find the final state (loss state or no-loss state). For simplicity, suppose the individuals who purchased full insurance. When the loss state occurs, the individuals feel rejoicing over purchasing full insurance. In contrast, when no-loss state occurs, the individuals feel regret over purchasing full insurance. This explanation indicates that foregone alternatives influence the individual's decision to purchase insurance.

Even though expected utility theory is a dominant tool for the model of insurance demand, it cannot be captured such feelings caused by foregone alternatives. Braun and Muermann (2004) is the first attempt to introduce regret from foregone alternatives for investigating the insurance demand. Their model succeeds to capture the aspect of regret from foregone alternatives in the theory of insurance demand. As a result, we believe Braun and Muermann (2004) is an influential study in the literature of insurance economics.

Regret theory is originally introduced by Bell (1982) and Loomes and Sugden (1982). Their original model considers a situation in which individuals face two alternatives, and they feel regret when the foregone alternative is better than the chosen, and rejoicing *vice versa*. When more than two alternatives are included, the original model causes intransitivity. Since there are more than two alternatives in most economic analyses including economic analysis in insurance demand, their model cannot be applied to those. Braun and Muermann (2004) solves this problem by setting the reference alternative which compares their chosen one as the best one. The utility form in Braun and Muermann (2004) is widely applied in the literature in insurance economics. Huang *et al*. (2015) and Fuji *et al*. (2016) are the recent studies through the utility form in Braun and Muermann (2004).

However, many models cannot capture the aspect of rejoicing. Although Fujii *et al*. (2016) is an exceptional model which contains rejoicing, either regret or rejoicing is contained in the model and then they do not simultaneously analyze in regret and rejoicing. In the models in previous studies, the reference to evaluate chosen alternatives is the highest or lowest wealth, but it seems to be extreme. Actually, individuals have the reference to evaluate chosen alternative is the "moderate" wealth. From that standpoint, this study introduces a hybrid model that individuals can feel both regret and rejoicing. Then, we examine the insurance demand under the model which includes the utility form by Braun and Muermann (2004) as an extreme case.

In order to reflect more actual case, we propose the reference as a convex combination of the highest and lowest alternatives. This reference is moderate compared with the highest and lowest alternatives. This utility form also avoids the violation of transitivity, and then we can analyze the insurance demand by using such moderate reference.

The organization of this article is as follows. In the next section, we build the model containing both regret and rejoicing. Section 3 derives optimal insurance coverage and investigates how mixture of regret and rejoicing affects optimal insurance coverage. Concluding remarks are in Section 4.

2. The model

Suppose an individual who feels both regret and rejoicing by comparing an actual and an ex-post outcome. The preference representation is given:

$$u(y) - g\left\{\left(\theta u(y^{\max}) + (1-\theta)u(y^{\min})\right) - u(y)\right\}, \qquad (1)$$

where y is the actual wealth, $y^{\max}$ and $y^{\min}$ are the highest and lowest wealth that the individual can achieve in a specific state, respectively. u is utility function with $u' > 0$ and $u'' < 0$. g is called regret-rejoicing function with $g' > 0$ and $g(0) = 0$. Let $UD = \left(\theta u(y^{\max}) + (1-\theta)u(y^{\min})\right) - u(y)$ be a utility difference where $\theta \in [0,1]$ represents weight on regret. When utility difference is positive, the individual feels regret and suffers disutility from the wrong decision. When it is negative, the individual feels rejoicing and gains utility from the right decision. We assume that $g''(UD) \geq 0$ for $UD > 0$ and $g''(UD) \leq 0$ for $UD < 0$. Bleichrodt *et al*. (2010) also provides an empirical support for $g''(UD) \leq 0$ for $UD < 0$. The regret-rejoicing function is also assumed to satisfy $g(|UD|) = g(-|UD|)$.

Its expected utility representation is written:

$$E\left[u(\tilde{y}) - g\left\{\left(\theta u(\tilde{y}^{\max}) + (1-\theta)u(\tilde{y}^{\min})\right) - u(\tilde{y})\right\}\right], \qquad (2)$$

The preference representation (2) can be viewed as a generalization of various existing representations:

- If g is a constant or a linear function, (2) is degenerated into the classical expected utility.
- If $\theta = 1$, (2) is degenerated into the regret theoretical expected utility by Braun and Muermann (2004), which is a modified version of the original model by Bell (1982) and Loomes and Sugden (1982).

Let us consider an individual who endows an initial wealth w and incurs a potential damage $D, w > D > 0$, with a probability $\pi \in (0,1)$. An individual determines the insurance coverage which is denoted by $\alpha \in [0,1]$. The individual can receive αD as the compensation when damage occurs. Insurance premium is assumed to be actuarially fair and then, $P = \pi D$ when an individual chooses full insurance. The individual's preference is represented by the form (1). Given the damage occurs, the highest and lowest wealth are achieved at $\alpha = 1$ and $\alpha = 0$, respectively. Given the damage does not occur, the highest and lowest wealth are achieved at $\alpha = 0$ and $\alpha = 1$, respectively. In this preparation, this individual determines the optimal insurance coverage to maximize the following objective function:

$$\max_{\alpha} V(\alpha) = \pi \left[u\big(W_L(\alpha)\big) - g\left(\Big(\theta u\big(W_L(1)\big) + (1-\theta) u\big(W_L(0)\big) \Big) - u\big(W_L(\alpha)\big) \right) \right] +$$
$$(1-\pi) \left[u\big(W_{NL}(\alpha)\big) - g\left(\Big(\theta u\big(W_{NL}(0)\big) + (1-\theta) u\big(W_{NL}(1)\big) \Big) - u\big(W_{NL}(\alpha)\big) \right) \right], (3)$$

where

$$W_L(\alpha) = w - D + \alpha(D - P),$$
$$W_{NL}(\alpha) = w - \alpha P.$$

The first-order condition for (3) can be derived as follows:

$$\mathrm{V}'(\alpha^*)$$
$$= \pi(1-\pi)D\left\{ u'\big(W_L(\alpha^*)\big)\big(1 + g'(UD_L)\big) - u'\big(W_{NL}(\alpha^*)\big)\big(1 + g'(UD_{NL})\big) \right\} = 0. (4)$$

The second-order condition is assumed to be satisfied through the analysis. Here,

$$UD_L = \Big(\theta u\big(W_L(1)\big) + (1-\theta) u\big(W_L(0)\big) \Big) - u\big(W_L(\alpha)\big),$$
$$UD_{NL} = \Big(\theta u\big(W_{NL}(0)\big) + (1-\theta) u\big(W_{NL}(1)\big) \Big) - u\big(W_{NL}(\alpha)\big).$$

From (4), the following relation is satisfied at the optimal insurance coverage, α^*:

$$V'(\alpha^*) = 0$$
$$\Leftrightarrow u'\big(W_L(\alpha^*)\big)\big(1 + g'(UD_L)\big) - u'\big(W_{NL}(\alpha^*)\big)\big(1 + g'(UD_{NL})\big) = 0$$
$$\Leftrightarrow \frac{u'\big(W_L(\alpha^*)\big)}{u'\big(W_{NL}(\alpha^*)\big)} = \frac{1 + g'(UD_{NL})}{1 + g'(UD_L)}. \qquad (5)$$

3. Optimal insurance coverage

It is known that the full insurance, $\alpha^* = 1$, is optimal under expected utility theory when insurance premium is actuarially fair that is assumed in our setting. We examine the effect of regret and rejoicing on insurance coverage by setting the full insurance a benchmark. It is noted that the full insurance is the interior solution under the expected utility theory. Since $\alpha^* > 1$ is prohibited in the law, the full insurance might be optimal as a corner solution, that is, $V'(\alpha = 1) > 0$. To distinguish between interior and corner solutions, we call the over-insurance in the case of corner solution. We find $u'\big(W_L(1)\big) = u'\big(W_{NL}(1)\big)$ since $W_L(1) = W_{NL}(1) = w - P$. From (5), the following is held:

$$\mathrm{sgn}\{V'(\alpha = 1)\} = \mathrm{sgn}\{g'(UD_L) - g'(UD_{NL})\}.$$

At $\alpha = 1$, we have

$$\begin{aligned} UD_L &= \left(\theta u\big(W_L(1)\big) + (1-\theta)u\big(W_L(0)\big)\right) - u\big(W_L(1)\big) \\ &= (1-\theta)\left(u\big(W_L(0)\big) - u\big(W_L(1)\big)\right) < 0, \\ UD_{NL} &= \left(\theta u\big(W_{NL}(0)\big) + (1-\theta)u\big(W_{NL}(1)\big)\right) - u\big(W_{NL}(1)\big) \\ &= \theta\left(u\big(W_{NL}(0)\big) - u\big(W_{NL}(1)\big)\right) > 0. \end{aligned}$$

Since g is an origin symmetric, the full insurance is optimal when $|UD_L| = UD_{NL}$. We denote $\bar{\theta}$ such that $|UD_L| = UD_{NL}$. $\bar{\theta}$ can be computed by

$$\begin{aligned} &-UD_L = UD_{NL} \\ &\Leftrightarrow (1-\bar{\theta})\left(u\big(W_L(1)\big) - u\big(W_L(0)\big)\right) = \bar{\theta}\left(u\big(W_{NL}(0)\big) - u\big(W_{NL}(1)\big)\right) \\ &\Leftrightarrow \frac{1-\bar{\theta}}{\bar{\theta}} = \frac{u\big(W_{NL}(0)\big) - u\big(W_{NL}(1)\big)}{u\big(W_L(1)\big) - u\big(W_L(0)\big)}. \end{aligned}$$

Then, if $\theta > (<)\bar{\theta}$,

$$\begin{aligned} &\frac{1-\theta}{\theta} < (>) \frac{u\big(W_{NL}(0)\big) - u\big(W_{NL}(1)\big)}{u\big(W_L(1)\big) - u\big(W_L(0)\big)} \\ &\Leftrightarrow (1-\theta)\left(u\big(W_L(1)\big) - u\big(W_L(0)\big)\right) < (>)\theta\left(u\big(W_{NL}(0)\big) - u\big(W_{NL}(1)\big)\right) \\ &\Leftrightarrow -UD_L < (>)UD_{NL}. \end{aligned}$$

When θ is large (small), $g'(UD_L) - g'(UD_{NL})$ is negative (positive) and $V'(\alpha = 1) < (>)0$ is found. Then, partial (over) insurance is optimal. We summarize the above argument into the following proposition:

Proposition 1.

Suppose that an individual follows the hybrid model of regret and rejoicing. The following is equivalent:

- The full (partial, over) insurance is optimal.
- θ is equal to (more than, less than) $\bar{\theta}$, where $\bar{\theta}$ is satisfied

$$\frac{1-\bar{\theta}}{\bar{\theta}} = \frac{u\big(W_{NL}(0)\big) - u\big(W_{NL}(1)\big)}{u\big(W_L(1)\big) - u\big(W_L(0)\big)}.$$

Now, we consider two extreme cases, that is, $\theta = 1$ and $\theta = 0$. In the case of $\theta = 1$, the individual always suffers disutility from regret by comparting the highest wealth with actual wealth. This case corresponds to the regret theoretical formation by Braun and Muerrmann (2004). Since $UD_L = 0$ and $UD_{NL} > 0$, an optimal insurance coverage is always partial insurance. In the case of $\theta = 0$, the individual always gains utility from rejoicing by comparting the lowest wealth with actual wealth. This case is considered in Fujii *et al.* (2016). Since $|UD_L| > 0$ and $D_{NL} = 0$, an optimal insurance coverage is always over insurance. We summarize this argument into the following corollary:

Corollary 1.
Suppose that an individual is regret (rejoicing) theoretical in the sense of $\theta = 1 (\theta = 0)$. An optimal insurance coverage is always partial (over) insurance.

Next, we consider the situation in which the threshold of $\bar{\theta}$ lies in the interval $[0,1]$. Suppose that $\pi \leq 1/2$. Then, we have

$$\frac{1}{2} \geq \pi \Leftrightarrow D \geq 2\pi D = 2P$$
$$\Leftrightarrow D - P \geq P.$$

Here applying $P = \pi D$ and $u'' < 0$, we have

$$W_L(1) - W_L(0) = D - P \text{ and } W_{NL}(0) - W_{NL}(1) = P,$$
$$u\big(W_L(1)\big) - u\big(W_L(0)\big) > u\big(W_{NL}(0)\big) - u\big(W_{NL}(1)\big).$$

Then, we obtain

$$1 > \frac{u\big(W_{NL}(0)\big) - u\big(W_{NL}(1)\big)}{u\big(W_L(1)\big) - u\big(W_L(0)\big)} = \frac{1-\bar{\theta}}{\bar{\theta}}$$
$$\Leftrightarrow \overline{\theta} > \frac{1}{2}.$$

We summarize this argument into the following proposition:

Proposition 2.
Suppose that the loss probability is less than $1/2$. The threshold, $\bar{\theta}$, is more than $1/2$.

A last question is whether the optimal insurance coverage is decreasing in θ. Since over insurance is optimal for $\theta \in [0, \bar{\theta}]$, the question is restricted in the range of $[\bar{\theta}, 1]$.
At $\bar{\theta}$, the full insurance is optimal, so $UD_L < 0$ and $UD_{NL} > 0$ hold since $W_L(1)$ is the highest and $W_{NL}(1)$ is the lowest. At the optimal insurance coverage,

$$u'\big(W_L(\alpha^*)\big)\Big(\big(1 + g'(UD_L)\big)\Big) - u'\big(W_{NL}(\alpha^*)\big)\Big(\big(1 + g'(UD_{NL})\big)\Big) = 0.$$

Since $W_L(\alpha) \leq W_{NL}(\alpha)$ for $\alpha \in [0,1]$, where the equality holds at $\alpha = 1$, so $u'\big(W_L(\alpha^*)\big) \geq u'\big(W_{NL}(\alpha^*)\big)$, we have

$$g'(UD_L) \leq g'(UD_{NL}) \Leftrightarrow |UD_L| \leq |UD_{NL}|.$$

Because $UD_L < 0$ and $UD_{NL} > 0$ at $\bar{\theta}$ and $|UD_L| \leq |UD_{NL}|$, UD_{NL} is always positive for $[\bar{\theta}, 1]$. On the other hand, UD_L is negative at $\theta = \bar{\theta}$, but it is positive at $\theta = 1$. Thus, there exists θ such that $UD_L = 0$. $\hat{\theta}$ denotes θ that satisfies $UD_L = 0$. From the implicit function theorem, we have

$$\frac{\partial \alpha^*}{\partial \theta} = u'(W_L)\Big(u\big(W_L(1)\big) - u\big(W_L(0)\big)\Big)g''(UD_L)$$
$$-u'(W_{NL})\Big(u\big(W_{NL}(0)\big) - u\big(W_{NL}(1)\big)\Big)g''(UD_{NL}).$$

For $\theta \in [\bar{\theta}, \hat{\theta}]$, $g''(UD_L) \leq 0$ for $UD_L \leq 0$ and $g''(UD_{NL}) > 0$ for $UD_L > 0$, we find

$$\frac{\partial \alpha^*}{\partial \theta} < 0,$$

that is, α^* is decreasing in θ for $\theta \in [\bar{\theta}, \hat{\theta}]$. In contrast, we cannot know whether the optimal insurance coverage is decreasing in θ for $\theta \in [\hat{\theta}, 1]$. In other words, α^* might not be a weakly decreasing function of θ. We summarize this argument into the following proposition:

Proposition 3.
The optimal insurance coverage has a following characteristic.

When $\theta \in [0, \bar{\theta}]$, optimal insurance coverage is over insurance and $\alpha^* = 1$ maintains.

When $\theta \in [\bar{\theta}, \hat{\theta}]$, optimal insurance coverage is decreasing in θ, where $\hat{\theta}$ denotes θ that satisfies $UD_L = 0$.

When $\theta \in [\hat{\theta}, 1]$, whether optimal insurance coverage is decreasing in θ is ambiguous.

4. Concluding remarks

This study investigated optimal insurance coverage under regret theory. Especially, this study proposed the reference as a convex combination of the highest and lowest alternatives for introducing not highest or lowest but moderate wealth. Then, this study mainly examined how the mixture of regret and rejoicing feelings affect optimal insurance coverage.

The main results of this study are as follows. First, the individuals who put more weight on regret (rejoicing) purchase partial (over) insurance. Second, the individuals who equally feel regret and rejoicing purchase over insurance when the accident probability is less than 1/2. Third, the optimal insurance coverage is over (full) insurance when weight on regret is small. Then, it is decreasing in weight on regret when weight on regret is moderate and partial insurance becomes optimal. Furthermore, whether the optimal insurance coverage is decreasing in weight on regret is ambiguous when weight on regret is large.

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