**An Investigation about the Effects of Different Scenarios Involving interest rate and Valuation Methods on Pension Fund Liabilities**

**Abstract**

The principles of present value and the determination of the correct discount rate as applied to pension funds, is the most important issue in Defined Benefit pension plan. As an important point, the choice of the discount rate should be in a way that precludes understatement of future pension obligations and pension funding gap.

We have considered three different discount rates in order to calculate future pension obligations which consist of: 1)5% more than the highest banking system’s deposit rate in Iran, 2) Technical interest rate of the Iranian’s Central Insurance and 3) Hypothetical discount rate.

We have also considered the application of different actuarial cost methods to show the differences between the two methods and the possibility to use alternative methods. In addition to Project Unit Cost method which is currently being used in Iranian’s pension funds, we have used Entry Age Normal method as an alternative method.

Our results illustrate that a one percent decrease in the discount rate will lead to 19% increase in Total Actuarial Liability under Project Unit Credit method. In the case of the Entry Age Normal, a one percent decrease in the discount rate will cause Total Actuarial Liability to increase by 17%.

Our findings indicate that a modification in valuation method or even a small change in discount rate can result a considerable change in actuarial liabilities. Consequently, we suggest a regulatory section for Iranian pension funds. By controlling important and vital parameters such as discount rate, huge deviations in estimating actuarial liability can be prevented.

**Key words:** discount rate, actuarial liability, pension funding gap, actuarial cost methods, Project Unit Cost method and Entry Age Normal method.

# 1. Introduction

Countries around the word are fast waking up to the fact that they have a major challenge on their hands with their pension schemes due to the combination of a rapidly changing population and fertility rates well below replacement rates, which has led to an increase in the dependency ratios in many countries. Moreover, the defined benefit (DB) pension plan is facing enormous challenges that has forced many large firms to begin to substitute defined contribution plan alternatives.

Pension obligations are among the largest obligations that organizations face, and taking accounting actions to improve the funded status of these obligations can reduce organizations expenditures. The discount rate is one of the most important parameters in the valuation of pension obligations although other factors, such as future coverage, salary growth, termination rates, and mortality rates also have a large impact. Choosing a discount rate should be in a way that avoids under or overstatement of pension obligations. In general, the higher the discount rate that is used, the lower the reported present value of the liabilities and the stronger the pension plan's funding position as reported in the accounting statements.

On an accrual basis, in Iran, both the Social Security Organization (SSO) and The Civil Servants Retirement Organization (CSRO) are insolvent*.* Financial projections under various economic scenarios show that the operational balances of the CSRO and the SSO will continue to deteriorate. This is not only the result of the gradual aging of the population but probably, more importantly, a reflection of the generosity of the system (World Bank). Under the rules of the audit organization of Iran, pension funds should discount their pension liabilities based on the expected rate of return. The use of expected rate of return as discount rate will allow states to understate their pension funding gaps, and thus lower their annual pension related expenditures (Naughton et al, 2015).

Wilcox (2006) notes that the link between discount rates and investment returns is "remarkable because it suggests that plan sponsors can reduce their funding obligations by investing in riskier securities, whereas conventional finance theory would suggest that a given level of benefit security can be maintained despite a shift to a riskier investment portfolio only by increasing, rather than reducing, contributions into the plan."

Plan sponsors and managers have incentives to invest more in risky assets in order to report a better funding status. The large, unfunded pension liabilities threaten the credibility of fiscal policy and the welfare of future generations. Today’s portfolios are illiquid and risky. Nonetheless, even if higher rates of return could be achieved, current reserves would not be sufficient to cover pension liabilities over the next decade. Thus, maintaining the current level of benefits implies accumulating a debt that future generations will have to finance, either by diverting resources from other sectors (e.g., education and health) or by increasing the tax burden (World Bank).

Understatement of pension funding gap or unfunded actuarial liability will causes severe short and long term problems. As a very effective factor, slight changes in discount rate will change the amount of pension liabilities by huge amount. Brown and Wilcox (2009) and Novy-Marx and Rauh (2011) suggest that promised pensions should be discounted at a rate that reflects the time value of money and the uncertainty of these liabilities. Brown and Pennacchi (2015) further argue that for funding purposes, pension funds should always use default-free discount rates.

In this study we have compared current discount rate, which is provided by the standard 27 rules of Iranian audit organization with some alternative choices. We have calculated the amount of Total Actuarial Liability under the application of two different actuarial cost method including: the Project Unit Cost method (PUC), which is currently used by pension funds in Iran and the Entry Age Normal (EAN( cost method as an alternative method in order to analyze the differences between their results.

The paper proceeds as follows. Section 2 reviews the literature. Section 3 describes theoretical framework. Section 4 describes interest rate and demographic analyses. Section 5 analyses the results. Section 6 concludes.

# 2. Literature Review

Throughout last decades numerous researchers have studied the important role of discount rate for pension valuations. At 1995 Gopalakrishnan et al argued that the status of the pension plan funding could influence the choice of the actuarial assumptions, particularly the discount rate. They concluded that firms with large unfunded pension liabilities and leverage are likely to choose higher discount rates to understate the magnitude of the reported liability. Peterson at this year stated that the correct discount rate should depend upon the type of risk inherent in the pension promise. In the United States, evidence from the 1980s showed that more profitable firms used lower rates of discount in assessing their future pension liabilities than less profitable ones (Bodie et al 1987). By inflating their future pension liabilities, firms were able to make larger deductible contributions, and thus lower their taxes, without reaching prescribed limits on overfunding. On the other hand, firms facing financial difficulties used high discount rates in order to understate their pension liabilities and lower their required contributions. A similar pattern was also reported for the 1990s (Asthana 1999). Godwin (1999) showed that firms with underfunded plans choose higher pension discount rates, although the difference declined over time. At 2000 Obinata investigated what factors affect the choice of pension discount rates. He showed that, the significant factor affecting the firms’ choice is not leverage, but profitability (return on equity). His results indicated that the firms, which chose lower pension discount rates, are valued higher.

At 2009 Brown et al stated that using the expected return on the assets held in the pension trust as discount rate would contrast sharply with finance theory. Their paper noted that the strong constitutional and other legal benefit protections make many defined benefit pension obligations virtually risk free. If governments discount liabilities in this way, it would reveal that state and local pensions are more underfunded than is generally reported. On the other hand, Stadler (2010) analyzed pension accounting choice in Germany. He showed that firms choose the pension discount rate and the treatment of actuarial gains and losses in order to smooth the effect of inherently volatile pension numbers. Vittas (2010) recommended the use of market valuations for assets and liabilities and the creation of dynamic risk buffers to shield pension plans from the large fluctuations in market values. At 2011, Novy et al measured state pension liabilities under a variety of different accrual methods and discount rates. Their main estimates focused on two primary measures of already-promised pension benefits. The first measure used a discount rate based on municipal bond yields.The second measure used a Treasury yield curve. They argued that investing public assets in risky securities may allow future generations to benefit from improved intergenerational risk sharing. But if future taxation to meet pension promises has nonlinear distortionary costs, then investing public pensions in risky assets with high expected returns and high volatility may impose a large expected cost of distortionary taxation on future generations.

In his paper, Babbel (2013) suggested to discount the liability stream by riskless interest rates instead of using corporate bond rates. Also Brown et al (2015) argued that if the objective is to measure pension under or overfunding, a default-free discount rate should always be used, even if the liabilities are themselves not default-free. If, instead, the objective is to determine the market value of pension benefits, then it is appropriate that discount rates incorporate default risk. At the same time Turner et al suggested to select a discount rate that is less than the expected rate of return on assets but greater than the risk free rate. Also Naughton et al (2015) found that the funding gap understatement is positively associated with higher future labor costs. Importantly, this association was primarily due to the Governmental Accounting Standards Board (GASB) methodology, which systematically understates the funding gap. This suggested that the GASB approach was associated with policy choices that had the potential to exacerbate fiscal stress. The GASB approach links pension liability discount rate to the expected return on assets, which gives pension funds incentives to invest more in risky assets in order to report a better funding status. Comparing public and private pension funds in the U.S., Canada, and Europe, Andonov et al (2016) found that U.S. public pension funds with a higher level of underfunding per participant, take more risk and use higher discount rates. The increased risk-taking by U.S. public funds is negatively related to their performance.

# 3. Theoretical Framework

Current practice for measuring the pension liabilities of pension plans provides information to plan sponsors and decision makers about how much it will cost over time to cover the financial obligations of participants. This is accomplished by calculating what is called an Actuarial Liability (AL), which is based on both current information and reasonable expectations of future events. AL is the portion of the present value of future benefits (PVFB) allocated to service accrued as of the valuation date. The AL measure not only takes into account the service and pay earned by employees, but also anticipates future service and pay raises, which will increase the plan’s obligations (Angelo, 2016). In addition to AL, the amount of Normal Cost (NC) is also needed to be calculated for the purposes of an actuarial valuation. NC is the portion of the PVFB that is attributed to the current year of service. Therefore, the PVFB, which is the basis for determining plan costs and liabilities, is equal to the Present Value of Future NC plus AL.

In this study we have calculate the amount of Total Actuarial Liability (TAL) under the application of most common actuarial cost methods including: ***projected unit credit* *method*** (PUC) and ***entry age normal cost method*** (EAN)***.***

## Project Unit Cost Method

Under the PUC method, the AL is the present value, at the valuation date, of the pension benefit accrued from the date of entry into the plan to the date of valuation. Through out this method annual pension expense calculations are based on future pension benefits that take projected salary levels into consideration. Assuming that each employee is entitled to retire at age *r* with an annual pension equal to *Br* , at age *x*, earlier than *r*, it has some intermediate value *Bx*which is called his accrued benefit. Based on the current rules in Iran, the amount of pension benefit accrued to age *x,* is equal to the average of the last two years’ salary multiple by total years of service.At age x, the present value of employee j’s accrued benefit is equal to , where represents probabilities of termination of employment before age r from all causes like mortality, resignation, discharge, disability, etc. However, in this study, we are going to just focus on the termination from mortality due to the lack of the data available for the calculation of the other probabilities. The amount of is calculated using the following formula:

The term is called a discount factor and calculates the probability that a person currently age x survive to the age of retirement, which is provided in a mortality table TD 88-90. The amount of the TAL is equal to the sumation of the Actuarial Liability for each participant. In this method the costs are low at the low ages, due to discounting from retirement back to these ages, but the costs usually increase with age (Aitken, 1996).

Under the rules of Iranian audit organization, pension funds must follow PUC method for their calculation. As an alternative approach we are going to use EAN method too. This method is most commonly used in the United States for both accounting and funding purposes (Brownlee, 1985).

## Entry Age Normal Cost Method

PUC method was built upon the premise that the accrued liability should equal the present value of accrued benefits at all times during an employee’s career, right up until retirement. The fact that the NC had this generally undesirable characteristic (that it tended to increase more rapidly than pay) was therefore a result of the way the method was constructed. The bad feature can be eliminated by defining the normal cost directly and that is the approach called the Entry Age Normal method (Anderson, 1985).

The EAN method (level dollar) is a cost method which start with the total value of each participant’s pension benefits (accruing from entry to retirement). The EAN (level dollar) method is such that, at age e, the present value of all future NC equals the present value of all future benefit. Symbolically we have:

|  |  |
| --- | --- |
| NCe . = Br . vr-e . r-epe .  |  |

*r-epe*states for the probability that a person aged e stays alive for r-e years.

For pension benefit based on salary, the actuarial cost method will be expressed as a level percentage of salary. A participant’s NC for a certain year is then a fixed percentage of his salary in that year, by using *s* as a salary scale:

AL at age x can be calculated in the usual way, which is equal to the PVFB minus the present value of future NC (Aitken, 1996).

# 4. Interest rate and demographic analysis

As mentioned above, a controversial issue is how pension liabilities should be valued. The pension liabilities are calculated as the present value of guaranteed future pension payments. To do these calculations, the choice of an appropriate discount rate plays a significant role.

In this study, we are going to calculate the Total Actuarial Liability under the application of 3 different discount rates. We are going to find the understatement of pension liabilities resulting from the difference between what Iranian pension funds used for their discount rate and the rate that if the state followed as an unbiased application of alternative discount rates. The three different discount rates that are being used in this study are:

1. 5% more than the highest banking system’s deposit rate in Iran
2. Technical interest rate of the Iranian’s Central Insurance[[1]](#footnote-2)
3. Hypothetical discount rate.

The first suggested discount rate is considered to be 20% because the highest banking system’s deposit rate in Iran is about 15% and Banks Employee’s Pension Fund uses 5 to 4 percent more than this rate for its calculations. This is the rate that is currently being used for discounting pension liabilities. The alternative discount rate is the rate based on the Technical interest rate of the Iranian’s Central Insurance. Based on this interest rate, the discount rate is equal to 18% for the first 5 years, and is equal to 15% for the second 5 years and for the remainder years this rate is equal to 10%. This rate is legislated by Life insurance and pension law of Iranian’s Central Insurance. The third option which is the hypothetical discount rate, is equal to 22%. We are going to find out that how much change will be caused in actuarial liabilities as a consequence of a 1% change in the discount rate.

For practical analysis we have used data available from one of the members of the Banks Employees’ Pension Fund for the year 1390. Table below shows a summary of the provided information:

 ***Table 4.1.***Demographic Information

|  |  |  |
| --- | --- | --- |
|  | **year** | **rial** |
|  | **Number of the employees** | **Number of the married employees** | **Average of the age of the employees** | **Average of the years of service** | **Average of the salary** |
| **Employees** | **Male** | **901** | **691** | **37** | **10** | **6832958.52** |
| **female** | **269** | **149** | **34** | **9** | **6291130.07** |

By using MATLAB software, we have calculated Total Actuarial Liability under two different cost methods and for three different discount rates. Based on the current rules, **Salary scale for retirees** is considered to be14.4 percent and **Salary scale for employees** is also considered to be 16.6 percent. And also the retirement age is equal to 60 years old.

# 5. Results

As we can see in table 4.2 and 4.3, a rate based on expected rate of return assumptions understates pension liabilities by huge amount. By using lower discount rate, the amount of the TAL increases. The TAL under the current discount rate which is being used by Iranian pension funds, has lower amount. Therefore, raising the discount rate lowers the present value of benefits, thereby reducing the required normal cost payment. The conventional wisdom is that a one percentage point increase in the discount rate reduces the present value at least by 20%. As we can see, our results admit that a one percent decrease in the discount rate will cause the TAL to increase by 19% in the PUC method and to increase by 17% in the EAN method. The third approach which has the lowest discount rates, has the highest amount of TAL. This will lead to the reduction in understating the TAL, which is provided by the current discount rate being used in pension funds.

***Table 4.2.*** Total Actuarial Liability under Project Unit Cost method

|  |  |  |  |
| --- | --- | --- | --- |
| **Discount rate** | **Total Actuarial Liability****(1 billion rials)** | **Differences between the TAL based on the current discount rate and based on the alternative discount rates****(1 billion rials)** | **Percentage difference** |
| **22%** | **1,889** | **-** | **-** |
| **20%** | **2,615** | **726** | **38%** |
| **For the first 5 years:18%****for the next 5 years:15%****for the remainder years:10%** | **5,552** | **3,663** | **194%** |

A defined-benefit plan is considered adequately **funded** if its assets equal or exceed the discounted value of its future pension liabilities. A DB "**funding ratio**" measures assets divided by liabilities. Most assets can be valued accurately, but as we have mentioned above, the valuation of liabilities is far more complex. Funding ratios above one will indicate the pension or annuity can cover all obligated payments. Ratios below one will reflect that it is unable to make payments or may be in danger of not being about to make payments at a later time. The funding ratio is a core indicator of a pension fund’s financial health. Higher TAL will indicate that the plan is facing challenges to pay these liabilities. Lower discount rate causes the TAL to increase which will lead to a decrease in funding ratio. This will help funds to avoid understating their future obligations and will also help them to assess their situation and conditions with open eyes.

***Table 4.3.*** Total Actuarial Liability under Entry Age Normal Cost method

|  |  |  |  |
| --- | --- | --- | --- |
| **Discount rate** | **Total Actuarial Liability****(10 million rials)** | **Differences between the TAL based on the current discount rate and based on the alternative discount rates****(10 million rials)** | **Percentage difference** |
| **22%** | **111** | **-** | **-** |
| **20%** | **148** | **37** | **33%** |
| **For the first 5 years:18%****for the next 5 years:15%****for the remainder years:10%** | **270** | **159** | **143%** |

Based on the standard 27 rules of Iranian audit organization, the accountants report’s information is used by the board of directors, the members of the funds and the plan’s sponsors. Each of these sectors acts differently when encountering this information.

From the perspective of the members of the fund and the board of directors, the realistic actuarial liability based on the rational discount rate is more favorable even if it causes fear. The more realistic discount rate avoids masking plan funding issues that could impact future generations of retirees and plan members. For example, if the assumption is too high, a funding shortfall could result that requires younger and future plan members to contribute more to the pension plan, receive lower benefits, or both. If the assumption is too low, current members could pay more than necessary for their pensions or benefits may be reduced more than necessary.

On the other hand, from the perspective of the plan’s sponsors higher actuarial liability is not favorable because they have to cover the shortfalls. So they explicitly prefer pension boards to use higher discount rate in order to decrease annual pension contributions because they believe that increasing discount rates reduces the cost of pensions. However, changing the discount rate does not reduce the true, long-term cost of a pension.

Given the results of the application of two actuarial cost methods, we can see that the TAL is lower in EAN method than in PUC method. Both methods create a distinction between past service liability, and current normal cost, as a part of the total projected liability. Entry age normal method allocates the present value of future benefits (PVFB) of a member over the working lifetime of that member, from his or her “entry age,” or date of membership, through his or her assumed exit age(s). Unit credit allocates the PVFB of a member based on benefits accrued as of the valuation date. If the benefit is pay-related, this allocation would reflect the impact of future pay increases and is referred to as projected unit credit.

Under the EAN cost method, the actuary projects the contributions needed each year to finance an employee’s benefits and then levels those contributions over the entire period the employee is expected to participate in the plan. Under the PUC method, contributions are made as benefits accrue, so they start low and increase each year. In addition, because the PUC method allocates a large portion of the required future contributions to normal cost than does the EAN method, it usually yields a substantially higher unfunded liability (Soto, 2008).

The EAN cost is higher than normal cost under PUC method upon entry, but it remains level as a percentage of pay throughout the member’s assumed working lifetime. Under PUC method, the normal cost increases dramatically as the member approaches retirement. PUC method produces an increasing cost over the member’s working lifetime, because each additional year of service is one year closer to the commencement of benefits.

As we have seen in table 4.1, the pension fund that we have used its data, is still at its early stages or in other words, it is a young fund (the average year of service for male employees is 10 years and for female employees is equal to 9 years). Consequently, the amount of Normal Cost for EAN method is greater that PUC method. Hence, the TAL for PUC method is larger than the TAL for EAN method, since:

|  |
| --- |
|  |

keeping in mind that the calculation of present value of future benefit is the same for both methods.

Based on financial theory, a discount rate should reflect the timing and riskiness of the promised value of future cash flows. Thus, pension funds that are more mature, whose liabilities have shorter duration and are more likely to be paid, should use lower discount rates than younger funds because the yield curve is generally upward sloping. Another reason why mature funds should use lower discount rates is because their projected liabilities are more akin to those of a (shorter duration) bond rather than of equity (Lucas and Zeldes, 2006; Benzoni et al, 2007).

The shift from the PUC method to EAN cost method results in higher costs early in a worker’s career and lower costs later when the fund matures (Munnell and Soto, 2004).

In foreign countries, a financial report is as important as the accountant’s report. They also let their companies take advantage of the best actuarial cost method that they consider suitable. As a consequence, competition between companies will also increase. However, in Iran, companies do not pay homage to their financial report. Also, based on the standard 27 rules of the Iranian audit organization, companies are obliged to follow PUC method for their calculations.

Our calculation consists of the application of different methods to show the differences between the two methods and the possibility to use alternative methods. An important fact which should be considered is that the information provided by these reports is the main source of decision making therefore, a slight change in the results will play an important role in the future of the funds and different sectors will respond differently to these changes. The more realistic the assumptions, the more rational the results will be.

**6. Conclusion**

In this study, we have analyzed the effects of different suggested discount rates on the amount of pension liabilities. Pension obligations are among the largest obligations organizations face, and taking accounting actions to improve the funded status of these obligations may reduce organizations expenditures.

Actuarial liability is the portion of the present value of benefits (PVFB) that is attributed to past service. Different cost methods calculate the AL differently, but it always reflects only past service. The calculation of the Total Actuarial Liability (TAL) under two different methods and for three different discount rates was done based on the information of the current employees of one of the members of the Banks Employees’ Pension Fund.

As a very effective factor, slight difference in discount rate would change dramatically pension expenses and pension liabilities. In general, the higher the discount rates, the lower the reported present value of the liabilities and the stronger the pension plan's funding position as reported in the accounting statements would be. The presence of a large unfunded pension liability could mean lower credit ratings and higher cost of debt for the firms. Therefore, a large unfunded pension liability could be regarded as undesirable by corporate managers and plan sponsors and managers have incentives to understate the magnitude of unfunded projected pension liabilities by increasing the discount rate or by decreasing the salary progression rate or both.

Based on the standard 27 rules of Iranian audit organization, the current accepted discount rate which is being used is equal to the expected rate of return. But, while this national trend is a common practice, it is not a prudent practice. Also, according to the standard 27 rules of Iranian audit organization, Project Unit Credit method (PUC) is currently being used in pension funds in Iran.

The first alternative discount rate that has been considered in this research is equal to 20% because the highest banking system’s deposit rate in Iran is about 15% (which is equal to the risk free rate of return) and Banks Employee’s Pension Fund usually use this rate for its calculation. The second alternative discount rate that has been suggested in this research is the rate based on the Technical interest rate of the Iranian’s Central Insurance. Based on this interest rate, the discount rate is equal to 18% for the first 5 years, and is equal to 15% for the second 5 years and for the remainder years this rate is equal to 10%. This rate is legislated in Life insurance and pension law of Iranian’s Central Insurance. The third option is the hypothetical discount rate which is equal to 22%.

For funding purposes, the two most common actuarial cost methods are entry age and unit credit cost methods. Actuaries use actuarial cost methods to allocate the PVFB to various time periods during the member’s working lifetime. Pension funds in Iran are obliged to follow PUC method, but in foreign countries pension funds can choose the best method that fits them. In this research we have calculated the results based on the application of two different methods to show the different results of these methods. As we have used data available from a pension fund which is still young and is still at its early stages, the amount of normal cost EAN method is greater that PUC method. Consequently the TAL for PUC method is larger than the TAL for EAN method.

Our results illustrate that a one percent decrease in the discount rate will lead to 19% increase in TAL under PUC method. In the case of the EAN method, a one percent decrease in the discount rate will increase TAL by 17%. Different methods have different results. In a competitive economy, companies can follow different methods for their calculations while in Iran they are obliged to follow PUC method. If companies have the permit to choose the method to follow they could choose the best method that fit them.

Different sectors will act differently when encountering the information provided by the accountant and actuarial reports. Company’s accountants and actuaries prefer the more realistic and rational results even if they cause fear. While managers and plan sponsors do not favor the high amounts that are the results of the more realistic discount rates. However, we should keep that in our mind that the information provided by these reports is the main source of decision making therefore, a slight change in the results will play an important role in the future of the funds and different sectors will respond differently to these changes. The more realistic the assumptions, the more rational the results will be.

Finally, it is worth mentioning that based on our findings, a change in valuation method or a small change in discount rate can result a huge change in actuarial liabilities. Hence, similar to the banking and insurance system in Iran, we need to have a regulatory body for Iranian pension funds. Hence, by controlling important and vital parameters, such as discount rate, huge deviations in estimating actuarial liability can be prevented.

Appendix A**:**

***Table 4.4.*** Total Actuarial Liability based on 22% discount rate

|  |  |  |
| --- | --- | --- |
| **Discount rate** | **Total Actuarial Liability** | **Percentage difference** |
| **PUC** | **EAN** |
| **22%** | **1,889** |  **110** | **94%** |

***Table 4.5.*** Total Actuarial Liability based on 20% discount rate

|  |  |  |
| --- | --- | --- |
| **Discount rate** | **Total Actuarial Liability** | **Percentage difference** |
| **PUC** | **EAN** |
| **20%** | **2,615** |  **148** | **94%** |

***Table 4.6.*** Total Actuarial Liability based on the discount rate which is equal to 18% for the first 5 years, 15% for the next five years and 10% for the remainder years

|  |  |  |
| --- | --- | --- |
| **Discount rate** | **Total Actuarial Liability** | **Percentage difference** |
| **PUC** | **EAN** |
| **For the first 5 years:18%****for the next 5 years:15%****for the remainder years:10%** | **5,552** |  **270** | **95%** |

Appendix B:

**Table de mortality TD 88-90**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **X** | **lx** | **dx** | **ex** | **X** | **lx** | **dx** | **Ex** |
| 0 | 100000 | 871 | 73 | 53 | 88791 | 780 | 25 |
| 1 | 99129 | 72 | 73 | 54 | 88011 | 846 | 24 |
| 2 | 99057 | 47 | 72 | 55 | 87165 | 924 | 23 |
| 3 | 99010 | 33 | 71 | 56 | 86241 | 985 | 22 |
| 4 | 98977 | 29 | 70 | 57 | 85256 | 1045 | 22 |
| 5 | 98948 | 27 | 69 | 58 | 84211 | 1128 | 21 |
| 6 | 98921 | 24 | 68 | 59 | 83083 | 1199 | 20 |
| 7 | 98897 | 21 | 67 | 60 | 81884 | 1282 | 19 |
| 8 | 98876 | 21 | 66 | 61 | 80602 | 1359 | 19 |
| 9 | 98855 | 20 | 65 | 62 | 79243 | 1436 | 18 |
| 10 | 98835 | 21 | 64 | 63 | 77807 | 1512 | 17 |
| 11 | 98814 | 21 | 63 | 64 | 76295 | 1575 | 17 |
| 12 | 98793 | 22 | 62 | 65 | 74720 | 1645 | 16 |
| 13 | 98771 | 26 | 61 | 66 | 73075 | 1709 | 15 |
| 14 | 98745 | 33 | 60 | 67 | 71366 | 1807 | 15 |
| 15 | 98712 | 45 | 59 | 68 | 69559 | 1904 | 14 |
| 16 | 98667 | 61 | 58 | 69 | 67655 | 2006 | 13 |
| 17 | 98606 | 86 | 57 | 70 | 65649 | 2106 | 13 |
| 18 | 98520 | 114 | 56 | 71 | 63543 | 2258 | 12 |
| 19 | 98406 | 129 | 55 | 72 | 61285 | 2374 | 11 |
| 20 | 98277 | 140 | 54 | 73 | 58911 | 2495 | 11 |
| 21 | 98137 | 150 | 53 | 74 | 56416 | 2598 | 10 |
| 22 | 97987 | 157 | 52 | 75 | 53818 | 2732 | 10 |
| 23 | 97830 | 153 | 51 | 76 | 51086 | 2835 | 9 |
| 24 | 97677 | 153 | 50 | 77 | 48251 | 2967 | 9 |
| 25 | 97524 | 151 | 50 | 78 | 45284 | 3081 | 8 |
| 26 | 97373 | 151 | 49 | 79 | 42203 | 3162 | 8 |
| 27 | 97222 | 152 | 48 | 80 | 39041 | 3217 | 7 |
| 28 | 97070 | 154 | 47 | 81 | 35824 | 3306 | 7 |
| 29 | 96916 | 157 | 46 | 82 | 32518 | 3298 | 6 |
| 30 | 96759 | 162 | 45 | 83 | 29220 | 3258 | 6 |
| 31 | 96597 | 168 | 44 | 84 | 25962 | 3182 | 6 |
| 32 | 96429 | 174 | 43 | 85 | 22780 | 3055 | 5 |
| 33 | 96255 | 184 | 42 | 86 | 19725 | 2882 | 5 |
| 34 | 96071 | 193 | 41 | 87 | 16843 | 2710 | 5 |
| 35 | 95878 | 202 | 40 | 88 | 14133 | 2508 | 4 |
| 36 | 95676 | 213 | 39 | 89 | 11625 | 2236 | 4 |
| 37 | 95463 | 226 | 38 | 90 | 9389 | 1951 | 4 |
| 38 | 95237 | 240 | 38 | 91 | 7438 | 1675 | 4 |
| 39 | 94997 | 251 | 37 | 92 | 5763 | 1413 | 3 |
| 40 | 94746 | 270 | 36 | 93 | 4350 | 1139 | 3 |
| 41 | 94476 | 294 | 35 | 94 | 3211 | 896 | 3 |
| 42 | 94182 | 314 | 34 | 95 | 2315 | 680 | 3 |
| 43 | 93868 | 353 | 33 | 96 | 1635 | 520 | 3 |
| 44 | 93515 | 382 | 32 | 97 | 1115 | 375 | 3 |
| 45 | 93133 | 406 | 31 | 98 | 740 | 287 | 2 |
| 46 | 92727 | 432 | 30 | 99 | 453 | 190 | 2 |
| 47 | 92295 | 462 | 30 | 100 | 263 | 118 | 2 |
| 48 | 91833 | 501 | 29 | 101 | 145 | 69 | 2 |
| 49 | 91332 | 554 | 28 | 102 | 76 | 39 | 2 |
| 50 | 90778 | 607 | 27 | 103 | 37 | 20 | 1 |
| 51 | 90171 | 660 | 26 | 104 | 17 | 10 | 1 |
| 52 | 89511 | 720 | 25 |  |  |  |  |

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1. Article 68 of Act on Establishment of Bimeh Markazi of Iran (Central Insurance of Iran) and Insurance Operation (Underwriting) [↑](#footnote-ref-2)