Heterogeneous Premiums for Homogeneous Risks? Asset Liability Management under Default Probability and Price-Demand Functions

Florian Klein, Hato Schmeiser*

June 21, 2017

Abstract

We consider an asset liability model under internal solvency constraint which includes default probability as well as price-demand functions and combine insights from empirical and theoretical research. Furthermore, as a result of policyholders' heterogeneous willingness to pay, we introduce heterogeneous premiums to maximize insurer's overall net present value and compare the results with an optimal homogeneous premium. For determining a reservation price for the insurer, we use the Margrabe-Fischer option pricing formula. Our numerical example documents that heterogeneous premiums for homogeneous risks improve the net present value when perfect expectations underlie and are vulnerable against a cost shift, but do not per se induce a decrease of the net present value. Moreover, we recognize that the optimal price setting under overall net present value maximization varies from the underwriting net present value maximization on the individual risk level. Hence, in practice, an overall asset liability management perspective should be in focus to reach the best results from the company's point of view.

Keywords: Default Probability, Heterogeneous Premiums, Asset Liability Management, Price-Demand Function, Risk and Insurance Management
JEL classification: G22; G28; G32

1 Introduction

In times of a period of low interest rates, financial institutions face the challenge to generate appropriate returns from their invested capital. In general, funds have more risky investments on the strategic map (see, e.g., Choi and Kronlund, 2015; Di Maggio and Kacperczyk, 2017), whereas insurer's investment behavior is rather restricted by regulatory requirements, such as Solvency II (see, e.g., Braun et al., 2014). However, improving the return on capital of an insurer is not only determined by the investment strategy. Underwriting profit generation, which we describe as policyholders' premiums minus claims, also represents an integral component for an improvement of the profitability. While various research focuses on optimal investment strategies (see, e.g., Braun et al., 2014, 2017; Eckert and Gatzert, 2017), we set underwriting profit generation in the center of our analysis. In contrast to Braun et al. (2017), we do not

*Florian Klein (florian.klein@unisg.ch) and Hato Schmeiser (hato.schmeiser@unisg.ch) are from the Institute of Insurance Economics, University of St. Gallen, Tannenstrasse 19, CH-9000 St.Gallen.

assume that the pool of policyholders of an insurance is given and hence is not a decision variable for the insurer. The environment in which insurers operate is dynamic and thus changing time-dependently. Therefore, we recognize the demand for insurance as well as policyholders' willingness to pay premium as important parameters for the profitability of an insurer.

Furthermore, we include insights about the inherent existence of default probability and the relation between default probability and willingness to pay premium in our analysis. Concerning the existence of default probability, previous research clarifies that a non-default assumption is not adequate as a result of the systemic risk within the financial world. The financial crisis has illustrated that even institutions which are regarded as too-big-to-fail can actually fail (see, e.g., Harrington, 2009; Brownlees and Engle, 2016). According to the relation between default probability and willingness to pay premium, Wakker et al. (1997) and Zimmer et al. (2009, 2016) emphasize with their empirical research that even a low default probability reduces the willingness to pay massively. Based on this research, we derive a relation between default probability and willingness to pay premium and embed it in our asset liability model. Moreover, we implement a price-demand function. In the context of substandard annuities, Gatzert et al. (2012) use a linear price-demand function. They analyze the optimal price-demand combination in a certain risk class and examine how different sub-populations influence the optimal price-demand combination. In addition, they include a misclassification of the risk class and the effect on the optimal price-demand combination in their investigation. Einav and Finkelstein (2011) also use a linear price-demand function in their analysis of adverse selection in the insurance market. In contrast, Zimmer et al. (2016) show that a convex price-demand function underlies within their empirical data set. Including both views in our approach, we firstly assume a linear price-demand function and later introduce the convex price-demand function which possesses in our case the same reservation premium and actuarilly fair premium than the linear function.

Derived from the insight that heterogeneous customers have a heterogeneous willingness to pay (see Zimmer et al., 2016), we extend previous research which considers an optimal premium-demand combination (see, e.g., Gatzert et al., 2012) and investigations that focus on the reduction of the willingness to pay if default may take place (see, e.g., Schlütter, 2014; Eckert and Gatzert, 2017). Hence, we introduce heterogeneous premiums and analyze how different premium levels influence the insurer's overall net present value. Moreover, we show in detail that the optimal demand varies between an isolated premium-costs consideration (as it is, e.g., done in Gatzert et al., 2012) and an overall perspective. Besides, we emphasize that heterogeneous premiums exhibit the potential to induce a shift of the mindset of an insurance enterprise. The insurance enterprise is adapting the insurance premium to the willingness to pay and not policyholder's willingness to pay to the insurance premium (which is only possible downwards). We evolve an internal solvency constraint and focus on a scenario under perfect expectations as well as a cost shift. In this context, we include uncertainty which is described as a simple variation of the expected costs from different points in time.

In summary, we develop an asset liability model based on default probability as well as price-demand functions and use heterogeneous premiums to maximize the net present value of the insurer. Net present values are calculated by using the Margrabe-Fischer option pricing formula. We combine insights from theoretical as well as empirical research and as outlined above, we extend previous research in several ways. Our aim is to critically reflect the use of heterogeneous premiums under default probability, price-demand functions, solvency constraint, and further model assumptions. This paper is organized as follows. In section two, we thematize the relation between default probability, price-demand function, and premium as basis for our asset liability model in section three. The structure of assets and liabilities, heterogeneous premiums, and the constraints are in focus of section three. Furthermore, in section four, we run a numerical analysis and compare the results under a scenario of perfect expectations as well as misjudging the expected costs. The economic implications of our findings are discussed in section five. Section six concludes with the main insights.

2 Influence of Default Probability and Price-Demand Function according to Insurance Premium

2.1 Central Assumptions for the Two Stakeholder Groups

In this paper, we strive for a shareholder value (overall net present value) maximization measured by the Margrabe-Fischer option pricing formula. In this context, we assume that insurance companies have access to a frictionless and complete capital market and can replicate future cash-flows. Hence, reservation prices for insurance contracts from the viewpoint of the provider are under the Margrabe-Fischer option pricing formula given by the price of the replication portfolio (risk-neutral valuation technique). We assume that policyholders cannot replicate future cash-flows and face a price-demand function. In addition, their willingness to pay for insurance depends on the default risk of the insurer (see, e.g., Wakker et al., 1997; Zimmer et al., 2009, 2016). The willingness to pay on the policyholders' side is – due to risk-aversion – typically higher than the risk-neutral price derived by the Margrabe-Fischer option pricing formula.

2.2 Default Probability and Insurance Premium

From the perspective of rational behavior, an increase of 1 percent default probability decreases for a given insurance amount the actuarially fair premium by 1 percent. However, individuals do not behave completely rational, even not when they intend to be rational. Instead, humans often act under the condition of bounded rationality (see Simon, 1957). As a result of restricted abilities, individuals over-weight or ignore very unlikely events (see Tversky and Kahneman, 1979). From the point of view of an insurance company, it is uncritical when a policyholder is ignoring (extreme) risk events which lead to a default of the insurance. More precisely, the willingness to pay a certain premium is not reduced by the existence of a potential risk when the policyholder does not recognize the risk, e.g., the potential default of an insurance contract is often not taken into consideration. However, previous research clarifies that if a low default probability is given and the policyholders are aware of such a risk, the willingness to pay a premium decreases substantially.

Wakker et al. (1997) analyze the willingness to pay an insurance premium and underlie either a default-free insurance or a 1 percent default probability. The authors demonstrate with a direct survey that the existence of 1 percent default probability leads to an average premium reduction of over 20 percent to compensate the potential default. Zimmer et al. (2009) generate similar results based on an online questionnaire. For 0.3 percent default probability, the average premium decreases by 14 percent related to a default-free insurance. 4.9 percent default probability leads to an average reduction of 26 percent. The findings of Zimmer et al. (2016) illustrate as well that the risk-averse behavior of policyholders is present in the context of probabilistic insurance[1]. Table 1 summarizes the above-mentioned empirical research.

Table 1: Empirical Research about Default Probability and Insurance Premium

Authors	Wakker et al. (1997)	Zimmer et al. (2009)	Zimmer et al. (2016)
Default Probability (%)	0; 1	0; 0.3; 4.9	0; 1; 2; 3
Average Premium Reduction (%)	higher 20 (1)	14 (0.3); 26 (4.9)	25 (1); 38.04 (2); 50 (3)
Technique	Direct survey	Online questionnaire	Laboratory experiment
Participants (number)	Students (230), Money managers (75)	No target audience; invited via e-mail (719)	Students (117), Employee (28), Other (36)
Type of Insurance	Fire insurane, Care insurance, International investment	Household insurance	Theft insurance

Based on the underlying empirical research, the question comes up how the willingness to pay a premium and default risk are related. Derived from the insights of Zimmer et al. (2009), Lorson et al. (2012) formulate a logarithmic regression to describe the relation between default probability DP_{ij} (which describes the default risk) and the premium reduction π^R_{mean}. More concretely, DP_{ij} is understood as the probability that an insurance contract among insurer i and policyholder j results in a default

$$\pi^R_{mean}(DP_{ij}) = 0.0419 \cdot \ln(DP_{ij}) + 0.3855 \ \ \forall DP_{ij}(0, 1]. \tag{1}$$

Thus,

[1] When a hazard or damage occurs, the probabilistic insurance does only pay off with a probability smaller than one (see Wakker et al., 1997). A probabilistic insurance can be induced by the insurance company itself, which structures the insurance contract in a way that there is only a payoff with a probability lower than one, or an inherent default probability (e.g., caused by systemic risk) leads to a probabilistic insurance.

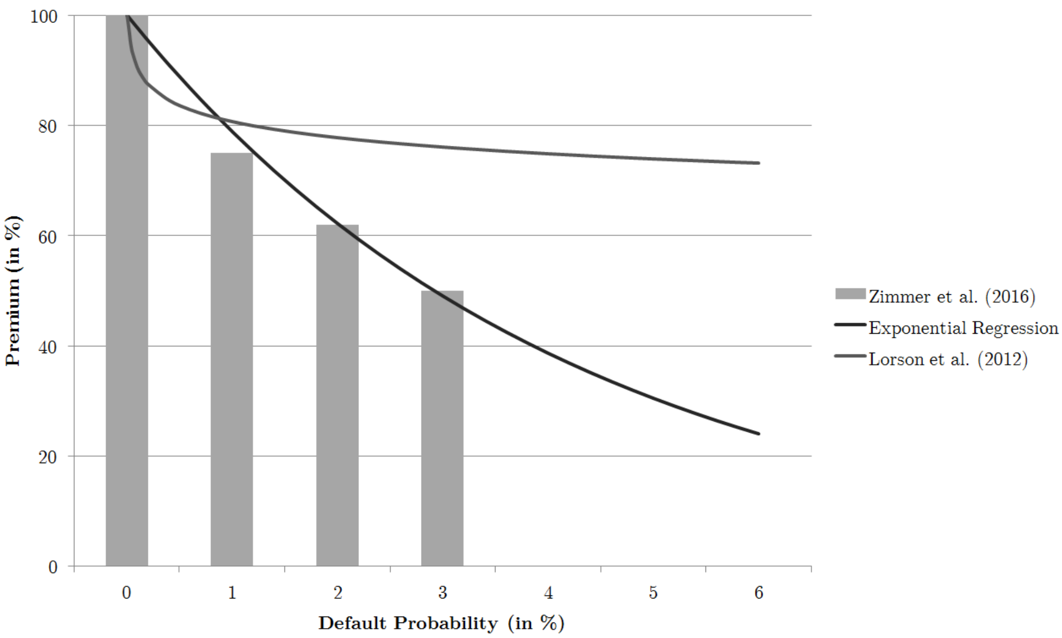

Figure 1: Default Probability and Premium

$$\pi_{mean}(DP_{ij}) = 1 - \pi^R_{mean}(DP_{ij}) = 1 - (0.0419 \cdot \ln(DP_{ij}) + 0.3855) \quad (2)$$

is defined as the mean premium π_{mean} which is paid dependent on DP_{ij}. Lorson et al. (2012) argue that this type of regression is a good fit for small default probabilities. However, the empirical study of Zimmer et al. (2016) shows that an exponential regression is more suitable to explain the dependency between default probability and premium. In the following chapters, we use the exponential regression because of two reasons. First, the amount of examined default probabilities is higher in Zimmer et al. (2016) than in Zimmer et al. (2009). Second, the interval between the analyzed default probabilities is equal in Zimmer et al. (2016). More precisely, the default probability increases by 1 percent per investigated level. In contrast, the examined default probability jumps in Zimmer et al. (2009) between 0, 0.3, or 4.9 percent. This non-linear default probability development impedes to indicate the actual willingness to pay for policyholders. Our exponential regression is based on the results of the mean premium

$$\pi_{mean}(DP_{ij}) = \exp(-23.75 \cdot DP_{ij}). \quad (3)$$

This implies that for a default probability of zero the premium is equal to 100 percent. However, the assumption that no default probability underlies is not realistic. The recent financial crisis has shown that even organizations and financial products with the highest ratings exhibit an inherent default probability (cf., e.g., Lehman Brothers and collateralized debt obligations (CDOs) (see, e.g., Harrington, 2009; Brownlees and Engle, 2016)). Figure 1 visualizes the logarithmic regression of Lorson et al. (2012) and our exponential regression which is derived from Zimmer et al. (2016).

2.3 Price-Demand Function and Optimal Insurance Premium on the Underwriting Level

Initially, we analyze the relation between price-demand function and optimal insurance premium on the underwriting perspective to clarify our basic idea. In this regard, it is important to mention that the results of an underwriting level are typically non-optimal from insurer's perspective which we will show later. Subsequently, we develop a model where the premium is dependent on the default probability DP_{ij} as well as a price-demand function PDF.

We consider a price-demand function of the insurer i and set i to 1 because we analyze in the following chapter the asset liability model of one specific insurance company. Including this notation, ensuing Gatzert et al. (2012) as well as Einav and Finkelstein (2011), at first a linear price-demand function is in focus to clarify our approach. Afterwards, we extend the model to non-linearity (see, e.g., Zimmer et al., 2016; Caginalp, 2005). The price-demand function exhibits the following form

$$\pi^{PDF}(\sum_{j=1}^{n} x_{1j}) = \pi_r - \theta \cdot \sum_{j=1}^{n} x_{1j}, \tag{4}$$

where $\pi^{PDF}(\sum_{j=1}^{k} x_{1j}) = 0$ and $k \geq n$.

Furthermore, π_r is the reservation price, $\sum_{j=1}^{n} x_{1j}$ the demand, where n represents the number of demanders, and θ determines the negative slope of the graph which remains constant. The expected cost function

$$E(c(\sum_{j=1}^{n} x_{1j})) = \pi_a \tag{5}$$

is not influenced by the number of insurance contracts and describes which premium is needed to compensate the expected costs of insurance (see also Gatzert et al., 2012). Hence, we consider risks which are homogeneous in respect to their distribution function. Let us in the first step maximize the expected underwriting profit $E(P_r)$ which can be generated through the premium-cost relation. Derived from the current economic situation, we suppose a risk-free interest rate of zero. With $x^N := \sum_{j=1}^{n} x_{1j}$, optimal demand x^{opt} and optimal premium π^{opt} can be determined through

$$\frac{\delta E(P_r(x^N))}{\delta x^N} = \frac{\delta(\pi^{PDF}(x^N) \cdot x^N)}{\delta x^N} - \frac{\delta(E(c(x^N)) \cdot x^N)}{\delta x^N} \stackrel{!}{=} 0. \tag{6}$$

Thus, the optimal point is

$$x^{opt} = \frac{\pi_r - \pi_a}{2\theta};\ \pi^{opt} = \pi_r - \theta \frac{\pi_r - \pi_a}{2\theta} = \frac{\pi_r + \pi_a}{2} \tag{7}$$

and this is a maximum for $\theta > 0$ because $\frac{\delta^2 E(P_r(x^N))}{\delta^2 x^N} < 0$. However, in the linear case the optimal premium-demand combination does only enable to realize 50 percent of the potential expected underwriting profit $\int_0^{x^S} E(P_r(x^N)) dx^N$, where x^S is the point of intersection between $\pi^{PDF}(x^N)$ and $E(c(x^N))$.

This insight is independent of θ and holds for $\pi_r > \pi_a$. Theoretically, it would be optimal to adapt the premium continuously to the demand quantity to reach the whole integral between $\pi^{PDF}(x^N)$ and $E(c(x^N))$.[2] However, a continuous premium adaptation is not realistic in practice.[3] Nevertheless, it is possible to increase the total expected underwriting profit in consequence of using heterogeneous premiums.[4] For instance, when an insurer chooses the right two additional premiums, the total expected underwriting profit increases by 50 percent. The two additional premiums are determined as follows:

$$\pi^{high} = \frac{\pi_r + \frac{\pi_r + \pi_a}{2}}{2}; \ \pi^{low} = \frac{\frac{\pi_r + \pi_a}{2} + \pi_a}{2}. \tag{8}$$

As a result of the linearity, the additional expected underwriting profit which results from π^{high} and π^{low} is equal. The following formula describes the dependency between expected underwriting profit and heterogeneous premiums which maximize $E(P_r^{total})$

$$E(P_r^{total}) = E(P_r^{opt}) + \sum_{l=1}^{m} \beta_l \cdot (\frac{1}{4})^l \cdot E(P_r^{opt}), \tag{9}$$

where β_l can exhibit values from 0 to 2^l. β_l increases until 2^l before l raises by 1. Heterogeneous premium levels are $\pi^{level} = 1 + \sum_{l=1}^{m} \beta_l$. Figure 2 visualizes how much of the potential underwriting profit can be generated through introducing different premium levels.

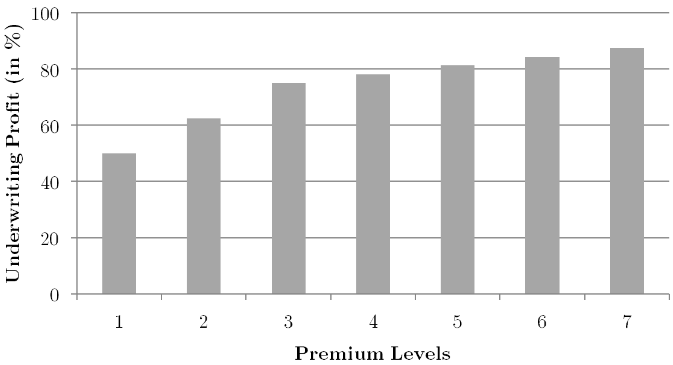

Figure 2: Underwriting Profit and Premium Levels

The idea behind is to absorb the willingness to pay a certain premium. From a perspective of perfect competition, such an increase of the potential profit is impossible. However, imperfect information restricts the degree of competition in the insurance market (see also Rothschild and Stiglitz, 1976). An information asynchronicity (asymmetry over time) can either result from the market side, e.g., market access

[2]Such a process is also known as price discrimination (see, e.g., Pindyck and Rubinfeld, 2008).
[3]Continuous premium adaptation typically reduces the acceptance of policyholders, induces reputational risks, additional transaction costs, and may not be achievable because of regulatory requirements.
[4]The pertinent literature considers heterogeneous costumers and beliefs in the context of asset pricing and strives for equilibrium models (see, e.g., Constantinides and Duffie, 1996; Brav et al., 2002; Basak, 2005). Moreover, Crocker and Snow (2013) examine risk classification with heterogeneous agents.

or intransparent markets, or the insurer-costumer relationship, e.g., various risk attitude of individuals[5] (see also Tversky and Wakker, 1995; Ellsberg, 1961), an adverse selection (see Rothschild and Stiglitz, 1997) or moral hazard (see Holmström, 1979), and leads to a heterogeneous willingness to pay. For instance, Zimmer et al. (2016) demonstrate that a heterogeneous willingness to pay premium is omnipresent and thus should be implemented from the insurer's point of view to maximize profits.

2.4 Interaction between Default Probability, Price-Demand Function, and Insurance Pricing

After we have analyzed the relation between default probability and premium as well as price-demand function and optimal premium, in a next step, both examinations will be combined. With $DP^A := \frac{1}{z} \cdot \sum_{j=1}^{z} DP_{1j}$, π_{mean}^{PDF} is described as

$$\pi_{mean}^{PDF}(DP^A, x^N) = (\pi_r - \theta \cdot x^N) \cdot \exp\left(-23.75 \cdot DP^A\right), \tag{10}$$

where π_{mean}^{PDF} is monotonically decreasing in DP^A because $DP^A \in (0, 1]$. Thus, the optimal point is

$$x_*^{opt} = \frac{\exp\left(-23.75 \cdot DP^A\right) \cdot \pi_r - \pi_a}{2\theta \cdot \exp(-23.75 \cdot DP^A)}; \tag{11}$$

$$\pi_*^{opt} = \frac{\exp(-23.75 \cdot DP^A) \cdot \pi_r + \pi_a}{2}. \tag{12}$$

The optimal premium and demand with default probability are smaller than without default probability $\pi_*^{opt} < \pi^{opt}$; $x_*^{opt} < x^{opt}$. Therefore, it is important to consider the default probability in regard to optimal premium. Moreover, price-demand function as well as default probability are necessary to understand the behavior of policyholders adequately. Our previous analysis clarifies that the premium is not a constant factor which is ex ante given (like it is considered, e.g., in Kahane (1977) as well as Kahane and Nye (1975)). In the next chapter, we will implement the insights according to the dependency between premium, default probability, and price-demand function in an asset liability framework to emphasize the necessity in the context of risk and insurance management.

[5]For instance, the risk preference of individuals varies with the underlying degree of uncertainty (see also Tversky and Wakker, 1995; Ellsberg, 1961) and intrapersonal.

3 Asset Liability Model Framework

For our analysis, we develop a one-period, non-life model which includes assets $A_t(DP^A, x^N)$ and liabilities $L_t(DP^A, x^N)$ to derive the equity capital $EC_t(DP^A, x^N)$. Including the fact that our model is generated for one-period, the following relation results when taking the limited liabilities of the shareholders into account

$$EC_1(DP^A, x^N) = max(A_1(DP^A, x^N) - L_1(DP^A, x^N), 0). \tag{13}$$

$A_1(DP^A, x^N)$ is the stochastic value of assets at point of time 1, where it stems from the product of assets at point of time 0 and the value change between $t = 0$ and $t = 1$. Given a complete and frictionless capital market, we have with

$$PV(A_1(DP^A, x^N)) = A_0(DP^A, x^N); PV(L_1(DP^A, x^N)) = L_0(DP^A, x^N) \tag{14}$$

a market equilibrium, where $PV(L_1(DP^A, x^N))$ stands for the present value of the liabilities (stochastic in $t = 1$). The initial capital $A_0(DP^A, x^N)$ is subdivided in the equity capital EC_0 as well as the premium $\pi^{PDF}_{mean}(DP^A, x^N) \cdot x^N$ at $t = 0$:

$$A_0(DP^A, x^N) = EC_0 + \pi^{PDF}_{mean}(DP^A, x^N) \cdot x^N. \tag{15}$$

This equation holds for the case that one premium level exists. We will later implement heterogeneous premiums. Our model extends the ideas of Eckert and Gatzert (2017) and assumes that the willingness to pay is affected by a price-demand function too. In general, our premium levels are higher than the risk-neutral equilibrium premium as a result of the assumed price-demand function. The expected liabilities $E(L_1(DP^A, x^N))$ consist of the expected claims of the policyholders $E(C_1(DP^A, x^N))$ at $t = 1$:

$$E(L_1(DP^A, x^N)) = E(C_1(DP^A, x^N)) = E(c_1(DP^A)) \cdot x^N, \tag{16}$$

where $C_1(DP^A, x^N)$ denotes the claims of all homogeneous risks. We suppose that $E(C_1(DP^A, x^N))$ is monotonically decreasing in DP^A. More precisely, when there underlies a probabilistic insurance and the probability occurs that the insurance does not pay the claims of the policyholder, $E(C_1(DP^A, x^N))$ is reduced. According to the price-demand function, we have described the expected cost function $E(c(x^N))$ as a constant factor in chapter 2. Claims of the policyholders generate costs and thus we define the claims of the policyholders as costs which have to be considered when selling an insurance contract. However, such claims are connected with uncertainty. Considering this fact, we define $E(C_1^0(DP^A, x^N))$ as the expected value of the claims for $t = 1$ from the perspective of $t = 0$ and furthermore ξ which explains the difference between the expected claims determined in $t = 0$ and $t = 1$. $\xi > 0$ exhibits an unexpected increase of the claims from perspective $t = 0$ and includes the risk of error which comes from forecasting and diagnostic risk (see, e.g., Kriele and Wolf, 2014).[6] Hence, in formal terms we define

$$E(C_1(DP^A, x^N)) = E(C_1^0(DP^A, x^N)) + \xi. \tag{17}$$

Based on these insights, we follow our previous definition and describe the expected cost (claim) function $E(c_1(DP^A))$ as independent of x^N. Furthermore, we take into consideration that the default probability is influencing the costs. We need an estimation of the claims for $t = 1$ from the perspective of $t = 0$ to calculate which premium might be profitable and which not. As a result of uncertainty, the threat exists that an estimated profitable premium results in an unprofitable case.

In a next step, we introduce our option pricing framework. The values of assets $A_t(DP^A, x^N)$ and liabilities $L_t(DP^A, x^N)$ follow geometric Brownian motions. Under a real-world probability measure P assets and liabilities are denoted by

$$\begin{aligned} dA_t(DP^A, x^N) &= \mu_A A_t(DP^A, x^N)dt + \sigma_A A_t(DP^A, x^N)dW_{A_t}^P, \\ dL_t(DP^A, x^N) &= \mu_L L_t(DP^A, x^N)dt + \sigma_L L_t(DP^A, x^N)dW_{L_t}^P, \end{aligned} \tag{18}$$

where the Wiener processes are correlated with $dW_{A_t}^P dW_{L_t}^P = \rho dt$. μ_A and μ_L describe the drifts, while σ_A and σ_L determine the volatility of the stochastic processes. We further consider a risk-neutral martingale Q which leads to the fact that the drifts are described by the risk-free interest rate r. For the sake of simplicity, we assume that under the real-world probability measure P the drift is equal to the drift under Q (risk-neutral market). At time $t = 1$, we reach the following solutions for the stochastic differential equations under Q (see, e.g., Björk, 2009)

$$\begin{aligned} A_1(DP^A, x^N) &= A_0(DP^A, x^N) \cdot exp[r - \sigma_A^2/2 + \sigma_A(W_{A_1}^Q - W_{A_0}^Q)], \\ L_1(DP^A, x^N) &= L_0(DP^A, x^N) \cdot exp[r - \sigma_L^2/2 + \sigma_L(W_{L_1}^Q - W_{L_0}^Q)], \end{aligned} \tag{19}$$

with $E(L_1(DP^A, x^N)) = L_0(DP^A, x^N) \cdot exp(r)$.

In this paper, we examine an insurer which maximizes the overall net present value between $t = 0$ and $t = 1$. Hence, we receive

$$\underset{DP^A \in (0,1], x^N \in \mathbb{N}}{\arg\max} \; NPV(EC_1(DP^A, x^N) - EC_0),$$

where

[6] Forecasting risk is connected with a wrong projection of the data of past to forecast the future, while diagnostic risk leads to a wrong distribution function, e.g., the distribution function of the past only exhibits an inadequate amount of data or the environment varies in a way that the distribution changes (e.g., extreme risk). Moreover, ξ focuses on the negative deviation of the expectations.

$$NPV(EC_1(DP^A, x^N) - EC_0) = PV(EC_1(DP^A, x^N)) - EC_0 \tag{20}$$

Using the Margrabe-Fischer option pricing formula based on two geometric Brownian motions (see Margrabe, 1978; Fischer, 1978), derived from equation (13), the present value of the equity capital EC_1 is described as follows:

$$\begin{aligned} PV(EC_1(DP^A, x^N)) &= E^Q[exp(-r)\ max(A_1(DP^A, x^N) - L_1(DP^A, x^N), 0)] \\ &= A_0(DP^A, x^N) \cdot \Phi(d_1) - L_0(DP^A, x^N) \cdot \Phi(d_2), \end{aligned} \tag{21}$$

where

$$d_1 = \frac{ln(\frac{A_0(DP^A, x^N)}{L_0(DP^A, x^N)}) + \frac{\sigma^2}{2}}{\sigma}; d_2 = d_1 - \sigma; \sigma = \sqrt{\sigma_A^2 + \sigma_L^2 - 2\rho \cdot \sigma_A \cdot \sigma_L}. \tag{22}$$

Moreover, Φ describes the standard normal distribution function. The idea here is to analyze how different premiums influence insurer's overall level. In this context, it is important to mention that when we consider E^Q-conditions, the optimal demand for underwriting net present value maximization $NPV(E(P_r(DP^A, x^N)))$ (premiums minus expected costs, see Appendix A) varies from the optimal demand for overall net present value maximization (without ξ)[7] $NPV^w(EC_1(DP^A, x^N) - EC_0)$. We state in the Appendix A the detailed net present value formula on the company's level and the underwriting profit for linear as well as convex price-demand function which we use to calculate the optimal demand numerically. Furthermore, we show in the Appendix A that EC_0 induces the deviation of the net present value calculation and hence the optimal demand between the overall and underwriting perspective varies. Thus, the results of a partial model are in general non-optimal from the overall asset liability perspective. According to insurance practice, we derive the insight that an insurer should not maximize the underwriting net present value because of the non-optimality. Instead, it should be concentrated on overall net present value maximization to reach optimality for the insurer. As a result of this insight, we focus in the following part on the overall perspective. Including the fact that the highest net present value is reached when the default probability is minimal given the line of reasoning by using the results of Zimmer et al. (2016), we consider a variation of default probabilities in our analysis and compare between the different default probabilities how the net present value maximizing parameters of our model change. Furthermore, the convex price-demand function exhibits the following form

$$\pi_{mean}^{PDF,con}(DP^A, x^N) = \frac{\pi_r}{\eta \cdot x^N + 1} \cdot \exp(-23.75 \cdot DP^A), \tag{23}$$

where $\pi_{mean}^{PDF,con}$ is monotonically decreasing in DP^A because $DP^A \in (0, 1]$.

[7] The optimal demand is calculated based on the expectations for $t = 1$ from the point of view of $t = 0$. Since ξ is only ex post known, it does not affect the optimal demands from the underwriting and overall perspective, which are determined in $t = 0$, but the optimal net present values are influenced.

Heterogeneous Premiums

In the presented model, we have focused on a homogeneous premium for all insurance costumers. The optimal premium π_{OP}^{opt} which can be derived through price-demand function and cost function leads to the optimal overall net present value under the condition that only one premium is existent. However, as we have seen in Zimmer et al. (2016), the willingness to pay for a homogeneous insurance product is highly heterogeneous. This insight is supported by the understanding of imperfect information in the insurance industry (see, e.g., Rothschild and Stiglitz, 1976, 1997; Holmström, 1979). Based on these findings, we extend our previous model and introduce different premium levels π^{level}. In this context, it is important to mention that we still strive after optimality. More concretely, each additional premium level is optimal under the underlying constraints to maximize the overall net present value. For the one premium case, the optimal aggregated premium is

$$\Pi^{total} = \pi_{OP}^{opt}(DP^A, x_{OP}^{opt}) \cdot x_{OP}^{opt}. \tag{24}$$

For heterogeneous premiums, the total premium is defined as

$$\Pi_*^{total} = \sum_{e=1}^{h} \Delta\Pi(\pi_e^{level}, x_e^{level}), \tag{25}$$

where $\Pi_*^{total} \geq \Pi^{total}$. In the case of heterogeneous premiums, the demand is equal to the sum of the demand which is generated with different premium levels. As mentioned before, heterogeneous premiums do not only include the chance for reaching higher aggregated premiums. When the expected costs $E(c_1^0(DP^A))$ are substantially lower than the expected costs $E(c_1(DP^A))$, it can happen that the additional premium level is $> E(c_1^0(DP^A))$ and $< E(c_1(DP^A))$. Figure 3 visualizes heterogeneous premiums (three premium levels) and the underwriting profit under perfect expectations.

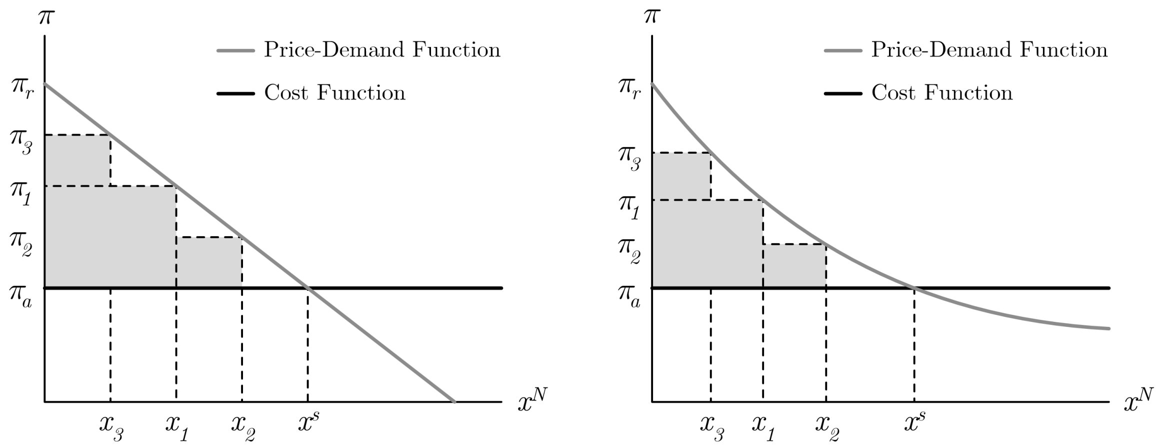

Figure 3: Heterogeneous Premiums under Linear and Convex Price-Demand Function

Constraints

In the next step, we define the constraints under which we consider our asset liability model. According to our first constraint, we introduce an internal solvency requirement which determines whether a certain premium level can be realized. The policyholders adapt their willingness to pay to the reported default probability DP^A. Since we regard that the actual default probability has to be smaller or equal than the reported default probability to protect policyholders and to avoid default probability discrimination, DP^A exhibits an upper bound for the default probability. In other words, when the actual default probability is smaller or equal than the reported default probability, the solvency constraint is satisfied. Our solvency requirement under the real-world probability measure P is structured as follows:[8]

$$P(A_1(DP^A, x^N) < L_1(DP^A, x^N)) \leq DP^A, \tag{26}$$

where

$$P(\frac{A_0(DP^A, x^N) \cdot exp[r - \sigma_A^2/2 + \sigma_A(W_{A_1}^P - W_{A_0}^P)]}{L_0(DP^A, x^N) \cdot exp[r - \sigma_L^2/2 + \sigma_L(W_{L_1}^P - W_{L_0}^P)]} < 1),$$

$$P(exp[\sigma_A(W_{A_1}^P - W_{A_0}^P) - \sigma_L(W_{L_1}^P - W_{L_0}^P) + \sigma_L^2/2 - \sigma_A^2/2] < \frac{L_0(DP^A, x^N)}{A_0(DP^A, x^N)}),$$

$$P(\sigma_A(W_{A_1}^P - W_{A_0}^P) - \sigma_L(W_{L_1}^P - W_{L_0}^P) < ln\left(\frac{L_0(DP^A, x^N)}{A_0(DP^A, x^N)}\right) - \sigma_L^2/2 + \sigma_A^2/2).$$

With Z:= $\sigma_A(W_{A_1}^P - W_{A_0}^P) - \sigma_L(W_{L_1}^P - W_{L_0}^P)$, Wiener processes which follow N(0,1), and uncorrelated assets and liabilities, we reach

$$\Phi(\frac{ln(\frac{L_0(DP^A, x^N)}{A_0(DP^A, x^N)}) - \sigma_L^2/2 + \sigma_A^2/2}{\sqrt{\sigma_A^2 + \sigma_L^2}}) \leq DP^A. \tag{27}$$

Equation (27) implies that a high asset-liability-ratio is needed to reach low default probabilities.

In addition, we introduce the constraint that in the homogeneous case the number of premium levels π^{level} is equal to 1 and in the heterogeneous case, the number of premium levels is less or equal 3 (see Figure 3). We use this constraint to differentiate the heterogeneous case clearly from a continuous premium adaptation. Besides, we argue that such a continuous premium adaptation is not economically adequate in consequence of several reasons. First, heterogeneous premium levels lead to classification costs (see, e.g., Gatzert et al. (2012)). Second, it underlies a reputational risk that policyholders recognize different premium levels and as a result change the insurance company.

[8]Note that we set the drifts of the stochastic processes under the real-probability measure P equal to r.

Excursus: Remarks in Respect to a Normative Price-Demand Function

Our previous considerations clarify for the assumed relation between default probability and willingness to pay that an insurance company is striving for a default probability of zero. In a next step, we will analyze whether for the normative price-demand function the insurer exhibits the incentive to reach the lowest reported default probability as possible if policyholders cannot replicate future cash-flows and thus are paying under perfect expectations more than the fair price for their insurance contract. We consider an application of the hybrid model with risk-averse policyholders ($a > 0$). The function is structured as follows (see, e.g., Rymaszewski et al., 2012):

$$\phi(w_1) = E(w_1) - \frac{a}{2}\sigma^2(w_1), \tag{28}$$

where $E(w_1)$ denotes the expected wealth of the policyholder in $t = 1$ and $\sigma^2(w_1)$ the variance of the wealth in $t = 1$. Under a full coverage of the claims, setting the wealth without insurance $\phi(w_1)^{NI}$ equal to the wealth with insurance $\phi(w_1)^I$, we derive the following relation for the premium π which is independent of the initial wealth

$$\pi = E(c) + \frac{a}{2}\sigma^2(c). \tag{29}$$

$E(c)$ represents the expected claims and $\sigma^2(c)$ the variance of the claims for the non-default case. Moreover, with a raising a the willingness to pay increases. The policyholder with the highest willingness to pay exhibits a risk parameter of a_1 and the policyholder with the second highest willingness to pay is described by a_2. Assuming linearity for the risk parameter, we can determine a normative price-demand function with these two points. Thus, in formal terms we reach

$$\pi = E(c) + \frac{a_1}{2}\sigma^2(c) - (\frac{a_1}{2} - \frac{a_2}{2})\sigma^2(c)(x^N - 1). \tag{30}$$

Based on our comprehensive numerical computations, we analyze how the overall net present value changes by a default probability affected standard deviation under stable risk attitudes. Therefore, we examine two effects which influence the overall net present value (see equation (21)). $\Phi(d_1)$ and $\Phi(d_2)$ vary by the default probability since the ratio of assets and liabilities changes and furthermore assets minus liabilities are also influenced. With $\sigma^2(c(DP^A)) = \sigma^2(c(1 - DP^A)) = \sigma^2(c)(1 - DP^A)^2$, we reach for a given demand

$$\begin{aligned}\frac{A_0(DP^A)}{L_0(DP^A)} &= \frac{EC_0 + x^N(E(c)(1 - DP^A) + \frac{a_1}{2}\sigma^2(c)(1 - DP^A)^2 - (\frac{a_1}{2} - \frac{a_2}{2})\sigma^2(c)(1 - DP^A)^2(x^N - 1))}{x^N E(c)(1 - DP^A)} \\ &= \frac{EC_0}{x^N E(c)(1 - DP^A)} + 1 + \frac{(\frac{a_1}{2} - (\frac{a_1}{2} - \frac{a_2}{2})(x^N - 1))\sigma^2(c)(1 - DP^A)^2}{E(c)(1 - DP^A)}.\end{aligned} \tag{31}$$

Since

$$\frac{EC_0}{x^N E(c)(1-DP^A)} > \frac{EC_0}{x^N E(c)}$$

and

$$\frac{(\frac{a_1}{2} - (\frac{a_1}{2} - \frac{a_2}{2})(x^N - 1))\sigma^2(c)(1-DP^A)^2}{E(c)(1-DP^A)} < \frac{(\frac{a_1}{2} - (\frac{a_1}{2} - \frac{a_2}{2})(x^N - 1))\sigma^2(c)}{E(c)} \tag{32}$$

$\frac{A_0}{L_0}$ can be smaller, greater, or equal than $\frac{A_0(DP^A)}{L_0(DP^A)}$ depending on the effects. Moreover, the reservation price is reduced by $E(c)DP^A + (a_1 - \frac{a_2}{2})(\sigma^2(c) - \sigma^2(c)(1-DP^A)^2)$ and the expected claims decrease by $E(c)DP^A$, while the maximal demand, which can be sold for the intersection point between costs and price-demand function, remains constant. Thus, when we maximize $A_0 - L_0 > A_0(DP^A) - L_0(DP^A)$.

$\Phi(d_1) - \Phi(d_2)$ can be greater, smaller, and equal to $\Phi(d_1(DP^A)) - \Phi(d_2(DP^A))$ depending on the available relation between assets and liabilities (see also Appendix B). Only if $\Phi(d_1) - \Phi(d_2) < \Phi(d_1(DP^A)) - \Phi(d_2(DP^A))$ the net present value under default relatively improves to the net present value under non-default, while $A_0 - L_0 > A_0(DP^A) - L_0(DP^A)$ leads to a higher net present value under non-default. It results a trade-off between both effects. Although we reach for all our numerical computations that NPV(non-default) > NPV (default), we cannot exclude that for at least one combination NPV (default) > NPV(non-default) might be possible.

4 Numerical Example

4.1 Scenario under Perfect Expectations

In the following chapter, we want to give additional insights to the model by means of different numerical examples. We vary the one year reported default probability (0.01, 0.5, 1 percent), premium levels (1, 2, 3), price-demand function (linear, non-linear (convex)), and the scenario (perfect expectations, cost shift). According to the default probabilities, since there does not underlie a default-free insurer in practice, we consider a reported default probability of 0.01 percent, include the maximal default probability under Solvency II (VaR confidence interval of 99.5 percent) (see, e.g., Braun et al., 2017), and a higher default probability in contrast to the others.

Input Parameters

At first, we describe a scenario with perfect expectations which means that the expected claims (from perspective of $t = 0$) $E(C_1^0(DP^A))$ (which represent the costs) do not deviate from the expected claims $E(C_1(DP^A))$. In other words, for this scenario $\xi = 0$ and the environment is stable.[9] According to the

[9]Time stability exists which assumes that fundamental conditions of the world do not or only slightly change over time. Thus, “the past is a statistically reliable, and hence unbiased, guide to the future” (which is the definition for objective probability by Davidson) (Davidson, 1991, p.131). We will show in the scenario under cost shift why such an assumption can be critical for insurance companies.

empirical data, we mainly refer to Eling et al. (2009) and Zimmer et al. (2016) but also include other data to enable comprehensive computations. Eling et al. (2009) consider a German non-life insurance company which is medium-sized and focuses on automobile as well as property-casualty insurance. In the numerical example of Eling et al. (2009), the stochastic claims of the policyholders after reinsurance are €1.171 billion. Due to the fact that we include a price-demand function in our model, we do not regard the total claims as fixed. Instead, it is increasing with a higher demand and decreasing with lower demand. Furthermore, we set the actuarially fair premium π_a, which is needed to compensate the costs of insurance, as €1000 per contract. Zimmer et al. (2016) show in their study that the maximum willingness to pay π_r is more than factor 5 of the fair premium. In contrast, we calculate more defensive and take factor 3 (€3000) as the maximum willingness to pay (non-default case). In the regard that the underlying insurance company is selling the contract for the optimal premium π^{opt} under underwriting profit maximization, $x^{opt} = 1.171$ million because the costs per contract are €1000. As a result of linearity of the price-demand function, θ is $8.5397 \cdot 10^{-4}$. Moreover, the equity capital at $t = 0$ is 1.25 billion. We further consider a correlation between the Wiener processes of zero. In addition, the standard deviation of the stochastic asset process is assumed as 0.05 and the standard deviation of the liability process as 0.4. Derived from the current economic situation, the risk-free interest rate is set to zero. Table 2 visualizes the input parameters.

Table 2: Input Parameters for the Perfect Expectation Scenario

Default probability in (%)	DP^A	$0.01, 0.5, 1$
Price-demand function (default case)	$\pi^{PDF}_{mean}(DP^A, x^N_*)$	$(3000 - 8.5397 \cdot 10^{-4} \cdot x^N_*)$ $\cdot \exp(-23.75 \cdot DP^A)$
Premium levels	π^{level}	1,2,3
Equity capital	EC_0	1.25 billion
Expected fair premium	$\pi_a = E(c^0_1(DP^A))$	$1000 \cdot (1 - DP^A)$
Correlation between Wiener processes	ρ	0
Deviation between expected claims over time	ξ	0
Standard deviation of the stochastic liability process	σ_L	0.4
Standard deviation of the stochastic asset process	σ_A	0.05
Risk-free interest rate	$\mu_A = \mu_L = r$	0

Note: For $\pi^{level} = 1$, x^N_* and x^N are equal. When π^{level} is two or three, $x^N_* = x^{level}_e$, where $e = 1, 2, 3$ (see Figure 3). In contrast, x^N is the sum of the demand which is realized with different premium levels.

Linear Price-Demand Function

At first, we underlie a linear price-demand function which is illustrated in Table 2. Afterwards, we also consider a convex price-demand function and compare the results. Subsequently, we will focus on the solvency constraint. Under perfect expectations $\xi = 0$, which means that the expected costs per x^N

never exceed the premium. We numerically determine the optimal demands for different premium levels under E^Q-conditions and overall net present value maximization. Table 3 clarifies that a raise of default probability reduces the net present value substantially. For the one-premium case, between 0.01 and 1 (0.5) percent default probability, the optimal premium decreases by 15.99 (8.36) percent. In addition, for the same relation the net present value is even reduced by 39.68 (21.18) percent. This results from the fact that an increasing default probability decreases the willingness to pay as well as the demand simultaneously. Furthermore, for a scenario under perfect expectations, an increase of the number of premium levels raises the net present value substantially. Besides, it is possible to overcompensate 0.5 percent default probability when choosing the right premium levels. For instance, comparing one-premium and three-premium level, for 0.01 (0.5; 1) percent default probability the net present value increases by 49.79 (50.22; 51.31) percent. Hence, the results of our model are close to the theoretical considerations of Figure 2. Figure 4 visualizes the net present value which is influenced by default probability and heterogeneous premiums.

Table 3: Linear Case: Net Present Value in Dependency of Default Probability and Premium

π^{level}	DP^A (in %)	Premium	x^N (in mio)	$\Phi(d_1)$	$\Phi(d_2)$	$NPV(EC_1 - EC_0)$ (in mio)
1	0.01	1994.40	1.172	0.9985	0.9950	1166.23
2	0.01	1994.40 1491.76	1.172 0.590	0.9940	0.9824	1459.73
3	0.01	2493.64 1994.40 1491.76	0.586 0.586 0.590	0.9962	0.9882	1746.91
1	0.5	1827.63	1.103	0.9982	0.9939	919.17
2	0.5	1827.63 1404.47	1.103 0.558	0.9923	0.9784	1151.57
3	0.5	2245.86 1827.63 1404.47	0.5515 0.5515 0.558	0.9948	0.9846	1380.75
1	1	1675.52	1.025	0.9979	0.9931	703.48
2	1	1675.52 1324.66	1.025 0.521	0.9909	0.9750	882.13
3	1	2020.65 1675.52 1324.66	0.5215 0.5215 0.521	0.9936	0.9816	1064.44

For the examined default probabilities, optimal demand for underwriting net present value maximization at most varies from optimal demand under overall net present value maximization by 5.37 percent. Underwriting net present value maximization refers to the relation of premiums minus expected costs (see Appendix A). An overall perspective for an insurer is necessary to sell the optimal quantity. Moreover,

$\Phi(d_1)$ and $\Phi(d_2)$ decrease with a higher default probability and a lower premium adaptation π^{low}, while both increase when a higher premium adaptation π^{high} underlies. In addition, the gap between $\Phi(d_1)$ and $\Phi(d_2)$ increases with a lower premium adaptation. Therefore, a lower premium adaptation leads to a higher net present value than higher premium adaptation (e.g., according to 0.01 percent default probability, €1491.76 results in a higher net present value than €2493.64).

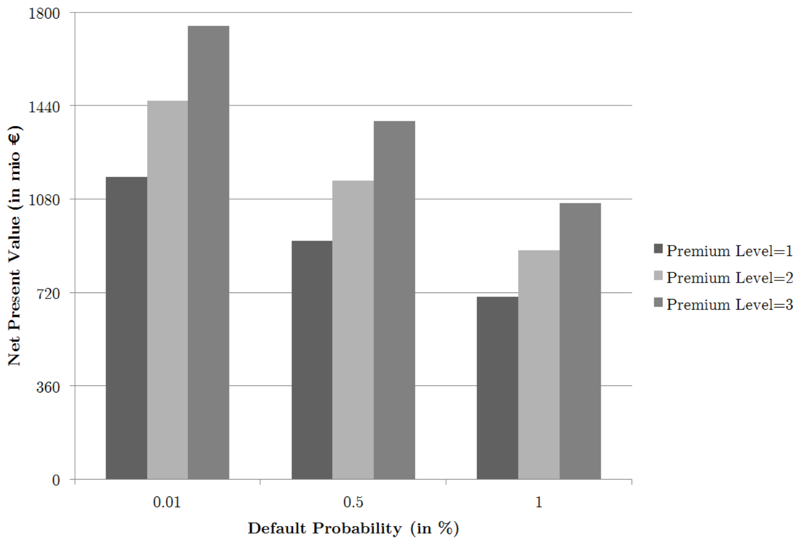

Figure 4: Linear PDF under Perfect Expectations: NPV and Premium Levels

Convex Price-Demand Function

In the pertinent literature is often assumed a linear price-demand function (see, e.g., Gatzert et al., 2012; Einav and Finkelstein, 2011). In contrast, Zimmer et al. (2016) show in their empirical analysis that a convex function describes the relation between premium and demand appropriately. Hence, we consider in the following part a convex price-demand function which is defined in equation (23) and compare the results with the linear case. With π_r, π_a, and the maximal demand (twice the optimal demand in the linear case under underwriting profit maximization), which we have determined in the previous subchapter, we develop a convex price-demand function and receive $\eta = 8.5397 \cdot 10^{-7}$. As a result, π_r, π_a, and maximal demand are equal for the linear and convex price-demand function. Furthermore, in consequence of these identical points and the convexity, $\int_0^{x^S} E(P_{r,convex}(x^N))dx^N < \int_0^{x^S} E(P_{r,linear}(x^N))dx^N$ and $x^{opt}_{convex} < x^{opt}_{linear}$ as well as $\pi^{opt}_{convex} < \pi^{opt}_{linear}$ (one-premium case). For three premiums, a lower and a higher premium adaptation are still optimal.

Table 4 clarifies the outcome under a convex price-demand function. In contrast to the linear case, as a result of convexity, the interval between different premium levels varies substantially. For instance, the interval between optimal premium (one-premium case) and π^{low} is enormously lower than the interval between optimal premium (one-premium case) and π^{high}. Moreover, the demand of π^{low} is higher than the demand of the other premiums (three-premium case). We will see in the next subchapter that the higher quantity of π^{low} is increasing the vulnerability against raising claims. Concerning one-premium case, between 0.01 and 1 (0.5) percent default probability, optimal premium decreases by 11.44 (5.83) percent and the net present value by 44.67 (24.42) percent. Hence, the relative net present value reduction is even higher than in the linear case, although the relative reduction of the optimal premium is lower. Under a default probability of 0.01 percent, comparing one-premium and three-premium level, the net present value increases by 65.03 percent. Moreover, for 0.5 (1) percent default probability the net present value raises by 63.47 (61.86) percent between one-premium and three-premium level. This is more than in the linear case. With a higher default probability, $\Phi(d_1)$ and $\Phi(d_2)$ increase since the ratio of asset and liabilities raises. In addition, considering the examined default probabilities, optimal demand for underwriting net present value maximization at most differs from optimal demand under overall net present value maximization by 18.75 percent. As a result of convexity, the impact of EC_0 is stronger than in the linear case. Figure 5 visualizes the development of net present value with heterogeneous premiums and default probability.

Table 4: Convex Case: Net Present Value in Dependency of Default Probability and Premium

π^{level}	DP^A (in %)	Premium	x^N (in mio)	$\Phi(d_1)$	$\Phi(d_2)$	$NPV(EC_1 - EC_0)$ (in mio)
1	0.01	1727.29	0.858	0.9989	0.9962	624.46
2	0.01	1727.29	0.858	0.9903	0.9735	829.77
		1302.85	0.661			
3	0.01	2274.28	0.370	0.9933	0.9808	1030.52
		1727.29	0.488			
		1302.85	0.661			
1	0.5	1626.51	0.747	0.9993	0.9972	471.96
2	0.5	1626.51	0.747	0.9923	0.9782	623.39
		1263.02	0.552			
3	0.5	2081.15	0.328	0.9944	0.9835	771.53
		1626.51	0.419			
		1263.02	0.552			
1	1	1529.73	0.640	0.9996	0.9982	345.54
2	1	1529.73	0.640	0.9946	0.9840	453.49
		1224.19	0.452			
3	1	1902.71	0.285	0.9959	0.9873	559.28
		1529.73	0.355			
		1224.19	0.452			

For the linear as well as the convex case, $\Phi(d_1)$ and $\Phi(d_2)$ are close to zero as a result of the high asset-liability-ratio which is necessary to be able to fulfill the solvency constraint. For the cost shift, the difference between $\Phi(d_1)$ and $\Phi(d_2)$ increases and hence affects the net present value.

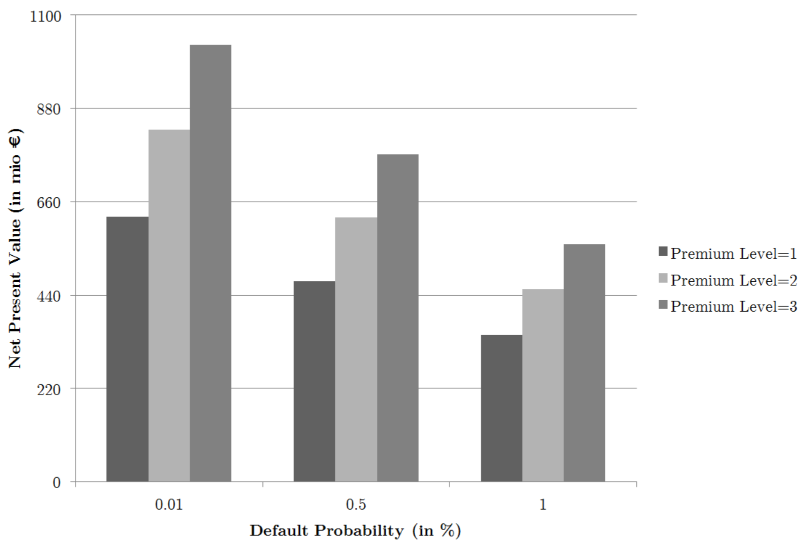

Figure 5: Convex PDP under Perfect Expectations: NPV and Premium Levels

4.2 Scenario under Cost Shift

In this subchapter, we will focus on a scenario under cost shift. In the scenario under perfect expectations, we have described the situation as stable over time. Hence, the past can be used as guide for the future. However, our business world and its environment are changing rapidly. Therefore, the future is widely unstable. We analyze how wrong estimations of the expected costs influence our asset liability model. Thus, we underlie a deterministic shift of the costs and examine the effect on the net present value of heterogeneous premiums. We assume that the deterministic shift ξ is increasing the costs per x^N by 50 percent. All other input parameters remain the same as before.

Linear Price-Demand Function

At first, we analyze how the net present value is affected by the deviation of the expected claims and afterwards we take the solvency requirement into consideration. Figure 6 clarifies how the net present value is influenced by the change of the costs in dependency of default probability and premium level. For the two-premium level, we consider both the higher and lower premium adaptation. Instead, in Figure 4, we focused on the one which results in the higher net present value. In the case under perfect expectations, the net present value of higher and lower additional premium varies only slightly. In contrast, in this case, we want to illustrate that choosing a different premium level is affecting the net present value substantially.

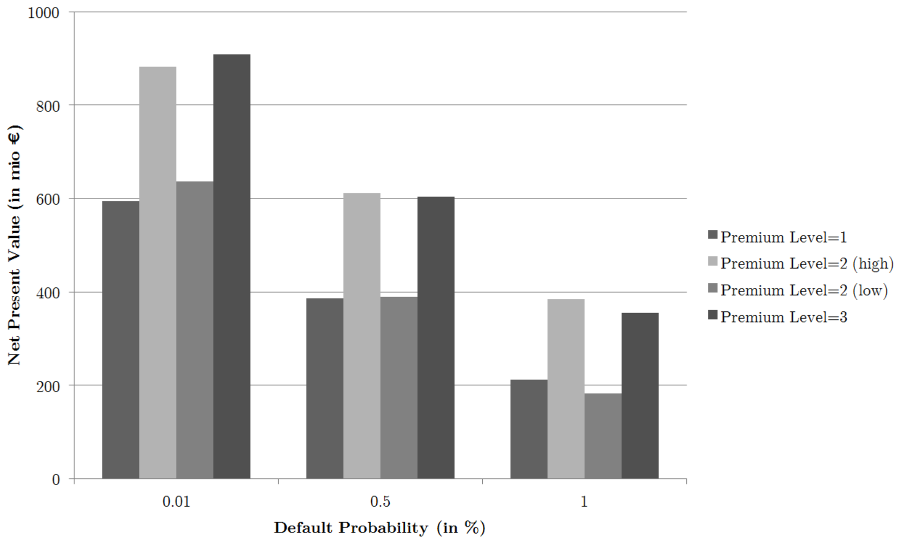

Figure 6: Linear PDF under Cost Shift: NPV and Premium Levels

As a result of increasing claims, it is intuitive that each insurance contract, which is sold below such claims, is connected with a decline of net present value. However, considering this relation, at first glance it might be surprising that even a lower premium adaptation where $\pi^{low} < E(c_1(DP^A))$ results in our analysis in a higher net present value for a default probability of 0.01 (0.5) percent. We can explain this development with the difference between $\Phi(d_1)$ and $\Phi(d_2)$. For $\sigma > 0$, $\Phi(d_1)$ is greater than $\Phi(d_2)$ and hence the assets are stronger weighted than the liabilities.[10] Moreover, a lower premium adaptation increases the gap between $\Phi(d_1)$ and $\Phi(d_2)$. Hence, as illustrated in Figure 6, the raising gap between $\Phi(d_1)$ and $\Phi(d_2)$ overcompensates $\pi^{low} < E(c_1(DP^A))$. Nevertheless, this relation does not necessarily underlie. With an increasing σ and a lower ratio of $\frac{A_0(DP^A,x^N)}{L_0(DP^A,x^N)}$, the difference between $\Phi(d_1)$ and $\Phi(d_2)$ raises and vice versa (see Appendix B). Furthermore, we recognize for a default probability of 0.5 percent, although both two-premium levels lead to a higher net present value than a homogeneous premium, three premiums are worse than two-premium level (high). This effect is induced by the increasing asset-liability-ratio in comparison to the two-premium level (low). Under 0.01 (0.5, 1) percent default probability, choosing the higher premium adaptation for the two-premium level is connected with a positive deviation of the net present value in relation to the lower premium adaptation by 245.15 (222.99, 202.01) million. Besides, three-premium level is the best alternative for a default probability of 0.01 percent and for 0.5 (1) percent default probability, the higher premium adaptation should be realized.

Convex Price-Demand Function

The net present value deviation under 0.01 (0.5, 1) percent default probability between π^{high} and π^{low} is 264.87 (226.30, 191.49) million and decreases with higher default probability. In addition, for a convex

[10]For a very low standard deviation, $\Phi(d_1)$ and $\Phi(d_2)$ are approximately equal (see Appendix B).

price-demand function, π^{low} generates higher demand under stable conditions than higher premium levels (three-premium case). Hence, a lower premium adaptation is under a convex price-demand function riskier than under a linear price-demand function and leads to a higher vulnerability against increasing costs. We reach for 1 percent default probability and a lower premium adaptation a negative net present value. π^{high} consequently leads to the best results under cost shift. Figure 7 illustrates that it is possible to overcompensate 0.5 percent default probability when choosing the right heterogeneous premiums.

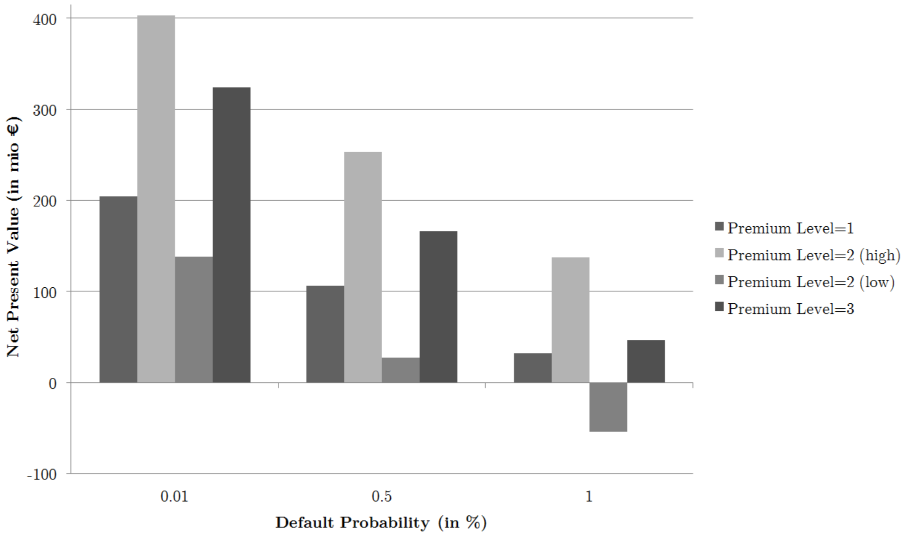

Figure 7: Convex PDF under Cost Shift: NPV and Premium Levels

In summary, the underlying example and the implications clarify that under E^Q-conditions heterogeneous premiums improve the net present value when perfect expectations underlie and are resilient against a cost shift when choosing the right heterogeneous premiums.

The following inherent problem underlies for net present value maximizing insurance enterprises. In our scenario under perfect expectations, an additional lower premium π^{low} leads to a higher net present value than an additional higher premium π^{high}. Hence, if only two-premium levels are feasible, each insurer has the incentive to realize the additional lower premium. However, this premium is connected with the risk that the conditions change to a scenario under cost shift and thus a higher premium adaptation would be substantially better. When one insurance enterprise decides to ignore the potential risk of lower premiums and realizes the profit, there would emerge a competitive advantage for this company as long as perfect expectations are present. In consequence, all other insurance companies are driven to do the same to strengthen the own market position relative to others. However, when the scenario switches from perfect expectations to cost shift and the market participants are behaving as illustrated, an unfavorable outcome results. This theoretical structure of action is known as a prisoner's dilemma[11] (see Nash, 1951).

Solvency Requirement

As we have demonstrated, a higher default probability reduces the net present value substantially under the assumptions taken in this paper. However, a solvency requirement, as presented in this paper, is useful to minimize the possibility that the actual default probability is higher than the reported default probability. In fact, solvency requirements restrict the risky behavior of an insurance company and enable that an insurance enterprise finds an appropriate balance between return and risk. Thus, the costumers are protected as a result of the constraints (see also Eling et al., 2007). Apart from this theoretical use of solvency requirements, we will consider in a next step whether our solvency constraint is fulfilled under perfect expectations and for the cost shift. In this context, we emphasize that a deviation between the expectations of claims over time is critical for the usefulness of solvency requirements. We visualize the reported default probabilities and the actual default probabilities in the following tables. If the reported default probability DP^A is smaller than the actual default probability in the linear (convex) case DP^{lin} (DP^{con}), the solvency requirement is violated.

Table 5: Perfect Expectations: Reported and Actual Default Probability in Percent

π^{level}	1	2(high)	2(low)	3	1	2(high)	2(low)	3	1	2(high)	2(low)	3
DP^A	0.01	0.01	0.01	0.01	0.5	0.5	0.5	0.5	1	1	1	1
DP^{lin}	0.15	0.08	0.61	0.39	0.19	0.11	0.78	0.53	0.21	0.13	0.92	0.65
DP^{con}	0.11	0.06	0.99	0.68	0.08	0.05	0.79	0.57	0.05	0.03	0.55	0.42

Table 6: Cost Shift: Reported and Actual Default Probability in Percent

π^{level}	1	2(high)	2(low)	3	1	2(high)	2(low)	3	1	2(high)	2(low)	3
DP^A	0.01	0.01	0.01	0.01	0.5	0.5	0.5	0.5	1	1	1	1
DP^{lin}	2.47	1.54	6.71	4.92	2.91	1.95	7.89	6.06	3.21	2.30	8.84	6.96
DP^{con}	1.96	1.25	9.25	7.19	1.52	1.05	7.94	6.38	1.04	0.76	6.24	5.18

Derived from Table 5, we receive that the solvency constraint is not fulfilled for the reported default probability of 0.01 percent under linear as well as convex price-demand function. Moreover, several lower premium adaptations and three-premium level also do not satisfy the constraint. Hence, the lower premium adaptation shows a higher vulnerability concerning the solvency constraint. Our solvency constraint exhibits a strong focus on the inherent threat of lower premium adaptation. For the cost shift, only under 1 percent default probability, the convex case satisfies the constraint for the higher premium adaptation. In all other cases, the solvency requirement is violated. A positive aspect which we derive

[11] A prisoner's dilemma stems from the non-cooperative behavior of market participants and collectively leads to an inappropriate outcome (see Nash, 1951).

is that solvency requirements restrict the risk to a certain degree and define a solid maximal default probability.

However, when we consider that the expected costs in $t = 1$ are not known from the point of time $t = 0$, the information that several premium-default combinations are not in line with the solvency condition is not influencing the previous decisions of the insurer in $t = 0$. The decision of the insurer bases on the expectation about the costs in $t = 1$ from the point of view of $t = 0$. Hence, only the default probability premium combinations, which are in conflict with the solvency constraint from the perspective of $t = 0$, affect the insurer's behavior. This insight clarifies that an unexpected shock is influencing the costs of the insurance company, while the solvency constraint does not have an appropriate answer because it is based on the expectation of $t = 0$. Of course, the future is only to a certain degree predictable. Therefore, a deviation between expectations is possible and probable. Including this fact in an internal solvency framework, stress-testing is an appropriate tool to analyze a miscalculation of distribution parameters.

False estimations of costs are often induced by external impacts, e.g., natural disasters (see, e.g., Lewis and Murdock, 1996; Harrington, 2009), political risks (see, e.g., Braun and Fischer, 2016; MIGA, 2014), or cyber attacks (see, e.g., World Economic Forum, 2016; Biener et al., 2015). Thus, it is important to develop an integrated approach, which includes the external development, such as a market analysis, in the internal solvency considerations. Furthermore, it is necessary to develop early warning systems (see, e.g., Brockett et al., 1994) and directly implement changes of the business environment in the solvency constraints to minimize miscalculations. Moreover, forward looking models, which anticipate future outcomes and focus on liability risk drivers, should be implicated (see SwissRe, 2016). Nevertheless, solvency requirements are still a framework of action which does not guarantee a successful development of an insurance enterprise. In addition, for instance, loss reserves (see, e.g., Dus and Maurer, 2001) are necessary to be able to pay the claims in $t = 1$, but an increase of loss reserves over a level, which is defined by the solvency constraint, to compensate a potential raise of claims, does not solve the problem of uncertainty and also leads to a lockup of more capital than needed under perfect expectations. Instead, it is more senseful to focus on the origins of deviations between expected costs in $t = 0$ and $t = 1$.

In summary, we have shown in our numerical example that the net present value is decreasing substantially with a higher default probability. Under perfect expectations heterogeneous premiums can compensate such a net present value reduction to a certain degree and thus heterogeneous premiums are profitable under stable conditions. When a cost shift underlies, a higher premium adaptation still improves the net present value, where a lower premium adaptation is only beneficial when premium greater costs or $\pi^{low} < E(c_1(DP^A))$ is overcompensated by the difference of $\Phi(d_1)$ and $\Phi(d_2)$. Solvency requirements define a solid maximum of default probability, however, in dynamic environments it is possible that the expected claims (based on the past), which are part of the solvency constraint, do not represent the reality. Hence, the solvency requirement does not or only delayingly protect for dramatic changes.

5 Economic Implications

5.1 Default Probability: Immanent Threat or Potential for Insurance Companies?

As we have defined before, a probabilistic insurance can either be induced by the insurance company itself, which sells insurance contracts with a payoff probability of lower than one, or an inherent default probability (e.g., caused by systemic risks) results in a probabilistic insurance. Our previous analysis clarifies that the risk-averse behavior of policyholders does not provide an incentive for the insurance company to structure insurance contracts in a way that a payoff with lower than one is existent (see also Wakker et al., 1997; Zimmer et al., 2009, 2016). Therefore, insurance companies always try to minimize the underlying default probability as a result of the enormous decrease of the willingness to pay of the policyholders. However, in the systemic financial world an inherent default probability is present and hence has to be reported. If the policyholders are aware of such a default probability, they typically adapt their willingness to pay. As we have shown with our numerical example, the potential premium, which can be generated under default probability, is substantially lower than under non-default. Thus, a higher default probability reduces the profit that can be reached through insurance contracts. Hence, an increasing default probability induced, e.g., by unstable conditions under which businesses operate, depicts an immanent threat for insurance enterprises.

In contrast, our asset liability model also expounds that if a perceived default probability is existent, there results the potential to improve the premiums in consequence of the high willingness to pay for a reduction of default probability and striving for certainty (see also Tversky and Kahneman, 1979, 1992). Therefore, for instance, the insurance industry has the incentive to offer additional insurance contracts which pay in the case when the first insurance contract does not pay the claims. Of course, also the second insurance exhibits an inherent default probability. Nevertheless, striving for certainty of the policyholders combined with the high willingness to pay elucidate the profitability of this approach.

In practice, according to the default probability, insurance contracts are very inflexible over time. More precisely, when there underlies at $t = 0$ a reported default probability DP^A and this default probability changes over time to DP^B, it would be consequent to adapt the premium to DP^B. However, this is not common in the insurance industry, although the default probability represents an important parameter of the insurance contract. In other words, policyholders gain from a decrease of default probability over time (after they have signed the contract) and are disadvantaged when the default probability increases over time (if such changes have not been rightly anticipated by the policyholders). Even if an insurance contract is not affected by a changing default probability, the behavior of policyholders is influenced by such a variation. For instance, when the default probability increases over time, some policyholders intend to switch the insurer as a consequence of a lower willingness to pay for the underlying default probability.

5.2 Heterogeneous Premiums: A Way for Customer Centricity

In fact, insurance customers are heterogeneous and heterogeneous customers consequently exhibit a heterogeneous willingness to pay even when homogeneous risks underlie (see also Zimmer et al., 2016). Hence, taking into consideration that such a heterogeneity is omnipresent, because of three reasons, it is suboptimal for an insurance enterprise to use a homogeneous premium. First, a homogeneous premium is ignoring the higher willingness to pay of a bundle of customers. Thus, it exists a gap between the willingness to pay and the actual acquired premium. Second, individuals with a lower willingness to pay than the homogeneous premium do not have the possibility to buy an insurance contract. Third, which is derived from the two reasons above, the customer is not in focus. Instead, a decoupling between the behavior of the customer and the insurance company is present.

In contrast, heterogeneous premiums minimize the gap between higher willingness to pay and actual acquired premium, enable individuals with a lower willingness to pay to buy the insurance, which results in a higher demand, and therefore focus on the customer. Hence, heterogeneous premiums lead to a shift of the mindset of an insurance enterprise. The insurance enterprise is adapting the insurance premium to the willingness to pay and not policyholder's willingness to pay to the insurance premium (which is only possible downwards). As a result, customer centricity enables the insurance company to increase its profits measured by the net present value. As we have stated before, the customer centricity has to be within the profitable restrictions under which the business operates.

According to the practicability of heterogeneous premiums in practice, it might be challenging at first glance. Given a set of different premiums for a homogeneous insurance product, we do not question that an individual which acts under bounded rationality (see Simon, 1957) prefers the one with the lowest premium because each individual behaves rationally within his or her limitations. However, we see a high potential for the practicability which stems from the limitations of each individual combined with the fact that individuals accept satisfying results (see Simon, 1955). In other words, it is possible for the insurer to realize premiums which are substantially higher than the fair premium. In combination with the limitation that no potential policyholder has perfect information, heterogeneous premiums are applicable. Hence, it results a collective improvement for insurance companies through heterogeneous premiums. Furthermore, our approach clarifies that no insurance company has an incentive to maximize the transparency of the insurance market. A higher level of transparency restricts the realization of heterogeneous premiums and therefore the potential profit. However, when we have a closer look to other industries, such as food industry[12], heterogeneous prices even work when there underlies a high degree of perceived transparency.

Another point why we hold the view that heterogeneous premiums are possible to implement is the increasing relevance of the digitization. Each potential policyholder has a digital footprint and such data are useful to improve the understanding about the consumer behavior and thus the willingness to pay.

[12]We refer, e.g., to a homogeneous product which is sold in the supermarket for heterogeneous prices, which is very common in our daily life.

6 Summary and Conclusion

In this paper, we develop an asset liability model which includes the interaction of default probability, price-demand function, and insurance premiums. Therefore, we do not consider the premium as a constant factor which is ex ante given and the insurer is not a pure price taker. Instead, different strategies to generate underwriting profit, from the relation of premiums minus costs, are influencing the overall net present value of an insurer enormously. In this context, we introduce heterogeneous premiums in consequence of policyholders' heterogeneous willingness to pay and compare the results with the outcome under a homogeneous premium. Furthermore, we separate between higher and lower premium adaption in regard to heterogeneous premiums. Moreover, we include an internal solvency restriction in our model. The numerical example illustrates the net present value development under homogeneous and heterogeneous premiums with varying default probabilities when perfect expectations and a cost shift are underlying.

Our findings include several economic implications. First, an increase of default probability decreases the willingness to pay substantially and therefore restricts the potential to improve the return on capital. An inherent default probability is consequently existing as a result of our systemic financial world. From the initial point that such an inherent default probability is present, policyholders' striving for certainty can be regarded as threat but also as potential. While an increase of default probability describes the threat, taking the high willingness to pay for a default probability reduction into consideration clarifies the potential. For instance, it results an incentive to offer additional insurance contracts which pay in the case when the first insurance contract does not pay the claims.

Second, we illustrate that heterogeneous premiums are useful to improve the net present value of an insurer under linear as well as convex price-demand function. It is inadequate from the perspective of the insurance company that policyholders adapt their heterogeneous willingness to pay to a homogeneous premium (which is only downwards possible). Instead, adapting the premium level to the heterogeneous willingness to pay is appropriate. We further emphasize that such a heterogeneous premium adaptation is supported by limitations of the customers (e.g., missing information) and the acceptance of satisfying results. While in our scenario under perfect expectations a higher and lower premium adaptation are profitable, under cost shift a lower premium adaptation is only profitable when the premium is higher than the costs or the effect of premium smaller costs is overcompensated by the difference of the standard normal distribution functions. Moreover, a higher premium adaptation is still beneficial.

Third, we recognize that an internal solvency constraint has to be structured as a forward looking approach, which anticipates the future to identify liability risk drivers, and to minimize the gap between cost expectations over time. An integrated approach is in this context useful to combine internal and external insights. Fourth, the optimal demand under overall net present value maximization varies from the net present value maximization of an isolated premium minus costs consideration. Hence, in practice, an overall perspective should be in focus to reach the best results from the company's point of view.

7 Appendix

Appendix A: Overall and Underwriting Net Present Value

Linear Price-Demand Function

$$\begin{aligned} NPV^w(EC_1(DP^A, x^N) - EC_0) &= PV^w(EC_1(DP^A, x^N)) - EC_0 \\ &= A_0(DP^A, x^N) \cdot \Phi(d_1) - L_0^w(DP^A, x^N) \cdot \Phi(d_2) - EC_0 \\ &= [EC_0 + (\pi_r - \theta \cdot x^N) \cdot exp(-23.75 \cdot DP^A) \cdot x^N] \cdot \Phi(d_1) \\ &- exp(-r)(1 - DP^A)E(c_1^0) \cdot x^N \cdot \Phi(d_2) - EC_0 \end{aligned}$$

where

$$d_1 = \frac{ln(\frac{A_0(DP^A, x^N)}{L_0^w(DP^A, x^N)}) + \frac{\sigma^2}{2}}{\sigma} = \frac{ln(\frac{EC_0 + (\pi_r - \theta \cdot x^N) \cdot exp(-23.75 \cdot DP^A) \cdot x^N}{exp(-r)(1-DP^A)E(c_1^0) \cdot x^N}) + \frac{\sigma^2}{2}}{\sigma},$$

$$d_2 = d_1 - \sigma; \sigma = \sqrt{\sigma_A^2 + \sigma_L^2 - 2\rho \cdot \sigma_A \cdot \sigma_L}.$$

$$\begin{aligned} NPV(E(P_r(DP^A, x^N))) &= PV(E(P_r(DP^A, x^N))) \\ &= (\pi_r - \theta \cdot x^N) \cdot exp(-23.75 \cdot DP^A) \cdot x^N \cdot \Phi(d_1) - exp(-r)\pi_a \cdot x^N \cdot \Phi(d_2), \end{aligned}$$

where

$$\pi_a = (1 - DP^A)E(c_1^0); d_1 = \frac{ln(\frac{(\pi_r - \theta \cdot x^N) \cdot exp(-23.75 \cdot DP^A) \cdot x^N}{exp(-r)\pi_a \cdot x^N}) + \frac{\sigma^2}{2}}{\sigma}$$

Convex Price-Demand Function

$$\begin{aligned} NPV^w(EC_1(DP^A, x^N) - EC_0) &= PV^w(EC_1(DP^A, x^N)) - EC_0 \\ &= A_0(DP^A, x^N) \cdot \Phi(d_1) - L_0^w(DP^A, x^N) \cdot \Phi(d_2) - EC_0 \\ &= [EC_0 + \frac{\pi_r}{\eta \cdot x^N + 1} \cdot \exp(-23.75 \cdot DP^A) \cdot x^N] \cdot \Phi(d_1) \\ &- exp(-r)(1 - DP^A)E(c_1^0) \cdot x^N \cdot \Phi(d_2) - EC_0 \end{aligned}$$

where

$$d_1 = \frac{ln(\frac{A_0(DP^A, x^N)}{L_0^w(DP^A, x^N)}) + \frac{\sigma^2}{2}}{\sigma} = \frac{ln(\frac{EC_0 + \frac{\pi_r}{\eta \cdot x^N + 1} \cdot \exp(-23.75 \cdot DP^A) \cdot x^N}{exp(-r)(1-DP^A)E(c_1^0) \cdot x^N}) + \frac{\sigma^2}{2}}{\sigma}.$$

$$NPV(E(P_r(DP^A, x^N))) = PV(E(P_r(DP^A, x^N)))$$
$$= \frac{\pi_r}{\eta \cdot x^N + 1} \cdot \exp\left(-23.75 \cdot DP^A\right) \cdot x^N \cdot \Phi(d_1) - exp(-r)\pi_a \cdot x^N \cdot \Phi(d_2),$$

where

$$\pi_a = (1 - DP^A)E(c_1^0); d_1 = \frac{ln(\frac{\frac{\pi_r}{\eta \cdot x^N + 1} \cdot \exp\left(-23.75 \cdot DP^A\right) \cdot x^N}{exp(-r)\pi_a \cdot x^N}) + \frac{\sigma^2}{2}}{\sigma}.$$

Appendix B: Standard Normal Distribution, Asset Liability Ratio, and Standard Deviation

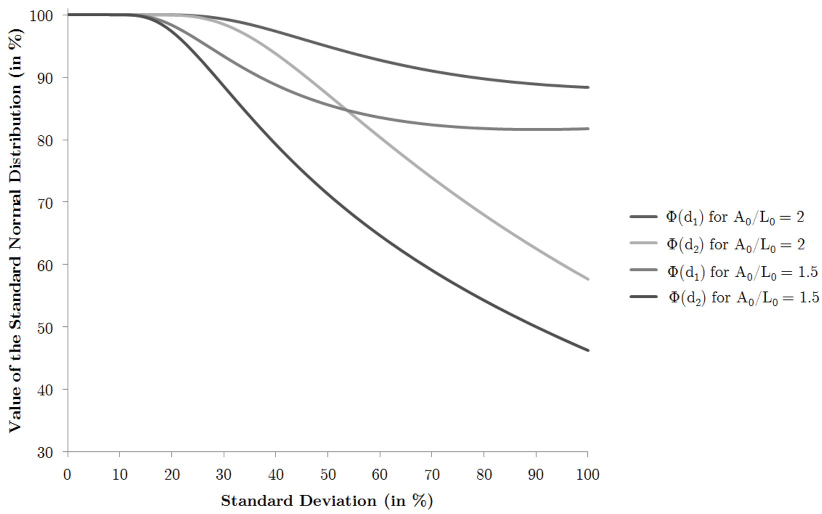

Figure 8: Influence of Asset Liability Ratio and Standard Deviation

References

Basak, S. (2005). Asset Pricing with Heterogeneous Beliefs, *Journal of Banking and Finance* **29**(11): 2849–2881.

Biener, C., Eling, M. and Wirfs, J. (2015). The Insureability of Cyber Risks: An Empirical Analysis, *Geneva Papers on Risk and Insurance* **40**(1): 131–158.

Björk, T. (2009). *Arbitrary Theory in Continuous Time*, 3rd edn, Oxford University Press, New York.

Braun, A. and Fischer, M. (2016). Determinants for the Demand of Political Risk Insurance: Evidence from an International Survey, *Working Paper* (August).

Braun, A., Schmeiser, H. and Schreiber, F. (2017). Portfolio Optimization under Solvency II: Implicit Constraints Imposed by the Market Risk Standard Formula, *Journal of Risk and Insurance* **84**(1): 177–207.

Braun, A., Schmeiser, H. and Siegel, C. (2014). The Impact of Private Equity on a Life Insurer's Capital Charges under Solvency II and the Swiss Solvency Test, *Journal of Risk and Insurance* **81**(1): 113–158.

Brav, A., Constantinides, G. M. and Geczy, C. C. (2002). Asset Pricing with Heterogeneous Consumers and Limited Participation: Empirical Evidence, *Journal of Political Economy* **110**(4): 793–824.

Brockett, P. L., Cooper, W. W., Golden, L. L. and Pitaktong, U. (1994). A Neural Network Method for Obtaining an Early Warning of Insurer Insolvency, *Journal of Risk and Insurance* **61**(3): 402–424.

Brownlees, C. and Engle, R. (2016). SRISK: A Conditional Capital Shortfall Index for Systemic Risk Measurement, *Review of Financial Studies* (forthcoming).

Caginalp, G. (2005). Nonlinear Price Evolution, *Quarterly Of Applied Mathematics* **63**(4): 715–720.

Choi, J. and Kronlund, M. (2015). Reaching for Yield by Corporate Bond Mutual Funds, *Working Paper* (October).

Constantinides, G. M. and Duffie, D. (1996). Asset Pricing with Heterogeneous Consumers, *Journal of Political Economy* **104**(2): 219–240.

Crocker, K. J. and Snow, A. (2013). The Theory of Risk Classification, *in* G. Dionne (ed.), *Handbook of Insurance*, Springer, New York et al., pp. 281–313.

Davidson, P. (1991). Is Probability Theory Relevant for Uncertainty? A Post Keynesian Perspective, *Journal of Economic Perspectives* **5**(1): 129–143.

Di Maggio, M. and Kacperczyk, M. (2017). The Unintended Consequences of the Zero Lower Bound Policy, *Journal of Financial Economics* **123**(1): 59–80.

Dus, I. and Maurer, R. (2001). Integrated Asset Liability Modelling for Property Casuality Insurance: A Portfolio Theoretical Approach, *Working Paper* (October).

Eckert, J. and Gatzert, N. (2017). Risk- and Value-Based Management for Non-Life Insurers under Solvency Constraints, *Working Paper* (January).

Einav, L. and Finkelstein, A. (2011). Selection in Insurance Markets: Theory and Empirics in Pictures, *Journal of Economic Perspectives* **25**(1): 115–138.

Eling, M., Gatzert, N. and Schmeiser, H. (2009). Minimum Standards for Investment Performance: A new Perspective on Non-Life Insurer Solvency, *Insurance: Mathematics and Economics* **45**(1): 113–122.

Eling, M., Schmeiser, H. and Schmit, J. T. (2007). The Solvency II Process: Overview and Critical Analysis, *Risk Management and Insurance Review,* **10**(1): 69–85.

Ellsberg, D. (1961). Risk, Ambiguity and the Savage Axiom, *Quarterly Journal of Economics* **75**(4): 643–669.

Fischer, S. (1978). Call Option Pricing When the Exercise Is Uncertain, and the Valuation of Index Bonds, *Journal of Finance* **33**(1): 169–176.

Gatzert, N., Hoermann, G. and Schmeiser, H. (2012). Optimal Risk Classification with an Application to Substandard Annuities, *North American Actuarial Journal* **16**(4): 462–486.

Harrington, S. E. (2009). The Financial Crisis, Systemic Risk, and the Future of Insurance Regulation, *Journal of Risk and Insurance* **76**(4): 785–819.

Holmström, B. (1979). Moral Hazard and Observability, *Bell Journal of Economics* **10**(1): 74–91.

Kahane, Y. (1977). Determination of the Product Mix and the Business Policy of an Insurance Company-A Portfolio Approach, *Management Science* **23**(10): 1060–1069.

Kahane, Y. and Nye, D. (1975). A Portfolio Approach to the Property-Liability Insurance Industry, *Journal of Risk and Insurance* **42**(4): 579–598.

Kriele, M. and Wolf, J. (2014). *Value-Oriented Risk Management of Insurance Companies*, Springer, London et al.

Lewis, C. M. and Murdock, K. C. (1996). The Role of Government Contracts in Discretionary Reinsurance Markets for Natural Disasters, *Journal of Risk and Insurance* **63**(4): 567–597.

Lorson, J., Schmeiser, H. and Wagner, J. (2012). Evaluation of Benefits and Costs of Insurance Regulation – A Conceptual Model for Solvency II, *Journal of Insurance Regulation* **1**: 125–156.

Margrabe, W. (1978). The Value of an Option to Exchange One Asset for Another, *Journal of Finance* **33**(1): 177–186.

MIGA (2014). *World Investment and Political Risk 2013*, The World Bank, Washington D. C.

Nash, J. (1951). Non-Cooperative Games, *Annals of Mathematics* **58**(1): 182–191.

Pindyck, R. and Rubinfeld, D. (2008). *Microeconomics*, 7th edn, Pearson Prentice Hall, New York.

Rothschild, M. and Stiglitz, J. (1976). Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information, *Quarterly Journal of Economics* **90**(4): 629–649.

Rothschild, M. and Stiglitz, J. E. (1997). Competition and Insurance Twenty Years Later, *Geneva Papers on Risk and Insurance Theory* **22**: 73–79.

Rymaszewski, P., Schmeiser, H. and Wagner, J. (2012). Under What Conditions Is an Insurance Guaranty Fund Beneficial for Policyholders?, *Journal of Risk and Insurance* **79**(3): 785–815.

Schlütter, S. (2014). Capital Requirements or Pricing Constraints?, *Journal of Risk Finance* **15**(5): 533–554.

Simon, H. A. (1955). A Behavioral Model of Rational Choice, *Quarterly Journal of Economics* **69**(1): 99–118.

Simon, H. A. (1957). *Models of Man: Social and Rational-Mathematical Essays on Rational Human Behavior in a Social Setting*, Wiley, New York.

SwissRe (2016). Liability Risk Drivers: Bringing a Forward-Looking Perspective into Liability Modelling, *Technical report*, SwissRe.

Tversky, A. and Kahneman, D. (1979). Prospect Theory: An Analysis of Decision under Risk, *Econometrica* **47**(2): 263–291.

Tversky, A. and Kahneman, D. (1992). Advances in Prospect-Theory - Cumulative Representation of Uncertainty, *Journal of Risk and Uncertainty* **5**(4): 297–323.

Tversky, A. and Wakker, P. (1995). Risk Attitudes and Decision Weights, *Econometrica* **63**(6): 1255–1280.

Wakker, P. P., Thaler, R. H. and Tversky, A. (1997). Probabilistic Insurance, *Journal of Risk and Uncertainty* **15**: 7–28.

World Economic Forum (2016). The Global Risks Report 2016 11th Edition, *Technical report*, World Economic Forum.

Zimmer, A., Gruendl, H., Schade, C. D. and Glenzer, F. (2016). An Incentive-Compatible Experiment on Probabilistic Insurance and Implications for an Insurer's Solvency Level, *Journal of Risk and Insurance* (forthcoming).

Zimmer, A., Schade, C. and Gründl, H. (2009). Is Default Risk Acceptable when Purchasing Insurance? Experimental Evidence for Different Probability Representations, Reasons for Default, and Framings, *Journal of Economic Psychology* **30**(1): 11–23.