Risk and Solvency Assessment of the Life Insurer: An Examination of the Interest-Sensitive Life Insurance Policies

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Abstract

In this study we examine the fair pricing of interest rate sensitive life insurance policies that are commonly sold in Taiwan. With the reference portfolio following Heston's stochastic volatility process, the payoff function of these policies consists of a series of forward-start options and a Bermudan option with a six-year lock-up period. Although the option to surrender are standard features of these policies, policyholders incur heavy penalties should they exercise such option. Our goal is also to obtain the fair pricing of the surrender option under the conditions as set out by the legislation and the Financial Supervisory Commission. Given certain policyholder behaviour, we study the impact of the surrender option, the minimum guaranteed interest rate, and the annually declared bonus rate on the issuing company's solvency.

Keywords: stochastic volatility, surrender option, default risk, interest rate guarantees.

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1 Introduction

The sharp decline of interest rates and certain events in the financial markets had elicited discussions and attention on the management, fair valuation, and default risks of interest sensitive life insurance policies, where most of these type of policies offer an explicit interest rate guarantee that the policyholder's account will be credited on an annual basis, together with any excess return from the reference portfolio. In the case of participating life policies, this excess return may be in the form of a certain percentage, say 70% of the return on the reference portfolio or in the case of interest sensitive life policies, the positive difference between the return on the reference portfolio and that of the guaranteed interest rate.

The study of this paper will focus on the life insurance policy with the highest annual gross premium income in Taiwan, namely, the interest sensitive life (ISL) insurance policy, where the minimum surrender conditions are dictated by law and its guidelines; however, our analysis can also be applied to similar insurance markets that offer life insurance policies with minimum interest rate guarantees and whose portfolio returns are benchmarked against interest rates. Given exceptional growth in new businesses written on ISLs and their share with respect to the entire product line of Taiwanese life insurers of 5%, 19%, and 44% in 2012, 2013, and 2014 respectively [1], it is imperative that the financial risks on these ISLs are fairly assessed.

The traditional life insurance policies in Taiwan had predominantly been participating or with-profits policies. Following the stock market drop after the financial crisis caused by the burst of the dot-com bubble and the sharp decline in the interest rate environment, from over 6% pre-2001 to under 2% in 2004 on the 2-year fixed deposit rate; life insurers were unable to meet the mandatory requirements, such as the 70% participating rate on excess profits to be distributed over the years on the participating policies. The Financial Supervisory Commission of Taiwan had ordered the life insurers to discontinue any sales of participating policies in 2004. Given the generally low interest rates and the investors' search for yield, the life insurers started offering ISL products with minimum guaranteed interest rates and bonuses that are much more competitive than fixed term deposits offered by the banks.

The observed behaviour of the market participants, the life insurer and the policyholder, is that the life insurer is in constant competition with the bank to offer competitive rates such that the policyholder would not exit the contract to enter

[1] According to the data from the Life Insurance Association of the Republic of China (LIAROC).

into more rewarding investment opportunities. This constant need to beat the market and the cost of the surrender option could potentially put the life insurer into ruin; whereas the policyholder's behaviour can be seen similar to that of an employee stock option (ESO) as described in Hull and White [2004], in the sense that (i) there is a *lock-up* period of 6 years, (ii) a penalty or surrender charge for exiting the contract during this period, (iii) it is probable that the policyholder will exercise the surrender option during this period, may it be personal liquidity reasons for example, and (iv) that the policyholder may choose to exit the contract after the lock-up period prior to maturity if the rate of return is lower than the fixed term deposit.

Extensive studies has been done on the fair valuation and risk management of life inurance policies with minimum interest rate guarantees, for example, Briys and de Varenne [1994, 1997], Bacinello [2001], Bacinello and Ortu [1996], Grosen and Jørgensen [2002], Jensen et al. [2001], Miltersen and Persson [2003], Tanskanen and Lukkarinen [2003], Barbarin and Devolder [2005], Bauer et al. [2006], Bernard et al. [2006], Gatzert and Kling [2007], Kling et al. [2007], and Graf et al. [2011] to cite a few. Examples of such life insurance policies with surrender options or early exercise features were considered and analyzed in Albizzati and Geman [1994], Grosen and Jørgensen [1997, 2000], Bacinello [2003a,b], Bacinello et al. [2009], Bernard et al. [2005], and more recently Le Courtois and Nakagawa [2013].

In this study we apply the arbitrage free pricing methodology to the ISL policies that are currently in-force in Taiwan and the conditions as set out by the Financial Supervisory Commission as a special case for the pricing of its surrender option. In particular, our reference portfolio follow a stochastic volatility process as described in Heston [1993]. As illustrated in Wilmott [2002], one would potentially underestimate the cost of the underlying when only constant volatility is considered. The payoff structure of the ISLs consists of an option of the cliquet type and a Bermudan option with a 6-year lock-up period.

2 The interest sensitive life policy

Although similar to participating policies at first glance, interest sensitive life insurance policies in Taiwan can generally be characterized as an endowment, whole life, or pension (retirement) with a guaranteed interest rate, namely the "headline rate" and the potential to earn in excess of the guaranteed rate. The shortest maturity currently seen on the endowment and retirement plans are 13 years, and the longest is 30 years. Currently these ISL policies also offer maturity or living benefits, death and funeral expense benefit, and disability benefit as standard policy packages.

The guaranteed interest rate is the minimum annual return set at the inception of the contract. This contractual guaranteed interest rate would remain fixed for the coverage period or the term of the contract. Any portfolio return with excess over the guaranteed interest rate would be credited to the policyholder's account. As such, these products with interest rate guarantees are not only sensitive to interest rate movements, but also the returns achievable under the prevailing market conditions.

3 The model framework

More recently, Gatzert and Schmeiser [2013] has provided a comprehensive overview of the different forms of traditional and innovative new life insurance products and Graf et al. [2012] in the methodologies in assessing these products that are commonly found in old-age provision products in practice.

Although the participating policies are not in issue anymore, however, they also offer minimum interest rate guarantees and similar in nature to ISL and given that the majority of the life insurance policies currently sold in Taiwan are ISLs, for the purpose of simplicity, we will categorise the existing participating policies as ISLs and assume that the balance sheet of a life insurance company would be a reflection of the assets and liabilities of the ISL. It is a statutory requirement that life insurance policies in Taiwan offer products with 100% guarantee of the contributions made. Hence, we do not consider products offering less or more than 100% guarantee.

3.1 The asset model

Policies underwritten by life insurers are often of a long-term nature, and it is the life insurer's obligation to manage its assets adequately to ensure its ability to meet future policyholder claims. It is not uncommon for life insurers to hold certain positions in fixed-income type assets and other risky assets on their balance sheet. We thus use the following three asset classes, bonds, stocks, and money market account in constructing a generic asset portfolio for the purpose of this study.

It is implied that the models and frameworks adopted in this paper are functions of time, t, unless otherwise stated. For example, we will drop the time index subscript and write $r = r_t$, and $Z_r = Z_{r,t}$ for notational convenience.

The value of the money market account M grows according to

$$\mathrm{d}M = rM\,\mathrm{d}t, \tag{1}$$

where r is the risk-free interest rate. The interest rate r follows the Cox et al. [1985] (CIR) process, and under the risk neutral probability measure, the interest rate dynamics of the CIR process is read as

$$\mathrm{d}r = \kappa_r(\theta_r - r)\,\mathrm{d}t + \sigma_r\sqrt{r}\,\mathrm{d}Z_r, \tag{2}$$

where $\kappa_r, \theta_r > 0$ represents the speed of adjustment and the long-run mean of the interest rate respectively, σ_r denotes the interest rate volatility, and Z_r denotes a Wiener process.

The bond price $B(t,T)$ at time t with maturity T in the CIR model is

$$B(t,T) = b_1(t,T)\exp\{-b_2(t,T)r\},$$

where (cf. Brigo and Mercurio [2006](3.25))

$$\begin{aligned}
b_1(t,T) &= \left(\frac{2he^{\left((\kappa_r+h)(T-t)\right)/2}}{2h + (\kappa_r + h)\left(e^{h(T-t)} - 1\right)}\right)^{\frac{2\kappa_r\theta_r}{\sigma_r^2}}, \\
b_2(t,T) &= \frac{2\left(e^{h(T-t)} - 1\right)}{2h + (\kappa_r + h)\left(e^{h(T-t)} - 1\right)}, \\
h &= \sqrt{\kappa_r^2 + 2\sigma_r^2}.
\end{aligned}$$

The differential form is written as

$$\frac{\mathrm{d}B(t,T)}{B(t,T)} = r\,\mathrm{d}t - b_2(t,T)\sigma_r\sqrt{r}\,\mathrm{d}Z_r. \tag{3}$$

We let the stock price follow a stochastic volatility process as described in Heston [1993]. Then under the risk-neutral measure, the process can be express as

$$\mathrm{d}S = \mu S\,\mathrm{d}t + \sqrt{v}S\,\mathrm{d}Z_S, \tag{4}$$

$$\mathrm{d}v = \kappa_v\left(\theta_v - v\right)\,\mathrm{d}t + \sigma_v\sqrt{v}\,\mathrm{d}Z_v, \tag{5}$$

where μ is the long-run mean or the drift process of the asset price, ν is the variance of the underlying asset price, which is a random variable. κ_v is the mean reversion speed for the variance. θ_v is the mean reversion level for the variance. The correlation coefficient between Z_S and Z_v is ρ, while Z_r is independent of Z_S and Z_v.

Following from the above, the reference asset portfolio A_t thus consists of bonds, stocks and money market account. We assume the life insurer invests a constant proportion of w_B in bonds, w_S in stocks, and the balance w_M in the money market account. These proportions are kept constant by continuous rebalancing, and $w_B + w_S + w_M = 1$. Let ϕ_B denote the number of units the life insurer holds in bonds, ϕ_S be the number of units held in stock and ϕ_M the number of units held in the money market account. This yields, $w_B = \frac{\phi_B B(t,T)}{A}$, $w_S = \frac{\phi_S S}{A}$ and $w_M = \frac{\phi_M M}{A}$. Thus we get $A = \phi_B B(t,T) + \phi_S S + \phi_M M$.

We further assume the life insurer's reference asset portfolio is self-financing, thus we obtain $\mathrm{d}A = \phi_B \mathrm{d}B(t,T) + \phi_S \mathrm{d}S + \phi_M \mathrm{d}M$. The dynamics of the reference asset portfolio, which is also the portfolio return r_A of the ISL can then be written as

$$r_A = \frac{\mathrm{d}A}{A} = w_B \frac{\mathrm{d}B(t,T)}{B(t,T)} + w_S \frac{\mathrm{d}S}{S} + w_M \frac{\mathrm{d}M}{M}. \tag{6}$$

3.2 The liability model

Let us consider a generic set up of an ISL product. The life insurer provides an annual minimum guaranteed interest rate for the term of the policy at inception; furthermore, at the policy's annual anniversary any excess return generated from its portfolio assets is distributed to its policyholders at the discretion of the life insurer's management. Thus, the annual portfolio return credited to the policyholders' account cannot be less than the guaranteed interest rate as stated in the contract. Any living or death benefit received would be the higher of the account value or a predetermined multiple of the initial premium paid. In practice, there is a "lock-in" period for these ISLs, and ranges between 6 to 10 years, where the surrender charge is much higher than the portfolio return credited. Which alternatively acted as a deterrent for early surrenders. This is not dissimilar to an European cliquet option with a maturity of T-years. The use of options to price corporate liabilities or life insurance contracts are not of a foreign nature, as can be seen in Black and Scholes [1973], Brennan and Schwartz [1976], Grosen and Jørgensen [1997, 2000, 2002], Bacinello [2001], and Bauer et al. [2006] etc. We thus use the fair valuation of a European cliquet option as our point of departure for the valuation of our liabilities.

Let P be the lump sum premium paid at the inception of the contract or policy and L be the liability at time t, such that $P = L_0$. r_P is the policy interest rate credited to the policy account in year t; it is determined at each of the valuation dates i, $i = 1, 2, \ldots, n$, as the lesser of the policy portfolio's asset return, r_A, in additional to any market related adjustments, ζ, and the policy portfolio's performance benchmark, K, plus any benchmark adjustments ξ. Define $r_{P,t} = \max\left(\min\left(K + \xi,\, r_{A,t} - \zeta\right),\, r_G\right)$ or $r_{P,t} = r_G + \max\left(\min\left(K + \xi,\, r_{A,t} - \zeta\right) - r_G, 0\right)$, then the policy interest rate and its relation with the liability can be expressed as

$$L_T = L_0 \left\{ 1 + \sum_{t=1}^{n} r_{P,t} \right\}. \tag{7}$$

$r_{P,t}$ is guaranteed to never fall below r_G, the guaranteed interest rate, which is specified in the policy contracts, and that $r_G \in [0.75\%, 1.50\%]$ as seen of the guaranteed interest rates currently declared amongst the life insurers in Taiwan. Depending on the life insurer's investment strategy, the benchmark K of these ISL products can be as short-dated as the 2-year fixed deposit rate to one with a longer term such as Taiwan's 10-year Government Bond. The market and benchmark adjustors $\zeta \in [0.00\%, 7.00\%]$ and $\xi \in [-3.00\%, 3.00\%]$ can vary considerably and are exercised at the management's discretion.

3.2.1 The liability reserve

In practice, the ISLs are subject to monthly valuations and bonus declarations, we thus implement this in our liability model set up. Let the value of the liability reserve (or the cost of writing an ISL) be p per unit dollar insured. Under the risk neutral pricing principle, this value of the bonus option at time 0 can be expressed as (cf. Graf et al. [2012])

$$\begin{aligned} p &= \mathsf{E}^Q \left\{ \exp\left(-\int_0^T r(\tau)\,\mathrm{d}\tau \right) \cdot \max\left[\min\left(K + \xi,\, r_{A,t} - \zeta\right),\, r_G \right] \right\} \\ &= \sum_{t=1}^{12T} \left\{ \mathsf{E}^Q \left[\exp\left(-\int_0^{\frac{t}{12}} r(\tau)\,\mathrm{d}\tau \right) \cdot r_{P,\frac{t}{12}} \right] \right\}. \end{aligned} \tag{8}$$

3.2.2 The bonus stabilization reserve

Since the value of bonus option is directly dependent on the investment performance in capital market, the policyholder has the right to claim from the life insurer at maturity, the life insurer would also need to assess its ability of providing for such terms. Introducing

$$r_{s,t} = \begin{cases} r_A - r_P & r_A \geqslant r_P, \\ r_A - r_G & r_A \leqslant r_G, \\ 0 & \text{otherwise} \end{cases}$$

we define p_s, the bonus stabilization reserve (BSR), as

$$p_s = \sum_{t=1}^{12T} \left\{ \mathsf{E}^Q \left[\exp\left(-\int_0^{\frac{t}{12}} r(\tau)\,\mathrm{d}\tau \right) \cdot r_{S,\frac{t}{12}} \right] \right\}. \tag{9}$$

For positive p_s, there exists a surplus after the distribution of the policy interest rate r_P given the level of r_G as set at the inception of the policy; for negative p_s the bonus stabilization reserve is in deficit.

4 Numerical simulation and illustration

4.1 Parameter estimation

All parameters appear in our models, namely the CIR process of interest rate and the Heston model of stock, should be estimated from actual market data before proceeding. The maximum likelihood estimation (MLE) method is applied to the parameter estimation problem of CIR process; the loss function approach is used for the Heston model.

4.1.1 Parameter estimation of the CIR process

The MLE method is taken from Iacus [2008], Kladıvko [2007]. Given the n observations of interest rate time series $\{r_{t_i}\}, i = 1, 2, \ldots, n$ at observation time t_i with equally spaced interval Δt, the likelihood function $F(\vartheta)$, $\vartheta \equiv (\hat{\kappa}_r, \hat{\theta}_r, \hat{\sigma}_r)$ is formed as

$$F(\vartheta) = \prod_{i=1}^{n} p(r_{t_{i+1}} | r_{t_i}; \vartheta), \tag{10}$$

where the conditional density $p(\cdot|\cdot)$ is (cf. Feller [1951])

$$p(r_{t_{i+1}}|r_{t_i};\vartheta) = c\, e^{-u-v} \left(\frac{v}{u}\right)^2 I_q(2\sqrt{uv}) \tag{11}$$

with

$$c = \frac{2\hat{\kappa}_r}{\hat{\sigma}_r{}^2(1-e^{-\hat{\kappa}_r\Delta t})} \qquad u = c\, r_{t_i} e^{-\kappa_r \Delta t}$$

$$v = c\, r_{t_{i+1}} \qquad q = \frac{2\hat{\kappa}_r\hat{\theta}_r}{\hat{\sigma}_r{}^2} - 1$$

and I_q, the modified Bessel function of the first kind and order q. The maximizer of $F(\vartheta)$ is the sought-after parameter set.

In practice one often use the iterative Newton-type algorithm to numerically optimize $\log F(\vartheta)$; the problems of inherent overflow in the modified Bessel function implementation and the proper selection of the initial value must be addressed. For the latter, Kladıvko [2007] suggests the ordinary least square method which runs as follows. Discretize the CIR process as

$$r_{t_{i+1}} - r_{t_i} = \kappa_r(\theta_r - r_{t_i})\Delta t + \sigma_r \sqrt{r_{t_i}}\, \epsilon_{t_i},$$

where ϵ is normally distributed with mean 0 and variance Δt. The above can be written as

$$\frac{r_{t_{i+1}} - r_{t_i}}{\sqrt{r_{t_i}}} = \frac{\kappa_r\, \theta_r}{\sqrt{r_{t_i}}} \Delta t - \kappa_r \sqrt{r_{t_i}} \Delta t + \sigma_r\, \epsilon_{t_i},$$

then the initial value $(\hat{\kappa}, \hat{\theta})$ is determined by

$$(\hat{\kappa}, \hat{\theta}) = \underset{\kappa,\theta}{\operatorname{argmin}} \sum_{i=1}^{n-1} \left(\frac{r_{t_{i+1}} - r_{t_i}}{\sqrt{r_{t_i}}} - \frac{\kappa\, \theta}{\sqrt{r_{t_i}}} \Delta t + \kappa \sqrt{r_{t_i}} \Delta t \right) \tag{12}$$

and the initial value of $\hat{\sigma}$ is derived as the standard deviation of the residuals after substituting $(\hat{\kappa}, \hat{\theta})$.

The data used in the calibration of the CIR process consist of 3,179 available daily observations on the 10-year Government Bond of Taiwan over the period of August 2002 to January 2015, as quoted from the Taiwan Economic Journal (TEJ) DataBank.

4.1.2 Parameter estimation of the Heston process

We use the loss function approach to estimate the parameters in the Heston process. The loss function is defined as the error between the market prices quoted and the prices that are computed from the model; model parameters are determined to minimize the value of this loss function, so that the model price are as close as possible to the ones as observed in the market. The loss function approach is demonstrated in e.g. Gilli and Schumann [2010], chapter 6 of Rouah [2013], which we follow closely hereafter.

To be precise, the price C of the European call option under the Heston process is (cf. Bakshi and Madan [2000])

$$C = e^{-q\tau} S_0 P_1 - e^{-r\tau} X P_2 \tag{13}$$

where S_0, X, r, q and τ are the spot price, the strike price, the risk free rate, the dividend yield and the time to expiration, respectively; P_1, P_2 are given as

$$P_1 = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \Re\left(\frac{e^{-iy\log X}\varphi(y-i)}{i\,y\,\varphi(i)} \right) \mathrm{d}y \tag{14}$$

$$P_2 = \frac{1}{2} + \frac{1}{\pi} \int_0^\infty \Re\left(\frac{e^{-iy\log X}\varphi(y)}{i\,y} \right) \mathrm{d}y \tag{15}$$

In the above integrals $\Re(\cdot)$ denotes the real part function, and the characteristic function $\varphi(y)$ is (cf. Heston [1993])

$$\varphi(y) = \exp\left\{ A_1(y) + A_2(y) + A_3(y) \right\} \tag{16}$$

where

$$A_1(y) = iyS_0 + iy(r-q)\tau \tag{17}$$

$$A_2(y) = \frac{\theta_v \kappa_v}{\sigma_v^2} \left(\left(\kappa_v - i\rho\sigma_v y - d \right) \tau - 2\log\left(\frac{1-ge^{-d\tau}}{1-g} \right) \right) \tag{18}$$

$$A_3(y) = \frac{\frac{v_0}{\sigma_v^2}\left(\kappa_v - i\rho\sigma_v y - d \right)\left(1 - e^{-d\tau} \right)}{1-ge^{-d\tau}} \tag{19}$$

and

$$d = \sqrt{\left(i\rho\sigma_v y - \kappa_v \right)^2 + \sigma_v^2 (iy + y^2)} \tag{20}$$

$$g = \frac{\kappa_v - i\rho\sigma_v y - d}{\kappa_v - i\rho\sigma_v y + d} \tag{21}$$

Now suppose we have a set of known market prices $C_{\mathrm{m}}(\tau_t, X_k)$ with the corresponding maturities τ_t, $t = 1, 2, \ldots, N_T$ and strikes X_k, $k = 1, 2, \ldots N_X$. Theoretical prices according to formula (13) with corresponding maturity-strike combinations and parameters to be determined $\kappa_v, \theta_v, \sigma_v, v_0, \rho$ are denoted by $C(\tau_t, X_k)$. The sought-after paramter estimation of the underlying Heston process is obtained via the minimization problem

$$\underset{\kappa_v,\theta_v,\sigma_v,v_0,\rho}{\operatorname{argmin}} \sum_{t,k} \frac{\left|C_{\mathrm{m}}(\tau_t, X_k) - C(\tau_t, X_k)\right|}{C_{\mathrm{m}}(\tau_t, X_k)} \tag{22}$$

The objective function appeared above is called the loss function. Apparently the loss function is complicated and not convex with respect to its arguments, hence traditional gradient-based methods will have difficulties finding the global minimizer. Here we adopt the differential evolution (Storn and Price [1997], Price et al. [2005]) heuristic to solve this problem. The differential evolution heuristic is a stochastic search strategy which updates the candidates by creating a new member according to some random scheme, compares the resulting objective values and selects the best among the candidates; the process repeats until some stopping criteria is met.

For the estimation of the Heston process, we use the data of TAIEX options [2]. The data consist of 48,938 entries, 248 trading days in 2014.

4.2 Simulation

Based on the discrete approximations of the continuous solution of the underlying stochastic differential equations, simulation methods try to depict the process trajectory and facilitate the computation of the expected value of certain functionals of the process. Iacus [2008] provides a detailed overview of this topic.

For the simulation of the CIR process, the Euler scheme (cf. section 2.1 of Iacus [2008]) is used. However, issues arise when one tries the same approach to the simulation of the Heston process: the slow convergence of the scheme and the occurrence of negative variances. Several dedicated schemes have been developed to mitigate the problem; chapter 7 of Rouah [2013] is an in-depth survey. Here we adopt the Quadratic Exponential sampling scheme of Andersen [2008] for the Heston process.

The simulation time step Δt is $\frac{1}{250}$, and with the long time span $T = 10$ years, $2,500$ terms are computed for each scenario; 5×10^4 scenarios are generated and

[2] `https://www.taifex.com.tw/eng/eng2/TXO.asp`

Table 1: Parameter definition and base values

Parameters	Descriptions	Values
r_0	Initial instantaneous interest rate	0.020
κ_r	Drift term of interest rate	0.032
θ_r	Mean reverting speed of interest rate	1.540
σ_r	Interest rate volatility	0.038
μ	Drift term of stock price	0.020
κ_v	Long-run mean of the stock price variation	0.607
θ_v	Mean reverting speed of the stock price variation	0.070
σ_v	Variation of v	0.293
v_0	Estimated initial value of v	0.065
ρ	Correlation coefficient of stock price and volatility	-0.757
w_B	Weight of bond	0.700
w_S	Weight of stock	0.200
η	TIGF coverage	0.900

stored for subsequent Monte Carlo computation, the total file size is 3.5 Gb after compression.

4.3 Numerical illustrations

In this section we present the results from the numerical analysis of our model. We set the parameters for the base case of our study as $P_0 = 100 = L_0, T = 10$ years, and $r_G = 1.50\%$, in line with the ISL products on offer in Taiwan. Current r_P declared by the life insurers are in the range of $[2.65\%, 2.89\%]$. Thus, for ease of comparison and without loss of generality, the market and benchmark adjustors ζ and ξ are set to 0% and $r_P \in [2.00\%, 6.00\%]$. The effect of benchmark K had been taken into account in r_P. According to the report by the Taiwan Insurance Institute (TII), the average stock and bond holdings of life insurers are approximately 20% and 70% respectively.

The preliminary results indicate that for an ISL policy with a minimum guarantee rate of $r_G = 1.50\%$ and declaring a policy interest rate of $r_P = 3.00\%$, the value of the liability reserve p is 18.4298 (cf. Table 3 and Table 2) and the life insurer is in deficit of 1.1999 (Table 4). In other words, for an ISL policy with a high guaranteed interest rate (upper bound) and declaring a relatively high policy can easily turn the life insurer into an under-funded position; whereas by lowering the policy interest rate r_P by 50 basis points, not only does it decreases the cost of liability from 18.4298 to 16.5756 by more than 11% but also turned the insurer's bonus stabilization reserve p_s from under-funded position into a surplus of 0.6543.

In Figure 1 and Figure 2, both the guaranteed interest rate r_G and the policy interest rate r_P appears to have a positive relation with respect to the value of liability reserves. This is not counter-intuitive as given a certain level of guaranteed interest rate, say $r_G = 1.50\%$, one would expect the option price of the liability reserve to cost more for a policy that declares a higher policy interest rate than one that declares a lower one. The same is true for a given level of policy interest rate and different levels of guaranteed interest rate. Figure 3 and Figure 4 shows r_G and r_P are decreasing functions of the bonus stabilization reserve p_s.

Given the guaranteed interest rate of ISL products in Taiwan are in the range of 0.75% to 1.50%, and their corresponding policy interest rates to be in the range of 2.65% to 2.85%, one can also interpret Table 4 as a guideline to a life insurer's policy rate declaration strategy. The results suggest that for an ISL product without any interest rate guarantee, the life insurer can declare up to 3.50% in policy interest without running the risk of being under-funded. However, as r_G increase, it is prudent not to over declare and over distribute of portfolio returns. Table 5 shows that for a life insurer offering ISL product to stay afloat, one should not over declare r_P for the different levels of r_G.

Furthermore, for $r_G = 1.50\%$ and an average r_P around 3.00%, the fair premium contribution towards the guaranty fund based on the total liability of ISL products is 11.3305. This is significantly more than the current contribution scheme as set out by the regulatory authorities.

5 Concluding remarks

Empirical studies have shown that an asset's log-return distribution is non-Gaussian. The fact that many popular models are still based on the assumption of normality is because of the simplicity that the Gaussian model presents. However, the use of Gaussian models when the distributions are not normal, could lead to the under estimation of extreme losses and hugely mispriced derivative products, see Jondeau et al. [2007]. We incorporated the Cox et al. [1985] interest rate model and Heston [1993]'s non-Gaussian stochastic volatility. We also empirically derived

Table 2: Liability reserve p for various levels of guaranteed interest rate r_G, given fixed level of policy interest rate r_P. Unit: %.

r_P	$r_G = 0.00$	0.50	1.00	1.50	2.00
2.00	10.3908	11.5881	12.9482	14.4917	16.2434
2.50	12.4748	13.6720	15.0321	16.5756	18.3273
3.00	14.3289	15.5262	16.8863	18.4298	20.1815
3.50	15.9609	17.1582	18.5183	20.0618	21.8135
4.00	17.3832	18.5804	19.9405	21.4840	23.2357
4.50	18.6089	19.8062	21.1663	22.7098	24.4615
5.00	19.6558	20.8531	22.2132	23.7567	25.5084
5.50	20.5447	21.7420	23.1021	24.6455	26.3973
6.00	21.2931	22.4904	23.8504	25.3939	27.1456

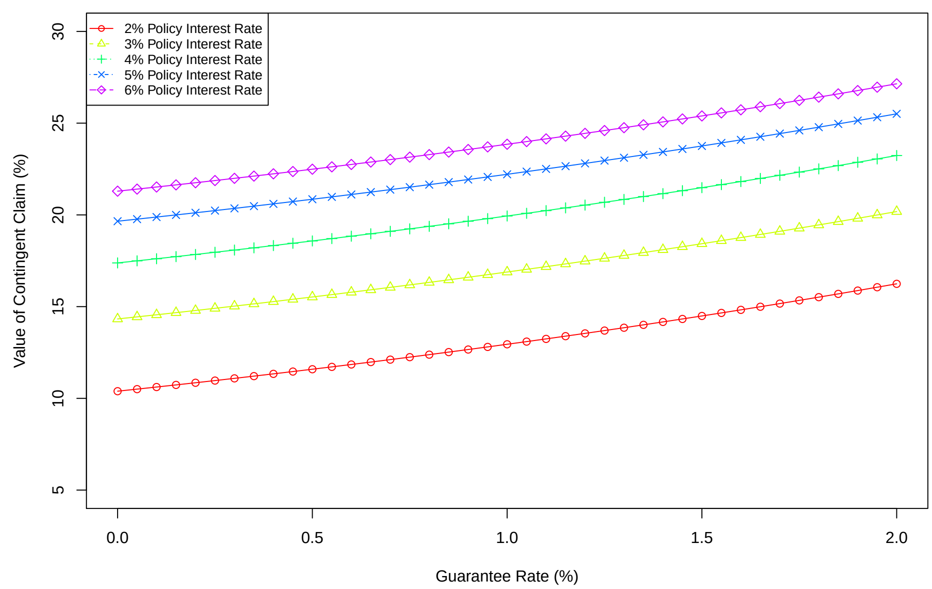

Figure 1: Liability reserve p for various levels of guaranteed interest rate r_G, given fixed level of policy interest rate r_P.

the parameters in our models fitted from Taiwanese data, and subsequently numerically computed the value of the liability reserve, the expected surplus / deficit of

Table 3: Liability reserve p for various levels of policy interest rate r_P, given fixed level of guaranteed interest rate r_G. Unit: %.

r_G	$r_P = 2.00$	3.00	4.00	5.00	6.00
0.00	10.3908	14.3289	17.3832	19.6558	21.2931
0.10	10.6178	14.5559	17.6101	19.8828	21.5200
0.20	10.8509	14.7889	17.8432	20.1158	21.7531
0.30	11.0903	15.0284	18.0826	20.3553	21.9926
0.40	11.3361	15.2742	18.3284	20.6011	22.2384
0.50	11.5881	15.5262	18.5804	20.8531	22.4904
0.60	11.8467	15.7848	18.8390	21.1117	22.7490
0.70	12.1118	16.0499	19.1041	21.3768	23.0140
0.80	12.3836	16.3216	19.3759	21.6485	23.2858
0.90	12.6623	16.6004	19.6546	21.9273	23.5646
1.00	12.9482	16.8863	19.9405	22.2132	23.8504
1.10	13.2413	17.1794	20.2336	22.5063	24.1435
1.20	13.5420	17.4801	20.5343	22.8070	24.4443
1.30	13.8504	17.7884	20.8427	23.1153	24.7526
1.40	14.1668	18.1049	21.1591	23.4318	25.0690
1.50	14.4917	18.4298	21.4840	23.7567	25.3939
1.60	14.8246	18.7627	21.8170	24.0896	25.7269
1.70	15.1661	19.1042	22.1584	24.4311	26.0684
1.80	15.5164	19.4545	22.5087	24.7814	26.4186
1.90	15.8754	19.8135	22.8677	25.1404	26.7777
2.00	16.2434	20.1815	23.2357	25.5084	27.1456

Table 4: Bonus stabilization reserve p_s for various levels of guaranteed interest rate r_G, given fixed level of policy interest rate r_P. Positive reserve values are indicated in blue. Unit: %.

r_P	$r_G = 0.00$	0.50	1.00	1.50	2.00
2.00	6.8390	5.6417	4.2817	2.7382	0.9865
2.50	4.7551	3.5578	2.1977	0.6543	-1.0975
3.00	2.9009	1.7037	0.3436	-1.1999	-2.9516
3.50	1.2689	0.0716	-1.2884	-2.8319	-4.5836
4.00	-0.1533	-1.3506	-2.7106	-4.2541	-6.0059
4.50	-1.3791	-2.5764	-3.9364	-5.4799	-7.2316
5.00	-2.4260	-3.6232	-4.9833	-6.5268	-8.2785
5.50	-3.3149	-4.5121	-5.8722	-7.4157	-9.1674
6.00	-4.0632	-5.2605	-6.6206	-8.1641	-9.9158

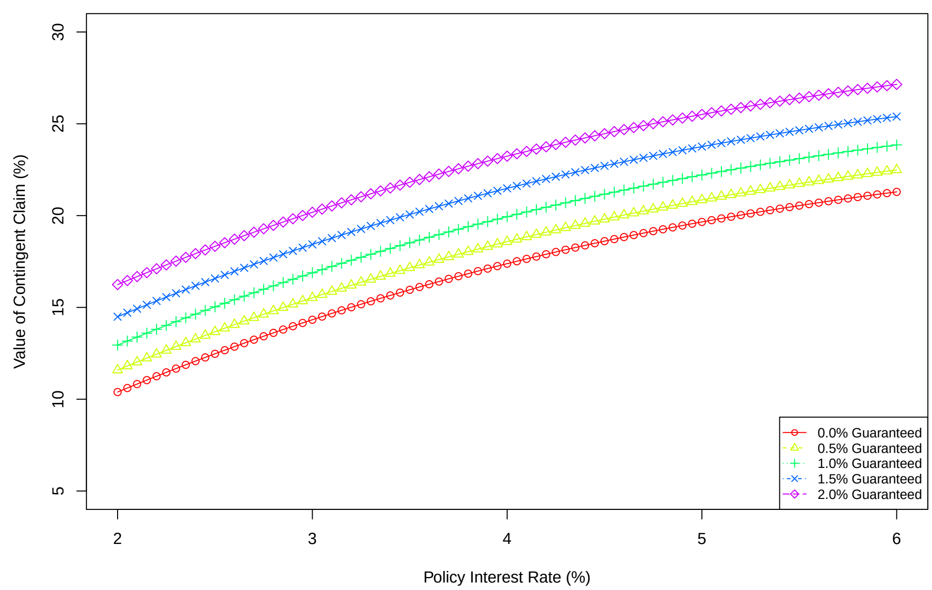

Figure 2: Liability reserve p for various levels of policy interest rate r_P given fixed level of guaranteed interest rate r_G.

the bonus stabilization reserve and other risk measures through the Monte Carlo

Table 5: Bonus stabilization reserve p_s for various levels of policy interest rate r_P given fixed level of guaranteed interest rate r_G. Unit: %.

r_G	$r_P = 2.00$	3.00	4.00	5.00	6.00
0.00	6.8390	2.9009	-0.1533	-2.4260	-4.0632
0.10	6.6121	2.6740	-0.3803	-2.6529	-4.2902
0.20	6.3790	2.4409	-0.6133	-2.8860	-4.5232
0.30	6.1395	2.2014	-0.8528	-3.1255	-4.7627
0.40	5.8937	1.9557	-1.0986	-3.3712	-5.0085
0.50	5.6417	1.7037	-1.3506	-3.6232	-5.2605
0.60	5.3831	1.4450	-1.6092	-3.8819	-5.5191
0.70	5.1181	1.1800	-1.8743	-4.1469	-5.7842
0.80	4.8463	0.9082	-2.1460	-4.4187	-6.0560
0.90	4.5675	0.6294	-2.4248	-4.6975	-6.3347
1.00	4.2817	0.3436	-2.7106	-4.9833	-6.6206
1.10	3.9886	0.0505	-3.0038	-5.2764	-6.9137
1.20	3.6878	-0.2503	-3.3045	-5.5772	-7.2144
1.30	3.3795	-0.5586	-3.6128	-5.8855	-7.5227
1.40	3.0631	-0.8750	-3.9292	-6.2019	-7.8392
1.50	2.7382	-1.1999	-4.2541	-6.5268	-8.1641
1.60	2.4052	-1.5329	-4.5871	-6.8598	-8.4970
1.70	2.0637	-1.8744	-4.9286	-7.2013	-8.8385
1.80	1.7135	-2.2246	-5.2788	-7.5515	-9.1888
1.90	1.3544	-2.5837	-5.6379	-7.9106	-9.5478
2.00	0.9865	-2.9516	-6.0059	-8.2785	-9.9158

Table 6: Fair premium of guaranty fund p_f for different levels of guaranteed interest rate r_G, given fixed level of policy interest rate $r_P = 3.00\%$. Unit: %.

r_P	$r_G = 0.00$	0.50	1.00	1.50	2.00
3.00	7.7138	8.7843	9.9853	11.3305	12.8394

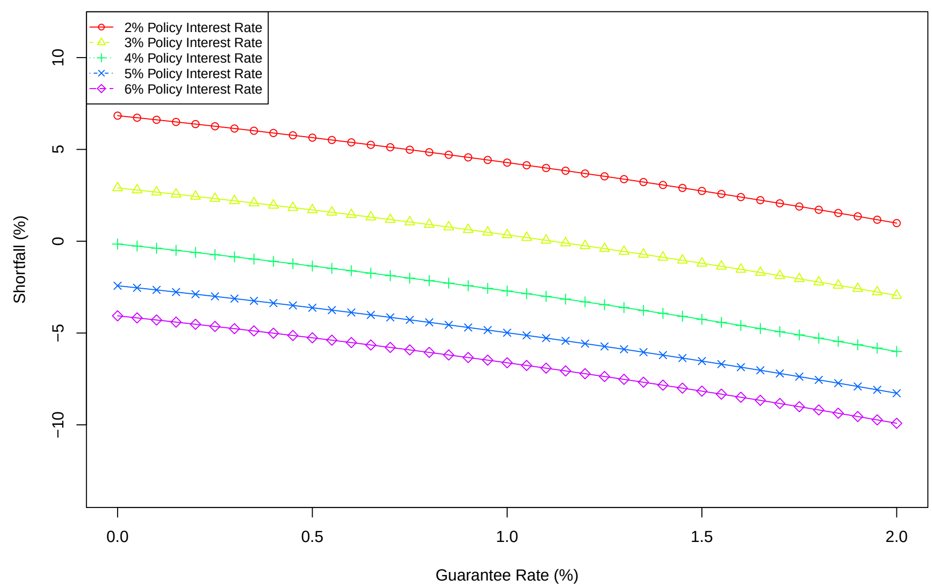

Figure 3: Bonus stabilization reserve p_s for various levels of guaranteed interest rate r_G, given fixed level of policy interest rate r_P.

methodology.

Our numerical results show that they are consistent with that of financial option pricing, in the sense that by offering an ISL policy with both higher guaranteed interest rate and policy interest rates, its liability reserves would also cost relatively more than one that does not, which is to say the cost of guarantee is higher for those offering higher rates. It also suggest that the current offerings by the life insurers are within bounds. Our model can be easily applied to other markets with similar products and our results be of interest to life insurers and the regulatory authorities.

References

M. O. Albizzati and H. Geman. Interest rate risk management and valuation of the surrender option in life insurance policies. *Journal of Risk and Insurance*, 61: 616–637, 1994.

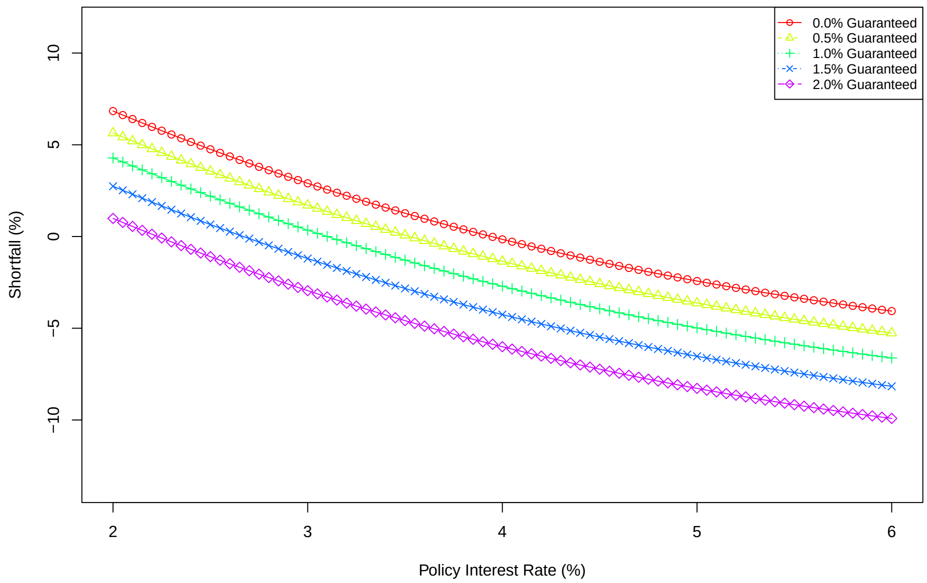

Figure 4: Bonus stabilization reserve p_s for various levels of policy interest rate r_P, given fixed level of guaranteed interest rate r_G.

L. Andersen. Simple and efficient simulation of the Heston stochastic volatility model. *Journal of Computational Finance*, 11:1–42, 2008.

A. R. Bacinello. Fair pricing of life insurance participating policies with a minimum interest rate guaranteed. *ASTIN Bulletin*, 31:275–297, 2001.

A. R. Bacinello. Pricing guaranteed life insurance participating policies with annual premiums and surrender option. *North American Actuarial Journal*, 7: 1–17, 2003a.

A. R. Bacinello. Fair valuation of a guaranteed life insurance participating contract embedding a surrender option. *Journal of Risk and Insurance*, 70:461–487, 2003b.

A. R. Bacinello and F. Ortu. Fixed income linked life insurance policies with minimum guarantees: Pricing models and numerical results. *European Journal of Operatoinal Research*, 91:235–249, 1996.

A. R. Bacinello, E. Biffis, and P. Millossovich. Pricing life insurance contracts with early exercise features. *Journal of Computational and Applied Mathematics*, 233:27–35, 2009.

G. Bakshi and P. Madan. Spanning and derivative-security valuation. *Journal of Financial Economics*, 55:205–238, 2000.

J. Barbarin and P. Devolder. Risk measure and fair valuation of an investment guarantee in life insurance. *Insurance: Mathematics and Economics*, 37:297–323, 2005.

D. Bauer, R. Kiesel, A. Kling, and J. Ruß. Risk-neutral valuation of participating life insurance contracts. *Insurance: Mathematics and Economics*, 39:171–183, 2006.

C. Bernard, O. Le Courtois, and F. Quittard-Pinon. Market value of life insurance contracts under stochastic interest rates and default risk. *Insurance: Mathematics and Economics*, 36:499–516, 2005.

C. Bernard, O. Le Courtois, and F. Quittard-Pinon. Development and pricing of a new participating contract. *North American Actuarial Journal*, 10:179–195, 2006.

F. Black and M. Scholes. The pricing of options and corporate liabilities. *Journal of Political Economy*, 81:637–654, 1973.

M. J. Brennan and E. S. Schwartz. The pricing of equity-linked life insurance policies with an asset value guarantee. *Journal of Financial Economics*, 3:195–213, 1976.

D. Brigo and F. Mercurio. *Interest Rate Models – Theory and Practice: With Smile, Inflation and Credit*. Springer-Verlag, Berlin, second edition, 2006.

E. Briys and F. de Varenne. Life insurance in a contingent claim framework: pricing and regulatory implications. *The Geneva Papers on Risk and Insurance Theory*, 19:53–72, 1994.

E. Briys and F. de Varenne. On the risk of insurance liabilities: Debunking some common pitfalls. *Journal of Risk and Insurance*, 64:673–694, 1997.

J. Cox, J. Ingersoll, and S. Ross. A theory of the term structure of interest rates. *Econometrica*, 53:385–407, 1985.

W. Feller. Two singular diffusion problems. *Annals of Mathematics*, 54:173–182, 1951.

N. Gatzert and A. Kling. Analysis of participating life insurance contracts: a unification approach. *Journal of Risk and Insurance*, 74:547–570, 2007.

N. Gatzert and H. Schmeiser. New life insurance financial products. In G. Dionne, editor, *Handbook of Insurance*, pages 1061–1095. Springer-Verlag, second edition, 2013.

M. Gilli and E. Schumann. Calibrating option pricing models with heuristics. In A. Brabazon, M. O'Neill, and D. Maringer, editors, *Natural Computing in Computational Finance: Volume 4*, pages 9–37. Springer-Verlag, New York, 2010. `http://papers.ssrn.com/sol3/papers.cfm?abstract_id=1566975`.

S. Graf, A. Kling, and J. Ruß. Risk analysis and valuation of life insurance contracts: Combining actuarial and financial approaches. *Insurance: Mathematics and Economics*, 49:115–125, 2011.

S. Graf, A. Kling, and J. Ruß. Financial planning and risk-return profiles. *European Actuarial Journal*, 2:77–104, 2012.

A. Grosen and P. L. Jørgensen. Valuation of early exercisable interest rate guarantees. *Journal of Risk and Insurance*, 64:481–503, 1997.

A. Grosen and P. L. Jørgensen. Fair valuation of life insurance liabilities: The impact of interest rate guarantees, surrender options, and bonus policies. *Insurance: Mathematics and Economics*, 26:37–57, 2000.

A. Grosen and P. L. Jørgensen. Life insurance liabilities at market value: An analysis of insolvency risk, bonus policy, and regulatory intervention rules in a barrier option framework. *Journal of Risk and Insurance*, 69:63–91, 2002.

S. Heston. A closed-form solutions for options with stochastic volatility. *Review of Financial Studies*, 6:327–343, 1993.

J. Hull and A. White. How to value employee stock options. *Financial Analysts Journal*, pages 114–119, 2004.

S. Iacus. *Simulation and Inference for Stochastic Differential Equations With R Examples*. Springer-Verlag, New York, 2008.

B. Jensen, P. L. Jørgensen, and A. Grosen. A finite difference approach to the valuation of path dependent life insurance liabilities. *The Geneva Papers on Risk and Insurance Theory*, 26:57–84, 2001.

E. Jondeau, S. H. Poon, and M. Rockinger. *Financial Modeling Under Non-Gaussian Distributions*. Springer-Verlag, 2007.

K. Kladıvko. Maximum likelihood estimation of the Cox-Ingersoll-Ross process: the MATLAB implementation. In *Technical Computing Prague*. working paper, 2007.

A. Kling, A. Richter, and J. Ruß. The interaction of guarantees, surplus distribution, and asset allocation in with-profit life insurance policies. *Insurance: Mathematics and Economics*, 40:164–178, 2007.

O. Le Courtois and H. Nakagawa. On surrender and default risks. *Mathematical Finance*, 23:143–168, 2013.

K. R. Miltersen and S. A. Persson. Guaranteed investment contracts: distributed and undistributed excess return. *Scandinavian Actuarial Journal*, 4:257–279, 2003.

K. Price, R. M. Storn, and J. A. Lampinen. *Differential Evolution: A Practical Approach to Global Optimization*. Springer-Verlag, New York, 2005.

F. Rouah. *The Heston Model and its Extensions in Matlab and C#*. John Wiley & Sons, Hoboken, N.J., 2013.

R. Storn and K. Price. Differential evolution – a simple and efficient heuristic for global optimization over continuous spaces. *Journal of Global Optimization*, 11:341–359, 1997.

A. J. Tanskanen and J. Lukkarinen. Fair valuation of path-dependent participating life insurance contracts. *Insurance: Mathematics and Economics*, 33:595–609, 2003.

P. Wilmott. Cliquet options and volatility models. *Wilmott Magazine*, December: 78–83, 2002.