A Markov-Switching Autoregressive Model for the Underwriting "Cycle"

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Abstract

The underwriting "cycle" is an important topic in insurance research. We apply a Markov-switching autoregressive model to country-level data to test the market asymmetry proposition predicted by the dynamic framework proposed by Henriet, Klimenko and Rochet (2016). The method of quasi-maximum likelihood is used to estimate parameters. The model outperforms a simple autoregressive model with both lower AIC and lower BIC. Empirical results show that asymmetry of markets does not exist in that there is no significant difference between the durations of the soft-market phase and the hard-market phase, and therefore does not support the asymmetry hypothesis that the soft market lasts longer than the hard market.

Keywords: Underwriting "Cycle"; Markov-Switching Auto-regressive Model; Asymmetry Hypothesis

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1 Introduction

Underwriting "cycle" is typically thought of as repeating, regular periods of the soft market and the hard market in the insurance field (Weiss, 2007). In the soft market, insurance products are abundant in supply, and therefore the prices of insurance products stay low. While in the hard market, insurance products are in short supply, and prices keeps relatively high. The alternate occurrence of the the soft market and the hard market gives rise to price fluctuations in the insurance market, rendering it hard for companies to make sensible business plans and for regulators to make timely judgment on the solvency of these companies. The underwriting "cycle" is an important topic in the scholarly research since its existence contradicts the efficient market hypothesis. Though extensively studied for more than 30 years, there has not been a consensus on the exact reason why the "cycle" occurs yet. Cummins and Outreville (1987) proposed the "arbitrage theory" attributing the ups and downs of insurance profitability to institutional lags and regulatory requirements such as data collection lags, regulatory lags, policy renewal lags and accounting rules. Therefore, countries with similar institutional characteristics should have similar underwriting patterns. Winter (1994) studied this problem from a supply side perspective, and argued that it is exogenous shocks to the markets that affected each companies' capacity level, which further influences profitability. Assuming that all insurers should hold enough equity to avoid insolvency. After raising enough capital, the insurers are able to issue more products, which drives down the prices. Therefore, different market regimes correlate with different status of companies' capital levels, and adjacent markets that are hit by the same shocks should behave similarly.

Using the combined ratio, which is defined as the percentage of the sum of incurred losses and earned expenses over earned premiums, and total premium as indicators of the insurance company's profitability, current research have been testing the existence and length of insurance cycles empirically. Figure 1 shows the combined ratio of US's insurance industry from 1967 to 2014.

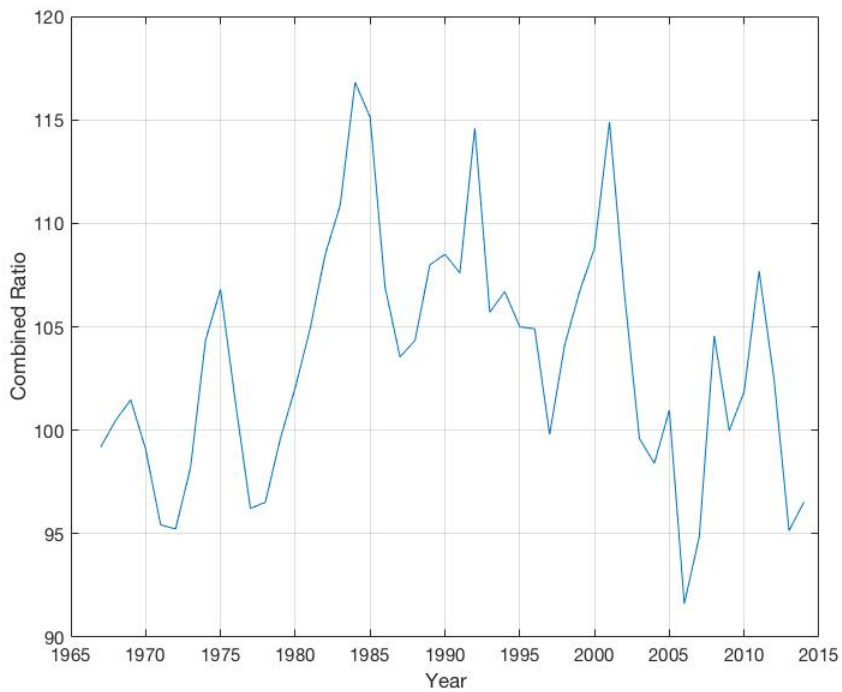

Figure 1: Combined Ratio of Insurance Industry in USA.

It can be seen from the figure that US's insurance industry has gone through roughly 6 ups and downs in this time period. The result is also consistent with existing research, which documents an estimation of cycle length of 6 to 7 years in US's insurance industry. (See Venezian, 1985; Cummins and Outreville, 1987 et al.) The lengths of insurance cycles varies from 4.7 years as in Australia to 8.7 years as in France. (See Cummins and Outreville, 1987).

Recently, Henriet, Klimenko and Rochet (2016, HKR hereafter) combined the arbitrage theory and the capacity constraint theories into a comprehensive general-equilibrium model, which rationalizes the dynamics of insurance prices in a competitive insurance market with financial frictions. The numerical results of HKR shows that insurance prices exhibit asymmetric reversals caused by the reflection of the aggregate capacity process at the dividend and recapitalization boundaries rather than true cycles.

This paper aims at testing this asymmetry of the soft- and the hard-market hypothesis proposed by HKR empirically with country-level data, which includes US insurance combined ratios and premiums data in the time period of 1967-2014. The dataset is from A.M Best's Global Insurance and Banking Database.

Instead of using a simple autoregressive model to characterize the underwriting cycles as existing research do (see Weiss (2007) for a review), we apply a Markov-switch autoregressive model to underwriting-"cycle" analysis. The results show that the model outperforms a simple autoregressive model with a lower AIC and a lower BIC. In addition, the model passes robustness checks in terms of the modeling and the choice of data, further strengthening the model's validity. Empirical results show that asymmetry of markets does not exist in that there is no significant difference between the durations of the soft- and hard-market phase, and therefore does not support the asymmetry hypothesis of HKR.

This paper contributes to the underwriting-"cycle" research in two aspects. Firstly, it is among the first studies that applies a Markov-switching autoregressive model to underwriting-"cycle" research, and its analysis is more comprehensive and rigorous than other existing studies. Secondly, the paper empirically tested HKR's market asymmetry hypothesis, and found that there is no significant difference between durations of the soft market and the hard market, thus does not support HKR's hypothesis.

2 Markov-Switching Auto-regressive Models

2.1 Auto-regressive Models

Current research mainly use auto-regressive models for the estimation of insurance cycles. Using AR(2) model, Cummins and Outreville (1987) and Chen et al. (1997) investigated into the insurance market in US and Asia, and proved the existence of cycles in the two markets, respectively. Meier(2006) and Meier and Outreville (2006) extended Cummins and Outreville (1987), and found that insurance cycles exist in Germany, Switzerland, France, but not in Japan.

On the other hand, Boyer et al. (2012) argued that the parameter estimates from AR models do not lead to any such inference since auto-regressive model puts strong prior conditions. Applying a number of filters, Boyer et al. (2012) shows that neither does the cycles exist in the sample of data, nor could the cycles be forecasted out of sample. However, Boyer et al. (2012) didn't consider the alternation of soft markets and hard markets, and thus fail to detect the existence of cycles in the insurance market. Therefore, applying the Markov-switching auto-regressive model stated in Section 2.2 in the analysis is necessary.

2.2 Markov-Switching Auto-Regressive Models

The application of Markov-switching models (also known as regime switching model) in economics starts in Goldfeld and Quandt (1973), and Hamilton (1989) provides a through analysis of the estimation of the parameters. The Markov-switching model has been widely applied in the area of macroeconomic research for the estimation of GDP, exchange rate, real interest etc. (See Engle and Hamilton, 1990; Garcia and Perron, 1996 et al.) The model is also applied in finance for modeling stock and bond returns, European option prices, etc. (See Guidolin and Timmermann, 2006; Buffington and

Elliott, 2002, etc.). While in the area of insurance studies, the model is mainly used for insurance product pricing such as Aase (2001) on catastrophe insurance futures and spreads.

As for insurance cycle modeling, Wang et al. (2011) used a UP / DOWN regime switching model to analyze underwriting cycles. However, the authors simply assigned the data points to each regime by an *ad hoc* criteria. The data point is regarded as belonging to the UP regime if the forward difference $Y_t - Y_{t-1} \geq 0$, and to the DOWN regime otherwise. Two models are fit to data points in the two regimes, respectively. However, this *ad hoc* criterion has its shortcomings as it might take the "noise" as a transition of regimes. In other words, a tiny variation of the data has the same weight as a shock as long as the forward difference has the same sign.

In this paper, we apply a bivariate Markov-switching model with a general $AR(k)$ dynamic structure in the analysis. The model is defined as below,

$$z_t = \alpha_0 + \alpha_1 s_t + \beta_1 z_{t-1} + \ldots + \beta_k z_{t-k} + \varepsilon_t, \tag{1}$$

where $s_t = 0, 1$ are the Markovian state variables, representing the soft market and hard market, respectively. ε_t are i.i.d. random variables with mean 0 and variance σ_ε^2. Denote the transition matrix of s_t as P_t as below,

$$P = \begin{bmatrix} P(s_t = 0|s_{t-1} = 0) & P(s_t = 1|s_{t-1} = 0) \\ P(s_t = 1|s_{t-1} = 0) & P(s_t = 1|s_{t-1} = 1) \end{bmatrix} = \begin{bmatrix} p_{00} & p_{01} \\ p_{10} & p_{11} \end{bmatrix} \tag{2}$$

Since $p_{00} + p_{01} = 1$ and $p_{10} + p_{11} = 0$, the transition matrix can be simplified as below,

$$P = \begin{bmatrix} p_{00} & 1 - p_{00} \\ 1 - p_{11} & p_{11} \end{bmatrix} \tag{3}$$

The transition matrix $P(t)$ can be time-dependent. Here we only consider constant transition matrix for simplicity.

2.3 Model Estimation

In this paper, we apply a quasi-maximum likelihood estimation (QMLE) method for parameter estimation following Hamilton (1989) and Kim and Nelson (1999). As for Eq. (1), the vector of parameters can be denoted as,

$$\theta = (p_{00}, p_{11}, \alpha_0, \alpha_1, \sigma_\varepsilon, \beta_1, \beta_2 \dots \beta_k) \tag{4}$$

Let $\mathcal{Z}^t = \{z_t, z_{t-1}, \dots z_1\}$ be the information set at time t which contains all the observed variables up to time t. Under the normality assumption, the density of z_t conditional on $\mathcal{Z}^{t-1}$ and $s_t = i\,(i = 0, 1)$ is

$$f(z_t|s_t = i, \mathcal{Z}^{t-1}; \theta) = \frac{1}{\sqrt{2\pi\sigma_\varepsilon^2}} exp\left\{ -\frac{(z_t - \alpha_0 - \alpha_1 i - \beta_1 z_{t-1} - \dots \beta_k z_{t-k})^2}{2\sigma_\varepsilon^2} \right\} \tag{5}$$

Given the *prediction probability* $P(s_t = i|\mathcal{Z}^{t-1}; \theta)$, the probability density function of z_t conditional on $\mathcal{Z}^{t-1}$ can be obtained as

$$\begin{aligned} f(z_t|\mathcal{Z}^{t-1}; \theta) &= P(s_t = 0|\mathcal{Z}^{t-1}; \theta) f(z_t|s_t = 0, \mathcal{Z}^{t-1}; \theta) \\ &+ P(s_t = 0|\mathcal{Z}^{t-1}; \theta) f(z_t|s_t = 0, \mathcal{Z}^{t-1}; \theta) \end{aligned} \tag{6}$$

For each state $s_t, t = 0, 1$, the *filtering probabilities* can be derived by the Bayes theorem as below,

$$P(s_t = i|\mathcal{Z}^t; \theta) = \frac{P(s_t = 0|\mathcal{Z}^{t-1}; \theta) f(z_t|s_t = 0, \mathcal{Z}^{t-1}; \theta)}{f(z_t|\mathcal{Z}^{t-1}; \theta)} \tag{7}$$

In addition, the relationship between the filtering probabilities and prediction probabilities is,

$$P(s_{t+1} = i|\mathcal{Z}^t; \theta) = p_{0i} P(s_t = 0|\mathcal{Z}^t; \theta) + p_{1i} P(s_t = 1|\mathcal{Z}^t; \theta) \quad i = 0, 1 \tag{8}$$

With initial prediction probabilities $P(s_k = i|\mathcal{Z}^{k-1}; \theta)$, the filtering probabilities $P(s_t = i|\mathcal{Z}^t; \theta), t = k, \dots T.$ and filtering densities

$f(z_t|\mathcal{Z}^t;\theta), t = k \ldots T$. can be derived recursively using Eqs. (5) - (8). Thus, the quasi-log-likelihood function can be derived as,

$$\mathcal{L}(\theta) = \frac{1}{T}\sum_{t=1}^{T} \log f(z_t|\mathcal{Z}^{t-1};\theta) \tag{9}$$

It can be easily seen that the log-likelihood function is a complex function of θ, which makes it difficult to find the analytical solution of the parameters. We used an EM algorithm for the derivation of parameters.

3 Data and Empirical Results

The dataset consists of industry-level information of US's insurance industry from 1967 to 2014 from A.M Best's Global Insurance and Banking Database. The combined ratio is defined as below,

$$Combined\,Ratio = \frac{Incurred\,Losses + Earned\,Expenses}{Earned\,Premiums} \tag{10}$$

The empirical analysis can be processed in several steps as below.

Step 1: Stationary Test The combined ratio itself is not stationary as it fails to reject the null hypothesis Augmented Dickey-Fuller (ADF) test. Let z_t be the differential of combined ratios shown in Figure 2, and the results show that it is stationary.

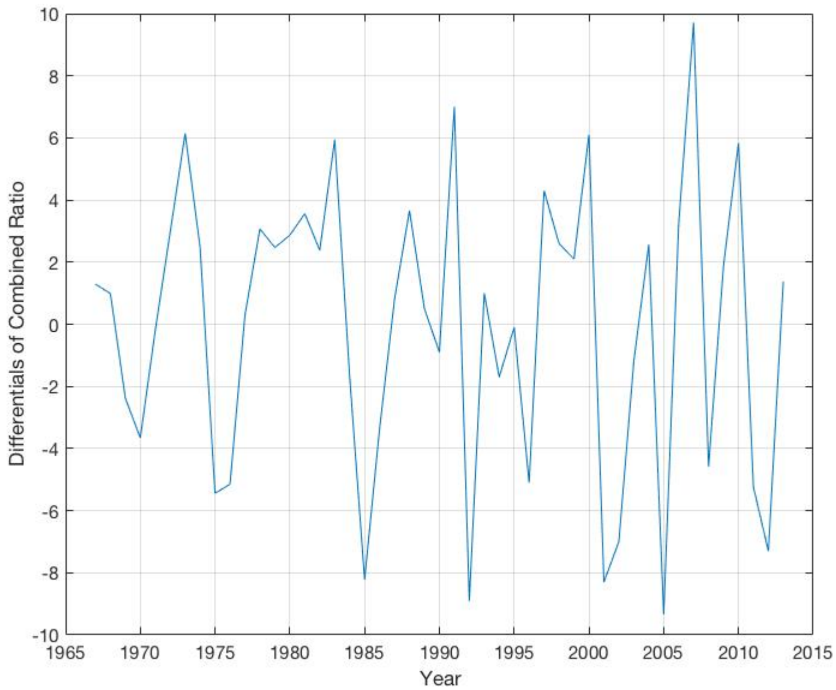

Figure 2: Differentials of Combined Ratio of Insurance Industry in USA.

Denote z_t as the differentials of combined ratios, and the analysis hereafter is based on z_t.

Step 2: Determine the structure of the model To determine the structure of the model, i.e. find the optimal number of lags of the Markov-switching AR(k) model, AIC and BIC could be used. The model with lowest AIC and BIC should be the model to use. AIC and BIC are defined as below, respectively.

$$AIC(k) = 2k - 2\log(\hat{L}) \tag{11}$$

$$BIC(k) = \log(n)k - 2\log(\hat{L}) \tag{12}$$

where k is the number of parameters and n is the number of observations. $\hat{L}$ is the maximized value of the likelihood function of the model.

By restricting the maximum lags to be 5, the AIC and BIC for the models are listed as below,

Table 1: AIC and BIC values of Different Models

No. of Lags	1	2	3	4	5
AIC	242.55	240.70	251.83	252.43	245.94
BIC	241.30	240.32	252.47	254.21	248.96

It can be easily seen from Table 1 that the model with lag 2 have both the lowest AIC value and lowest BIC value among the 5 models. Therefore, we use the following model for further analysis,

$$z_t = \alpha_0 + \alpha_1 s_t + \beta_1 z_{t-1} + \beta_2 z_{t-2} + \varepsilon_t \tag{13}$$

In addition, the AIC and BIC of model above

Step 3: Estimate the parameters Using the method in Section 2.3, the parameters can be estimated using QMLE. The estimated values are listed in Table 2 as below,

Table 2: Estimated Parameters

Parameter	p_{00}	p_{11}	α_0	α_1	σ_e	β_1	β_2
Coefficient	0.68	0.72	-4.67	8.30	2.47	-0.24	-0.44
Std.Dev	0.16	0.12	0.73	0.92	0.30	0.13	0.11
P-Value	0.00	0.00	0.00	0.00	0.00	0.03	0.00

From Table 2, all the p-values are below 5%, thus all the parameters are significant at the 5% level. Therefore, the differentials of the combined ratios in US's insurance industry follows the following process,

$$z_t = -4.67 + 8.30 s_t - 0.24 z_{t-1} - 0.44 z_{t-2} + \varepsilon_t, \quad s_t = 0, 1 \tag{14}$$

where $\varepsilon_t \sim N(0, 2.47^2) = N(0, 6.10)$, and the transition matrix is

$$P = \begin{bmatrix} 0.68 & 0.32 \\ 0.28 & 0.72 \end{bmatrix} \tag{15}$$

The expected duration of the state 0 (soft market) can be calculated as below,

$$L_0 = E(Duration_0) = \sum_{k=1}^{\infty} k p_{00}^{k-1}(1 - p_{00}) = \frac{1}{1 - p_{00}} = 3.13\,Years$$

Similarly, the expected duration of the state 1 (hard market) can be calculated as below,

$$L_1 = E(Duration_1) = \sum_{k=1}^{\infty} k p_{11}^{k-1}(1 - p_{11}) = \frac{1}{1 - p_{11}} = 3.57\,Years$$

The difference between the expected durations is,

$$D = L_0 - L_1 = -0.44\,Years$$

In the next section, we will test HKR's asymmetry hypothesis by examining whether the durations of soft market and hard market are significantly different, i.e. whether D is significant different from 0.

4 Testing HKR's Hypothesis

Hhenriet et al. (2016) developed a general-equilibrium model for continuous insurance sector. The numerical results of HKR shows that the market exhibits alternating periods where premium and profitability rise (hard markets) and fall (soft markets). The average duration of hard markets is shorter than that of soft markets, provided that the elasticity of the demand for insurance is not too low.

In this section, we first derive the theoretical test statistics for testing the hypothesis $H_0:\ D = 0$.

From Section 3, the difference between the expected durations of two states is,

$$D = \frac{1}{1-p_{00}} - \frac{1}{1-p_{11}} \tag{16}$$

For an estimated value $\hat{D}$,

$$\hat{D} = g(\hat{p}_{00}, \hat{p}_{11}) = \frac{1}{1-\hat{p}_{00}} - \frac{1}{1-\hat{p}_{11}} \tag{17}$$

Applying the Lagrange mean value theorem, for some $\delta \in (0,1)$,

$$\begin{aligned}
\hat{D} - D &= g(\hat{p}_{00}, \hat{p}_{11}) - g(p_{00}, p_{11}) \\
&= g_1\left(\hat{p}_{00} + \delta(p_{00} - \hat{p}_{00}), \hat{p}_{11} + \delta(p_{11} - \hat{p}_{11})(\hat{p}_{00} - p_{00})\right) \\
&+ g_2\left(\hat{p}_{00} + \delta(p_{00} - \hat{p}_{00}), \hat{p}_{11} + \delta(p_{11} - \hat{p}_{11})(\hat{p}_{00} - p_{00})\right) \\
&= g_1\left(\overline{p}_{00}, \overline{p}_{11}\right)\left(p_{00} - \hat{p}_{00}\right) + g_2\left(\overline{p}_{00}, \overline{p}_{11}\right)\left(p_{00} - \hat{p}_{00}\right) \qquad (18)
\end{aligned}$$

where $g_1 = \frac{\partial}{\partial p_{00}} g(\cdot), g_2 = \frac{\partial}{\partial p_{11}} g(\cdot)$ are the partial derivatives with regard to p_{00} and p_{11}, respectively. In the last line, $\overline{p}_{00} \in (\hat{p}_{00}, p_{00}), \overline{p}_{11} \in (\hat{p}_{11}, p_{11})$.

Since QMLE estimators are consistent, we have,

$$plim\, \hat{p}_{00} = p_{00}, \quad plim\, \hat{p}_{11} = p_{11} \tag{19}$$

Therefore,

$$plim\, \overline{p}_{00} = p_{00}, \quad plim\, \overline{p}_{11} = p_{11} \tag{20}$$

Applying the continuous mapping theorem,

$$plim\, g_1\left(\overline{p}_{00}, \overline{p}_{11}\right) = g_1(p_{00}, p_{11}), \quad plim\, g_2\left(\overline{p}_{00}, \overline{p}_{11}\right) = g_2(p_{00}, p_{11}) \tag{21}$$

The asymptotic variance-covariance matrix for $\hat{p}_{00}, \hat{p}_{11}$ is,

$$\sqrt{n} \begin{pmatrix} \hat{p}_{00} - p_{00} \\ \hat{p}_{11} - p_{11} \end{pmatrix} \sim N \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix} \tag{22}$$

where ρ is the correlation coefficient of p_{00} and p_{11}.

Therefore, the asymptotic variance of $\hat{D}$ can be calculated as,

$$\begin{aligned} \sqrt{n}(\hat{D}-D) &= \sqrt{n}g_1\left(\overline{p}_{00},\overline{p}_{11}\right)\left(p_{00}-\hat{p}_{00}\right)+\sqrt{n}g_2\left(\overline{p}_{00},\overline{p}_{11}\right)\left(p_{00}-\hat{p}_{00}\right) \\ &\sim N\Big(0,\left[g_1\left(p_{00},p_{11}\right),g_2\left(p_{00},p_{11}\right)\right]^T\cdot\Sigma\cdot\left[g_1\left(p_{00},p_{11}\right),g_2\left(p_{00},p_{11}\right)\right]\Big) \\ &= N\Big(0,\nabla g\left(p_{00},p_{11}\right)^T\cdot\Sigma\cdot\nabla g\left(p_{00},p_{11}\right)\Big) \end{aligned} \tag{23}$$

where

$$\nabla g\left(p_{00},p_{11}\right)=\left[\frac{1}{(1-p_{00})^2},-\frac{1}{(1-p_{11})^2}\right] \tag{24}$$

Thus, the asymptotic variance of $\hat{D}$ is,

$$Var(\hat{D})=\nabla g\left(p_{00},p_{11}\right)^T\cdot\Sigma\cdot\nabla g\left(p_{00},p_{11}\right)=\frac{\sigma_1^2}{(1-p_{00})^4}-\frac{2\rho\sigma_1\sigma_2}{(1-p_{00})^2(1-p_{11})^2}+\frac{\sigma_2^2}{(1-p_{11})^4} \tag{25}$$

From Section 3, the estimated values are,

$$\sigma_1^2=0.0244,\sigma_2^2=0.0140,\rho=0.3240$$

and

$$p_{00}=0.68,p_{11}=0.72$$

The asymptotic variance of $\hat{D}$ can be calculated as,

$$Var(\hat{D})=2.9704$$

Therefore, the p-value is 0.34, thus the null hypothesis can not be rejected. In other words, the results does not support HKR's asymmetry hypothesis that the soft market lasts longer than the hard market.

5 Conclusions

The underwriting "cycle" is an important topic in insurance research. Assuming that the insurance market has both soft market phase and hard market phase, we apply a Markov-switching auto-regressive model to country-level data to test the market asymmetry proposition predicted by the dynamic framework proposed by Henriet, Klimenko and Rochet (2016) in this paper. The method of quasi-maximum likelihood is used to estimate parameters. Empirical results show that asymmetry of markets does not exist in that there is no significant difference between the durations of the soft-market phase and the hard-market phase, and therefore does not support the asymmetry hypothesis that the soft market lasts longer than the hard market.

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